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Tamal Roy
Ranjit Kumar Barai

Robust Control-Oriented Linear Fractional Transform Modelling

Applications for the μ -Synthesis Based
 H_∞ Control

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Preface

The interest in model-based control has been growing due to the developments of new techniques for the design of advanced control law and availability of tools for mathematical modelling of complex systems. However, the design approach of some particular control laws demands a specific form of mathematical modelling. The popular μ -synthesis-based H_∞ controller design necessitates a particular form of model known as the linear fractional transformation (LFT) model. Although lot of literature was available at various sources, a comprehensive text that elaborates the general theoretical techniques to derive the LFT model of complex systems in a systematic manner was absent. The authors felt the need for a comprehensive text during their research on application of robust control in complex systems like robots and mechatronic systems.

This book covers a new paradigm of system modelling—the robust control-oriented linear fractional transformation (LFT) modelling. A dynamic system expressed in LFT modelling framework paves the way for the application of modern robust controller design techniques like μ -synthesis method for H_∞ controller design. This book covers the generalized robust control-oriented LFT modelling representation of the MIMO system depending upon the uncertainty structure, system dynamics, and the dimensions of the input–output. The modelling framework results in a compact and manageable representation of uncertainty modelling in the form of feedback-like structure that is suitable for design and implementation of the robust control technique like μ -synthesis-based H_∞ control theory. This book also describes the application of the proposed methodology in a variety of advanced mechatronic systems like the laboratory-scale helicopters and wheeled mobile robots with and without slip.

We hope that the contents of the book would be useful for teaching and research. Any suggestions for further improvements to the book are most welcome. We are grateful to all who encouraged and helped us directly or indirectly during the preparation of the manuscript.

Kolkata, India
August 2022

Tamal Roy
Ranjit Kumar Barai

Contents

1	Introduction	1
1.1	Motivation	1
1.2	Organization of This Book	2
	References	3
2	Mathematical Modelling of Real Physical Systems	5
2.1	Introduction	5
2.2	Mathematical Modelling of Real Physical Systems	6
2.3	Model-Based Control System	6
2.4	Uncertainty Modelling	7
2.5	Linear Fractional Transformation Modelling	8
2.6	Important Observations	9
2.7	Chapter Summary	10
	References	10
3	Control-Oriented Linear Fractional Transformation	13
3.1	Introduction	13
3.2	Control-Oriented Modelling	14
3.3	Uncertainty Modelling	16
3.3.1	Unstructured Uncertainties	17
3.3.2	Parametric Uncertainty	20
3.3.3	Structured Uncertainty	21
3.4	Linear Fractional Transformation	21
3.4.1	Basic Principles	21
3.4.2	State-Space Realization of LFT Modelling	25
3.4.3	Interconnection of LFT	26
3.5	Reasons for Adopting LFT Model for Control-Oriented Modelling	28
3.6	Chapter Summary	29
	References	29

4	μ-Synthesis-Based H_∞ Control Theory	31
4.1	Introduction	31
4.2	Small-Gain Theorem	32
4.3	H_∞ Optimization	34
4.4	H_∞ Sub-optimal Controller Design	35
4.5	H_∞ Control Problem	36
4.6	μ -Synthesis	36
4.7	Chapter Summary	38
	References	38
5	Generalized Control-Oriented LFT Modelling of a Coupled Uncertain MIMO System	41
5.1	Introduction	41
5.2	Generalized Control-Oriented LFT Modelling of Linear MIMO System	42
5.2.1	Problem Formulation	42
5.2.2	Control-Oriented LFT Modelling Approach for Multiplicative Uncertainty Structure	43
5.3	Generalized Control-Oriented LFT Modelling of Nonlinear MIMO System	54
5.3.1	Problem Formulation	54
5.3.2	Control-Oriented LFT Modelling Approach for Polytopic Uncertainty Structure	54
5.4	Chapter Summary	63
	References	63
6	Control-Oriented LFT Modelling of a Two-DOF Spring–Mass–Dashpot Dynamic System	65
6.1	Introduction	65
6.2	Mathematical Modelling of 2DOF SMD System	66
6.3	LFT Modelling	68
6.3.1	Control-Oriented Linear Fractional Transformation Modelling of Two-DOF Spring–Mass–Dashpot Dynamic System	68
6.4	H_∞ Controller Design	73
6.4.1	Weighting Function	73
6.4.2	System Interconnection	74
6.4.3	Simulation Results	74
6.5	Chapter Summary	78
	References	78
7	Control-Oriented LFT Modelling and H_∞ Control of Twin Rotor MIMO System	81
7.1	Introduction	81
7.2	Mathematical Modelling of Twin Rotor MIMO System	82
7.2.1	Lagrangian Model of Twin Rotor MIMO System	83

