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Yu Guo · Albert C. J. Luo

Periodic Motions to Chaos in a Spring-Pendulum System

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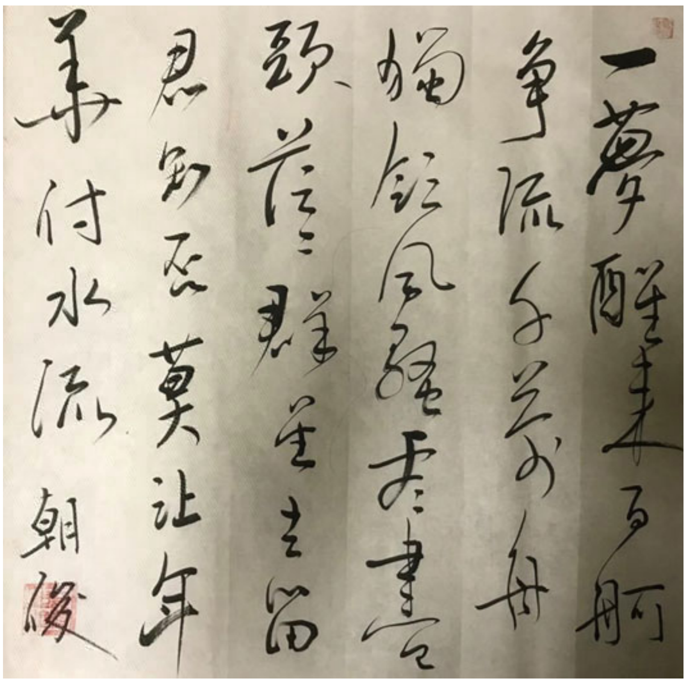
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Preface

In 1933, such a spring-pendulum system was proposed by Witt and Gorelik as a two-degree-of-freedom parametric oscillator, and the spring-pendulum system possesses an internal parametric resonant system. Such a system was used for explanation of the internal resonance phenomena. Traditionally, the Taylor series was used for expanding sinusoidal function of the pendulum, and the perturbation method was employed for approximate dynamical behaviors in the spring-pendulum. The linear results were extensively used to explain physical phenomena. However, such an approximated spring-pendulum cannot provide accurate solutions and cannot adequately explain the real physical phenomena as well. Thus, this book presents accurate, semi-analytical solutions of the non-approximated spring-pendulum, and the bifurcation trees of periodic motions to chaos in such a nonlinear spring-pendulum are presented. The harmonic frequency-amplitude characteristics of periodic motions to chaos are presented. However, the semi-analytical results of periodic motions for the nonlinear spring-pendulum cannot be obtained through perturbation methods.

In this book, periodic motions to chaos in a nonlinear spring-pendulum are discussed. In Chap. 2, the implicit mapping method is presented as a semi-analytical method for periodic motions in nonlinear dynamical systems. In Chap. 3, the mathematical modeling of a nonlinear spring-pendulum is developed, and the corresponding discretization of the non-approximated differential equations is completed for the implicit discrete maps. In Chap. 4, periodic motions in the periodically forced nonlinear spring-pendulum are studied through the mapping structures, and the finite-Fourier series is also presented for periodic motions. The bifurcation tree of period-1 motions to chaos varying with excitation frequency is presented in Chap. 5. In Chap. 6, higher-order periodic motions to chaos varying with excitation amplitude are discussed through period-3 motion to chaos.

Finally, the authors hope the materials presented herein can last long for science and engineering.

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Yu Guo is Associate Professor at Midwest State University. His research lies in nonlinear dynamical systems, discontinuous dynamics, nonlinear dynamical systems control. He is an associate editor for *Journal of Vibration Testing and Systems Dynamics.*

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1 Introduction

The spring-pendulum system possesses an internal parametric resonant system. One used such a system to investigate the internal resonance phenomena. Traditionally, one used the Taylor series to expand sinesoidal function of the pendulum, and the perturbation method was adopted to approximately determine dynamical behaviors in the spring-pendulum. The linear results were extensively used to explain physical phenomena. However, such linearized spring-pendulum cannot provide accurate solutions and cannot adequately explain the real physical phenomena as well.

