

Mo-Lin Ge
Yang-Hui He *Eds.*

Dialogues Between Physics and Mathematics

C.N. Yang at 100

 Springer

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Mo-Lin Ge • Yang-Hui He
Editors


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*Opto, domine, et hunc natalem et plurimos
alios quam felicissimos agas.*

- Pliny, Sec. Ep. Triiano Imp.

敬祝楊先生福如東海，壽比南山。

Preface

The breadth and depth of Professor Chen-Ning Frank Yang's contributions to theoretical physics have been legendary. His Nobel Prize-winning work on parity violation in 1957 is but a glimpse into his profound insights and influences. Though his initial contributions were to theoretical particle and nuclear physics, C. N. Yang's work had bestowed lasting effects on high energy physics, condensed matter physics, mathematical physics, as well as to pure mathematics, especially to topology, algebraic geometry, differential geometry, and group theory.

This volume, comprised of essays from world-renowned mathematicians and physicists, many of whom are friends and colleagues of C. N. Yang over the decades, is a celebration of his 100th birthday. After a brief summary of Prof. Yang's life and achievements, the book begins with recollections of his years at Stony Brook by Prof. Peter van Nieuwenhuizen, who succeeded Yang in 1999 as the director of the Institute for Theoretical Physics, now named the *C. N. Yang Institute for Theoretical Physics* at Stony Brook. Prof. van Nieuwenhuizen is one of the founders of the super-gravity theory, which is a brainchild between supersymmetry, Yang-Mills theory, and general relativity.

Chapter 2 is by the Fields Medalist Prof. Vladimir Drinfeld, entitled "A stacky approach to crystals", which is in the field of arithmetic geometry. It initiates the stacky approach to so-called crystalline cohomology (algebraic stacks are generalizations of algebraic varieties in that points on them form a groupoid rather than a set, and may thus have nontrivial automorphism groups). This is a p -adic version of de Rham cohomology theory, well-known to geometers and physicists, that governs differential forms entering Yang-Mills gauge theories. It is the hope of many leading mathematicians such as Minhyong Kim, Don Zagier, et al., that arithmetic geometry and number theory will play an increasing role in fundamental physics.

Chapter 3 is from the topologist and Norbert Wiener awardee Prof. Louis Kauffman, entitled "The Potts Model, the Jones Polynomial and Link Homology", and explores the relation between the Potts model in statistical mechanics, the Temperley-Lieb algebra, as well as the Jones polynomial and Khovanov homology in knot theory. This involves two subjects close to Prof. Yang's heart: (1) models

from statistical mechanics such as Ising and Potts and (2) groups from braids and knots.

Chapter 4 is from the physics Nobel Laureate Prof. Sir Anthony Leggett, entitled “The Penrose-Onsager-Yang Approach to Superconductivity and Superfluidity”. As he states in the abstract, “While the work of Professor C. N. Yang in condensed matter physics may not be quite as universally known as that in the area of particle physics, it includes a number of seminal papers, of which the one I like best is the 1962 Reviews of Modern Physics paper entitled *Concept of off diagonal long range order and the quantum phases of liquid He and of superconductors....* This paper in effect asks the general question “what exactly is it that superfluid helium and superconductors have in common, and what is at the root of their anomalous properties?” Although this question has also been answered by others in rather different terms, I believe that the answer given by Yang is by far the clearest and most intuitive.”

Chapter 5 is by the mathematician Prof. Yuri Manin, recipient of numerous prizes, such as Nemmers, Schock, Bolyai, etc. Joint with N. Combe and M. Marcolli, the chapter is entitled “Quantum Operads” and introduces the study of quantum symmetries of operads. Quantum groups and braid groups are intimately related to the Yang-Baxter equations. From a categorical point of view, the study of operads that generalize the structure of groups should offer new insights into the construction of quantum codes and thus into quantum information theory.

Chapter 6 is from the physicist Prof. Jian-Wei Pan, a pioneer in experimental quantum sciences and winner of many awards such as the Zeiss Prize, the International Quantum Communication Award, R. W. Wood Prize, the Future Science Prize. It is a review on quantum computational advantage, bringing together mathematics, computational complexity, and quantum optics. Prof. Yang has had a long interest in the development of the quantum computer, both the theoretical and experimental aspects, and has appreciated the works of Pan.