- 7.3 Control-Oriented LFT Modelling of Twin Rotor MIMO System 84
- 7.4 Design Specifications 98
 - 7.4.1 Weighting Function 98
 - 7.4.2 System Structure 101
- 7.5 H_∞ Controller Design 102
- 7.6 Simulation Results 103
- 7.7 Chapter Summary 106
- 7.8 Notation 107
- References 108
- 8 Control-Oriented LFT Modelling and H_∞ Control of Differentially Driven Wheeled Mobile Robot 111**
 - 8.1 Introduction 111
 - 8.2 Mathematical Modelling of Differentially Driven Wheeled Mobile Robot 112
 - 8.2.1 Kinematic Modelling 112
 - 8.2.2 Dynamic Modelling 114
 - 8.3 Control-Oriented LFT Modelling of Differentially Driven Wheeled Mobile Robot 115
 - 8.4 Design Specifications 123
 - 8.4.1 Weighting Functions 124
 - 8.4.2 System Interconnections 126
 - 8.5 H_∞ Controller Design 128
 - 8.6 Simulation Results 129
 - 8.7 Chapter Summary 131
 - 8.8 Notation 132
 - References 132
- 9 Control-Oriented LFT Modelling and H_∞ Control of Differentially Driven Wheeled Mobile Robot with Slip Dynamics 135**
 - 9.1 Introduction 135
 - 9.2 Differently Driven Wheeled Mobile Robot with Slip Dynamics ... 136
 - 9.3 Control-Oriented LFT Modelling of Differentially Driven Wheeled Mobile Robot with Slip Dynamics 141
 - 9.4 Design Specifications 147
 - 9.4.1 Weighting Function 147
 - 9.4.2 System Interconnections 149
 - 9.5 H_∞ Controller Design 151
 - 9.6 Simulation Results 152
 - 9.7 Comparison of the Performance of WMR Without and with Slip Dynamics 155
 - 9.8 Chapter Summary 155

9.9 Notation	156
References	156
Index	159

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Chapter 1

Introduction



1.1 Motivation

This book aims to deliberate a systematic approach of mathematical modelling of complex physical system that would enable the control system engineer to effectively design H_∞ robust controller with guaranteed performance. It is a well-known fact that the design approach of some particular control law demands specific mathematical model. For example, classical SISO control requires transfer function model, state feedback control requires state-space model, back-stepping control requires strict feedback form, fuzzy logic control requires fuzzy logic model, adaptive control requires linear parametric model and so on. The popular μ -synthesis-based H_∞ control requires linear fractional transformation (LFT) [1] model of the given plant to utilize the available design tools.

From the modelling point of view, this book introduces a modelling technique to achieve optimum control performance by including the control objective in the system modelling. Instead of attempting to apply general mathematical modelling technique, which is quite often complicated, control-oriented modelling of the real-world system results into a compact and practicable representation of the mathematical modelling. In fact, control-oriented system modelling and identification [2] is a convenient modelling approach that leads to a useful representation of the physical system with an ultimate objective of control system design for it.

The performance of the robust controller largely depends on the uncertainty representation of the system. A systematic way for the uncertainty modelling of the dynamic systems in the closed-loop structure is in the form of LFT modelling by the norm bounded unstructured, dynamic perturbation matrix. The book deals with control-oriented LFT modelling of complex dynamic systems with the aim to utilize the LFT model for μ -synthesis-based H_∞ [2] control law design. The H_∞ control law design employing μ -synthesis technique has been a popular control system design

paradigm for the last decade. However, widespread application of this μ -synthesis-based H_∞ control law design for arbitrarily complex systems was afflicted by lack of any systematic derivation of LFT model of such systems. The contents presented in this text will largely fill this gap.