The early studies on periodic motion in nonlinear systems were mainly based on the perturbation methods. In 1788, Lagrange [1] investigated the three-body problems as a perturbation of the two-body problems. In 1899, Poincare [2] developed the perturbation theory for the periodic motions of celestial bodies. In 1920, van der Pol [3] studied the periodic solutions of an oscillator circuit through the method of averaging. In 1928, Fatou [4] proved the asymptotic validity of the method of averaging based on the solution existence theorems of differential equations. In 1935, Krylov and Bogoliubov [5] extended the method of averaging to nonlinear oscillations in nonlinear vibration systems. In 1964, Hayashi [6] presented the perturbation methods including averaging method and the principle of harmonic balance method for nonlinear oscillations. In 1969, Barkham and Soudack [7] employed the extended Krylov–Bogoliubov method for the approximate solutions of a second-order nonlinear autonomous differential equations. In 1987, Garcia-Margallo and Bejarano [8] used a generalized harmonic balance method for the approximate solutions of nonlinear oscillations. Yuste and Bejarano [9, 10] used the elliptic functions instead of trigonometric functions to improve the Krylov–Bogoliubov method on nonlinear oscillators. In 1990, Coppola and Rand [11] obtained the approximation of limit cycles through the method of averaging via elliptic functions.

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In 1933, such a spring pendulum system was proposed in Witt and Gorelik [12] as a two-degree-of-freedom parametric oscillator. In 1976, Olson [13] discussed the auto-parametric resonance of a spring pendulum using the harmonic balance method. In 1978, Falk [14] studied the recurrence effects of a parametric spring pendulum using the perturbation method. In 1984, Lai [15] investigated the recurrence of a resonant spring pendulum system; and the trajectories of motions are correlated with experiments. In 1990, numerical investigations on chaos in a spring pendulum system without damping were presented (e.g., Nunez-Yepez et al. [16], Cuerno et al. [17]). In 1993, Bayly and Virgin [18] used the Taylor series to expand the sinusoidal function up to the cubic terms, and the periodic motion was assumed to the first-order harmonic term with constant for (1:2)-internal resonance. In addition, the experimental studies were completed. In 1994, Lee and Hsu [19] used harmonic balance method to obtain the approximate stead-state solutions of a spring pendulum; and the global domains of attraction for the periodic motions were presented using cell-to-cell mapping method. In 1996, Weele and Kleine [20] presented a detailed order-chaos-order sequence of such a spring pendulum system. In 1999, Lee and Park [21] proposed a second-order approximation for the chaotic responses in a harmonically excited spring pendulum. In 2003, Eissa et al. [22] studied the resonance and stability of a non-linear spring pendulum under harmonic excitation and Popov [23] used Poincare maps to investigate the chaotic motions of a spring pendulum, providing an insight to nonlinear shell vibrations. In 2006, Alasty and Shabani [24] studied the chaotic responses of a spring pendulum system through bifurcation diagram and Poincare maps. In 2008, Eissa et al. [25] studied the vibration reduction of a nonlinear spring pendulum through the perturbation method. In 2009, Amer and Bek [26] investigated the chaotic responses of a harmonically excited spring pendulum moving in a circular path. In 2011, Markeyev [27] studied the rotating motions and the stability in the spring pendulum using the perturbation method. In 2018, Sousa et al. [28] studied the energy distributions of a spring pendulum system. However, the traditional perturbation method does not provide an effective route towards analytical predictions of the nonlinear dynamical behaviors of such a system.

On the other hand, researchers have been interested in analytical predictions of periodic motions for nonlinear oscillatory systems. In 2012, Luo [29] developed an analytical method for analytical solutions of periodic motions in nonlinear dynamical systems, which was based on the generalized harmonic balance. The detailed discussion can be found in Luo [30–32]. Luo and Huang [33] used the generalized harmonic balance method for the analytical solutions of period-1 motions in the Duffing oscillator with a twin-well potential. Luo and Huang [34] also employed a generalized harmonic balance method to find analytical solutions of period-m motions in such a Duffing oscillator. The analytical bifurcation trees of periodic motions to chaos in the Duffing oscillator were obtained (also see, Luo and Huang [35–40]). Such analytical bifurcation trees showed the connection from periodic motions to chaos analytically. Luo and Yu [41] also employed the generalized harmonic balance method for approximated analytical solutions of period-1 motions in a nonlinear quadratic oscillator. Analytical solutions of periodic motions in