Chapter 7 is from the physics Nobel Laureate Prof. Sir Roger Penrose, which begins with a review of how twistor theory is important in describing classical and quantum particles with spin and helicity. This twistor formalism has been over the years, a novel and complexified perspective on the structure of space-time and quantum gravity. There has been much recent activity on relating this approach to scattering amplitudes in the Yang-Mills and string theory. In this chapter, the quantum twistor algebra is shown to have a hitherto-unnoticed split-octonion algebra and G_2^* structure.

Chapter 8 is from the physicist Prof. Alexander Polyakov, winner of numerous awards such as the Breakthrough Prize, the Planck Medal, the Lorenz Medal, etc. Based on joint work with F. Popov, it is on “Kronecker anomalies and gravitational striction”, which studies quantum field theories in which the number of degrees of freedom changes discontinuously across the momentum space. This leads to a concept which they call “Kronecker anomaly” and which leads to non-local effective actions that can be represented as a theory with random, self-tuning coupling constants.

Chapter 9 is by the physics Nobel Laureate Prof. Gerard 't Hooft, on “Projecting Local and Global symmetries to the Planck Scale”. It is based on a recent series of papers by the authors on a novel approach to reconciling quantum mechanics with general relativity, a topic that had been in Prof. Yang’s mind for decades. Very appropriately, the chapter begins with the words “In 1954, Chen-Ning Yang and Robert L. Mills published their idea that Maxwell’s equations for the electromagnetic force could be generalised....”

Chapter 10 is by the physics Nobel Laureate Prof. Frank Wilczek, entitled “Gauge Symmetry in Shape Dynamics”, and is on the subject of gauge symmetry—one of Prof. Yang’s key contributions to physics—on deformable bodies. Indeed, as a tribute, the abstract begins with “C. N. Yang’s ideas about local gauge symmetry and non-integrable phases have been enormously fertile sources of inspiration in fundamental physics and in the quantum theory of matter....”

Chapter 11 is by the Fields Medalist Prof. Edward Witten, provocatively entitled “Why Does Quantum Field Theory in Curved Spacetime Make Sense? And What Happens to the Algebra of Observables in the Thermodynamic Limit?” It provides a gentle introduction to some classical results about quantum field theory in curved spacetime and about the thermodynamic limit of quantum statistical mechanics. This fundamental question, unifying gauge theory and general relativity, has long been on the mind of C. N. Yang, as exemplified by his 1974 paper on the pure space formalism.

Chapter 12 is by the experimental physicist Prof. Qikun Xue, inaugural winner of the Chinese Future Science Award and of the Fritz London Memorial Prize. It is on the “Quantum Anomalous Hall Effect”. Prof. Yang has long appreciated the work of Prof. Xue and has been a colleague of his at Tsinghua University. It was at Tsinghua that Prof. Xue made the experimental discovery of the quantum anomalous Hall effect (QAHE).

Finally, Chap. 13 is from the physicist Prof. Zhongxian Zhao, winner of the State Supreme Science and Technology Award and the Bernd T. Matthias Prize. It is based on joint work with J. Hu and X. Dong, entitled “Magic superconducting states in Cuprates”, where superconductivity is stronger and more robust. Indeed, as discussed in Chap. 4, Prof. Yang had a long interest in superconductivity.

We have included a plenitude of photographs featuring Prof. Yang over the years, his times in the USA and China, his long service to Nankai and Tsinghua, as well as his life-long friendship with the late Professor Shiing-Shen Chern (1911–2004), who along with Prof. Yang stands as one of the giants of modern mathematics and physics. Indeed, the friendship between Prof. Chern and Prof. Yang, personally and intellectually, deserves a biography of its own. Yang’s father was a teacher to Chern, and in return, Chern became a teacher to Yang. One of the crowning achievements of Prof. Yang, and indeed, of modern science, is the Yang-Mills theory, which is intimately related to Chern’s theory of connections on bundles over manifolds.