1.2 Organization of This Book

A step-by-step systematic approach has been adopted in this book to elaborate the concept and applications of the main theme of this book. The important and notable aspects related to general modelling techniques are elaborated at first. Then the origin and representation of model uncertainty have been explained. Then the method of casting the uncertainty in the LFT framework has been elaborated. A generalized approach of control-oriented LFT modelling has also been presented. Finally, several applications of control-oriented LFT modelling and subsequent H_∞ control law design employing μ -synthesis based on the derived LFT models have been elaborated.

Chapter 2 covers a comprehensive survey on the mathematical modelling of real physical systems.

Chapter 3 presents a theoretical discussion for H_∞ control-oriented LFT modelling of nonlinear multivariable systems.

Chapter 4 covers a comprehensive survey on the μ -synthesis-based H_∞ controller design technique. This chapter extensively discussed two major aspects of the H_∞ controller, mainly optimal and sub-optimal control techniques. Finally, the H_∞ control law is implemented by using μ -synthesis to obtain optimal control performance.

Chapter 5 deals with the generalized control-oriented LFT modelling of a nonlinear MIMO system with an equal number of input–output combinations in the first approach. In the second approach, a generalized representation of the proposed modelling for the nonlinear MIMO system for any number of input–output combinations is derived by considering the system parameters varying within certain intervals. The state matrix representation in the functional form for the parametric model is used to realize the uncertain parameters. Finally, the uncertain parameters of the system are expressed as affine functions to derive the generalized control-oriented LFT modelling for a nonlinear MIMO system.

Chapter 6 presents application of the concepts of the previous chapters to derive robust control-oriented LFT modelling for the 2DOF spring-mass-damper system by considering all possible parametric uncertainties and external disturbance signals. Finally, H_∞ control law is implemented to validate the effectiveness of the derived modelling by μ -analysis-based robust stability and performance analysis. The transient response of the system during time domain analysis exhibits excellent performance both in the normal and in the presence of external noise signals.

Chapter 7 covers robust control-oriented LFT modelling approach for the laboratory-scale helicopter for further design of the H_∞ controller by μ -synthesis.

The linearized mathematical modelling has been formulated for the twin rotor MIMO system considering all its cross-coupled dynamics to recast the system modelling in terms of uncertainty representation by truly integrating the control objectives.

Chapter 8 mainly discussed a systematic approach towards the control-oriented LFT modelling of a nonlinear multivariable mobile robotic system, i.e. wheeled mobile robot (WMR). The LFT modelling of the wheeled mobile robot is derived for the H_∞ controller to achieve robust stability and performance even in the presence of all possible uncertainties.

Chapter 9 addressed the improved modelling quality of the WMR by introducing the longitudinal slip dynamics into the robust control-oriented LFT modelling structure.

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2. Chen, J., Gu, G.: Control-Oriented System Identification: An H_∞ Approach. Wiley (2000)

Chapter 2

Mathematical Modelling of Real Physical Systems



2.1 Introduction

Mathematical model depicts functional correspondence between the input variables and output variables of a real physical system. These mathematical functions may be linear or nonlinear and may contain a room for uncertainty depending upon the complexity of the real physical system. Modelling uncertainty may arise due to the limited knowledge about the actual physical phenomena involved in the system and associated laws of nature during the manifestation of the dynamic behaviour of the system. Many physical phenomena may be involved simultaneously in one single system. For example, even a simple system like electric motor involves electrical principles, magnetic principles, principles of mechanics, thermal effects, etc. More accurate model may involve the principles of tribology, principles of vibration, earth's magnetic and capacitive influence and so on. More the physical laws are involved to describe the dynamics of the physical system, more complex will be the structure of the model and the number of the parameters and variables required to describe the model will be large. Even after a thorough investigation of the involvement of natural phenomena with a system, at a certain stage, we will have to halt due to the limitation of understanding of the nature at that moment. Thus uncertainty is bound to exist in any mathematical model of a real physical system.

Thus, control system engineer faces a big challenge to find out the mathematical model appropriate for analysis and synthesis of the control system for a physical system. He can overcome this challenge by application of his engineering skill and engineering common sense to estimate the level of complexity needed in the mathematical model to meet the requirements and objectives of the analysis and synthesis of the control system satisfactorily. This decision making is very important before any modelling task is undertaken.