This friendship is immortalized at the entrance of the *Chern Institute of Mathematics* at Nankai University, where a mural captures a moment of the countless dialogues between Professors Chern and Yang. This image, in part, is the inspiration of the title of this volume, celebrating Prof. Yang’s birthday, and embracing the “dia-

logue” between physics and mathematics. Coincidentally, last year was the 110th anniversary of Prof. Chern’s birth, and the *Nankai Symposium on Mathematical Dialogues* was held, of which we had been two of the organizers.¹

As we pay our tributes to Prof. Chern, we also fondly remember three other dear friends of Prof. Yang, who had been waiting to celebrate this birthday and who had wished to contribute chapters. Sadly, they have of late been recalled to eternity. They are Professors Sir Michael Atiyah² (1929–2019), Freeman Dyson (1923–2020), and Ludvig Faddeev (1934–2017); we have included their photos in this volume. *Requiescant in pace, et lux perpetua luceat eis.*

We wish, above all, a heartfelt Happy 100th Birthday to Professor Chen-Ning Frank Yang! May you, and your beloved dialogues between mathematics and physics, be as in *Shih-Ching*, the Book of Odes:

如月之恆，如日之升。如南山之壽，不騫不崩。如松柏之茂，無不爾或承。
《詩經·小雅·天保》

Tianjin, China
Oxford, UK
August, 2022

Mo-Lin Ge
Yang-Hui He

¹ See eponymous Proceedings, Springer, 2022, ISBN 9789811923272.

² We were fortunate that Sir Michael was able to contribute, just before his passing, to a volume which we had the honour to co-edit with Prof. Yang: C.N. Yang, M.-L. Ge and Y.-H. He, *Topology and Physics*, World Scientific, 2019, ISBN 978-9813278509.

Chen Ning Yang 楊振寧

A Brief Curriculum Vitae

- Sept 22, 1922 Born in Hefei, Anhui, China
 1938–1942 Undergraduate, Southwest Associated University³
 1942–1944 Graduate Student, SAU, China
 1946–1948 PhD, University of Chicago, USA (Advisor: Edward Teller)
 1949–1966 Institute for Advanced Study, Princeton, USA
 1966–1999 Albert Einstein Professor, Institute of Theoretical Physics,⁴ Stony Brook, SUNY, USA
 (Emeritus 1999–)
 1986 Established the Theoretical Physics Section at Nankai Institute of Mathematics,⁵ China (Invited by S.-S. Chern)
 1986– Distinguished Professor-at-large, Chinese University of Hong Kong
 1995 Founded the Center for Advanced Studies of Tsinghua University, China
 1998– Professor of Tsinghua University, China

Main Awards

- 1957 Nobel Prize
 1980 Rumford Premium
 1986 National Medal of Science, USA
 1993 Benjamin Franklin Medal
 1994 Bower Award
 1995 Einstein Medal
 1996 N. Bogoliubov Prize
 1997 Small Planet #3421 named after Chen Ning Yang
 1999 Lars Onsager Prize
 2001 King Faisal International Prize
 2006 *You Bring Charm to the World* Award—Prize of Success for Life, China

Key Contributions

- 1948 Angular distribution in nuclear reaction
 1949 Fermi-Yang model
 1949 Selection rules of a particle decaying into two photons
 1950 Reflection properties of Spin-1/2 fields (with J. Tiomno)
 1951 Exact calculation of spontaneous magnetization of 2-dimensional Ising model

³ SAU was a consortium, due to WWII, of Peking, Tsinghua and Nankai Universities, (1938–1946).

⁴ Now named the C. N. Yang Institute of Theoretical Physics.

⁵ Now dubbed the Chern Institute of Mathematics.

- 1952 Statistical theory of equations of state and phase transition I and II (circle theorem) (with T. D. Lee)
- 1954 Yang-Mills theory (with R. L. Mills)
- 1956 Parity non-conservation in weak interaction (with T. D. Lee)
- 1957 Pseudo potential calculation of hard-sphere Bosonic particles (with K. Huang and T. D. Lee)
- 1960 Imperfect Bose system
- 1969 Quantization of magnetic flux in superconducting cylinders (with N. Byers)
- 1962 Off-diagonal long-range order (ODLRO)
- 1963 Critical point in liquid-gas transition (with C. P. Yang)
- 1964 Phenomenology in CP violation (with T. T. Wu)
- 1966 Bethe hypothesis for anisotropic spin-spin chain (with C. P. Yang)
- 1967 Foundation of “Yang-Baxter equation” (Partly with C. P. Yang)
- 1975 Gauge fields and fiber bundles (with T. T. Wu)
- 1977 Self-dual gauge fields
- 1989 η -pair (with S. C. Zhang)
- 1990 $SO(4)$ symmetry in Hubbard model (with S. C. Zhang)
- 2003 Selected papers II
- 2008 曙光集 (“The Twilight Collection”) collected works (in Chinese)
- 2018 晨曦集 (“The Dawn Collection”) collected works (with Wong Fan, in Chinese) Second edition, 2021.

Membership/Foreign Membership of Academies

U. S. Academy of Sciences
 Chinese Academy of Sciences, Beijing
 Russian Academy of Sciences
 Royal Society (London)
 American Philosophical Society
 Academia Sinica, Taiwan
 Brazilian Academy of Sciences
 Venezuelan Academy of Sciences
 Polish Academy of Sciences
 Royal Spanish Academy of Science
 Korean Academy of Science and Technology
 Pontifical Academy of Sciences
 Japan Academy

Chairmanships

1965 Panel of Theoretical Physics, Physics Survey Committee, National Academy of Sciences

1972–1976 Division of Particles and Fields of the International Union of Pure and Applied Physics

1970–1971 Division of Particles and Fields of the American Physical Society

1980–1983 Fachbeirat of the Max Planck Institute of Physics, Munich

Presidencies

- 1977–1980 National Association of Chinese Americans (NACA)
 1989–1994 Association of Asia Pacific Physical Societies
 1996– Asia Pacific Center for Theoretical Physics (APCTP)

Board Memberships

- Governing Council of the Courant Inst. of Mathematical Science 1963–
 Science Advisory Committee of IBM 1966–1971
 High Energy Physics Advisory Panel (HEPAP) 1968–1970
 Board of Trustees of Rockefeller University 1970–1976
 Woods Hole Oceanographic Institution 1962–1978
 Board of Trustees of Ben Gurion University 1981–
 Board of Directors of the AAAS 1975–1979
 Board of Trustees of the Salk Institute 1978–1989
 Board of Directors of the Neuroscience Institute 1983–1988
 Board of Directors of the Scientific American Inc. 1983–1990
 Council of Scholars, The Library of Congress 1990–
 Board of Trustees, Stony Brook Foundation 1981–1999
 Board of Trustees, Brookhaven Science Associates 1998–1999
 Board of Shaw Prize Foundation, Hong Kong 香港邵逸夫獎基金會 2002–2014

Lectureships

- Gibbs Lecturer, American Mathematical Society
 Loeb Lecturer, Harvard University (1957, 1972 and 1997)
 Vanuxem Lecturer, Princeton University
 Lincoln Lecturer, Bd. of Foreign Scholars (Fulbright Bd.) of the State Department
 Pauli Lecturer, ETH, Zurich
 Courant Lecturer, New York University
 Fermi Lecturer, Scuola Normale Superiore, Pisa
 Oppenheimer Memorial Lecturer, Los Alamos
 Lorentz Professor, Leiden
 Klein Memorial Lectures, Stockholm
 Ta-You Wu Lecturer, University of Michigan
 Goudsmit Lecturer, University of Michigan
 Wu-Zhi Yang Lecturer, Fudan University, Shanghai
 Primakoff Lecturer, APS
 Oppenheimer Lecture, Berkeley

Honourary Doctorates

- Princeton University (1958), Brooklyn Polytechnic Institute (1965), University of
 Wroclaw, Poland (1974), Gustavus Adolphus College, Minnesota (1975), Univer-
 sity of Maryland (1979), University of Durham, England (1979), Fudan University,
 China (1984), Eidg. Technische Hochschule (ETH, 1987), Moscow State University

(1992), Drexel University (1995), Tsing Hua University, Taiwan (1996), Chiao Tung University, Taiwan (1996), Chinese University, Hong Kong (1997), University of Michigan (1998), State University of New York, Stony Brook (1999), Washington College (1999), Baptist University, Hong Kong (1999), Chung-Cheng University, Taiwan (2000), Hong Kong University of Science and Technology (2002), Ecole Normale Supérieure, Paris (2003), Nanyang Technological University, Singapore (2008), Taiwan University (2015), University of Macau (2015)

Published Books and Collections

- Selected Papers 1945–1980 With Commentary, Freeman and Company, 1983
- 《讀書教學四十年》，香港三聯書店 (*Forty Years of Reading and Teaching*, SDX Joint Pub. Hong Kong), 1985
- 《讀書教學再十年》，台北時報出版公司 (*Ten More Years of Reading and Teaching*, Taipei Times Pub), 1995
- 《楊振寧文集》，上、下冊，上海華東師範大學出版社 (*Collected Works, C.N. Yang, Vol. I & II*, East China Normal U. Press), 1998
- 《曙光集》，(簡體字版)，北京三聯書店；(繁體字版)，八方文化創作室 (*The Twilight Collection*, SDX Joint Pub. Beijing), 2008
- 《曙光集十年增版》，北京三聯書店 (*The Twilight Collection, Extended*, SDX Joint Pub. Beijing), 2018
- *Selected Papers II With Commentaries*, World Scientific Publishing Co, 2013
- 《六十八年心路》，北京三聯書店 (*Six-eight Years of my heart's journey*, SDX Joint Pub. Beijing), 2014
- 《楊振寧選集》，華中科技大學出版社 (*Selected Works of C.N. Yang*, Central China Normal U. Press), 2016
- 《晨曦集》，商務印書館；八方文化創作室 (*The Dawn Collection*, Commercial Press, Hong Kong), 2018
- C. N. Yang, M. L. Ge and Y. H. He, *Topology and Physics*, (楊振寧、葛墨林、何楊輝，《拓撲和物理》) World Scientific, 2019
- *Selected Papers, 1945–1980, with Commentary*, 商印 Reprint 2020
- 《晨曦集》增版，商務印書館 (*The Dawn Collection, Extended*), 2021



Private Collection



Private Collection



C. N. Yang speaking at the 2019 Xplorer Prize Ceremony (Private Collection)

The Friendship Between Professors S. S. Chern and C. N. Yang over the Years



C. N. Yang and S. S. Chern at dinner ca. 1990 (Private Collection)



C. N. Yang and S. S. Chern in Nankai University, ca. 1990 (Private Collection)



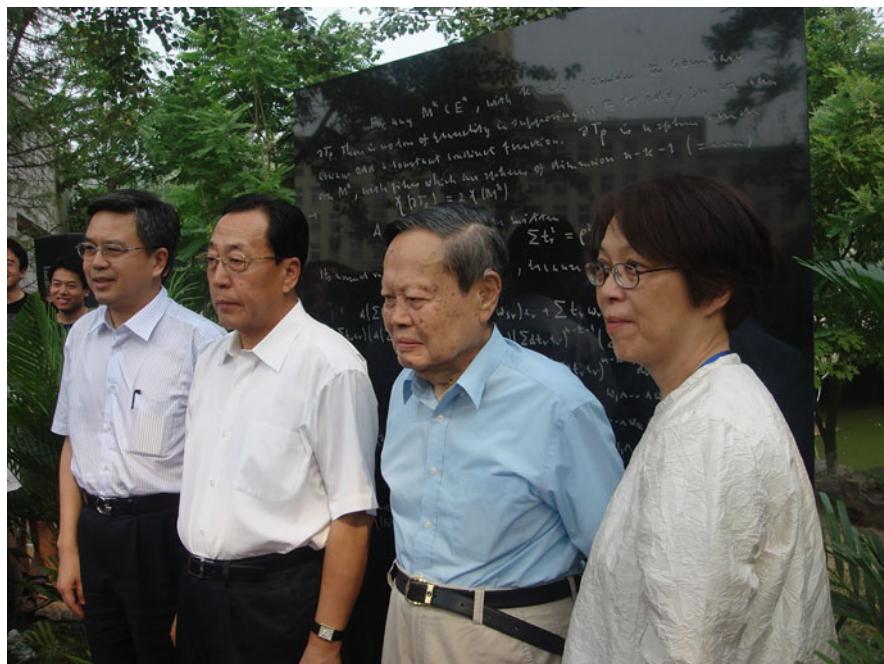
C. N. Yang and S. S. Chern in Chern's house, Nankai University, 1999 (Private Collection)



C. N. Yang and S. S. Chern in Nankai University, 2002 (Private Collection)



C. N. Yang's 70th birthday celebration at Nankai, 1992 (front row from left: Mrs. Chern, S. S. Chern, Ta-You Wu, T.-R. Wu, Mrs. S. Chen-Wu; back row from left: M.-G. Guang, C. N. Yang); (Private Collection)



C. N. Yang et al. at S. S. Chern's grave, Nankai University, 2005 (Private Collection)

Professor C. N. Yang and Various Friends



Sir & Lady Michael Atiyah at the Great Wall, circa 2000 (Photo by Mo-Lin Ge)



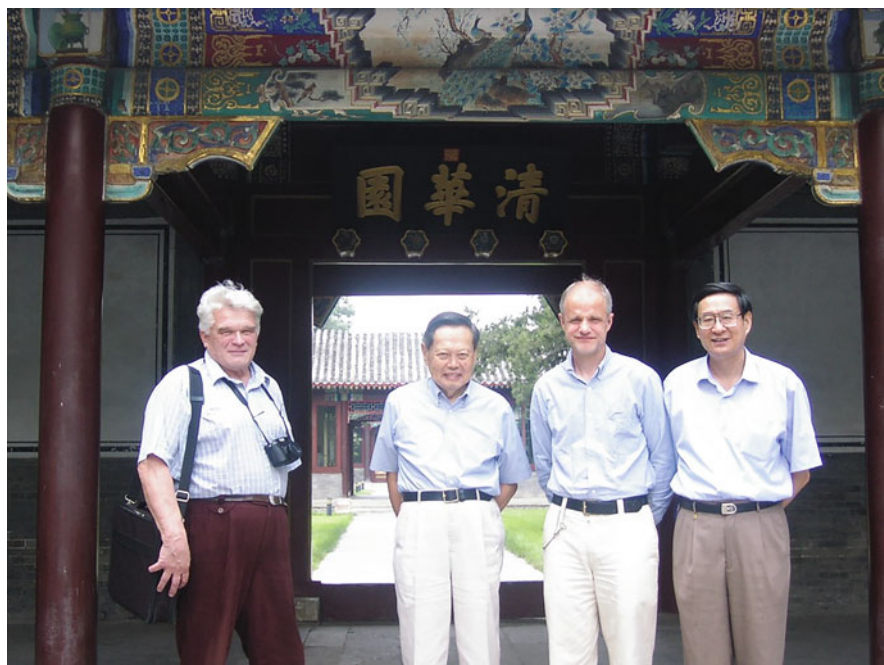
Sir M. Atiyah with M.-L. Ge at Nankai, circa 2000 (Private Collection)



Sir M. Atiyah with Y.-H. He in China, 2009 (Private Collection)



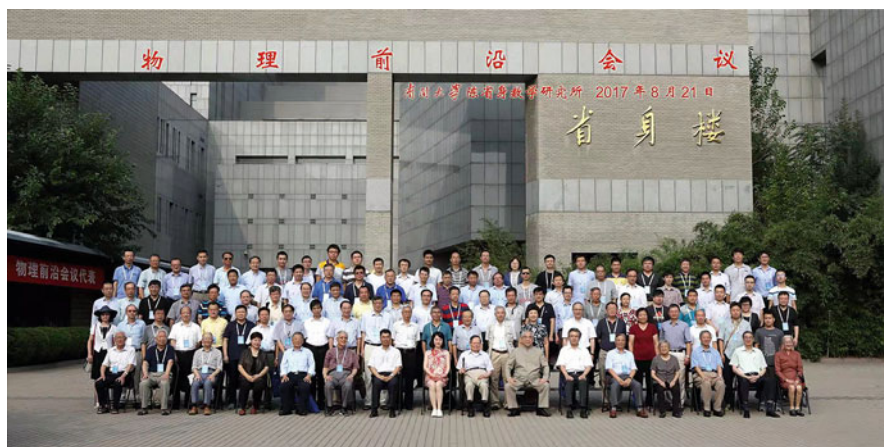
C. N. Yang and Ludwig Faddeev, 2008 (Photo by Mo-Lin Ge)



L. Faddeev, C. N. Yang, A. Niemi, M.-L. Ge, 2008, Tsinghua University Campus (Private Collection)



Freeman Dyson and M.-L. Ge before the portrait of S.S. Chern in the Chern Museum, Nankai, circa 2000 (Photo by Mo-Lin Ge)



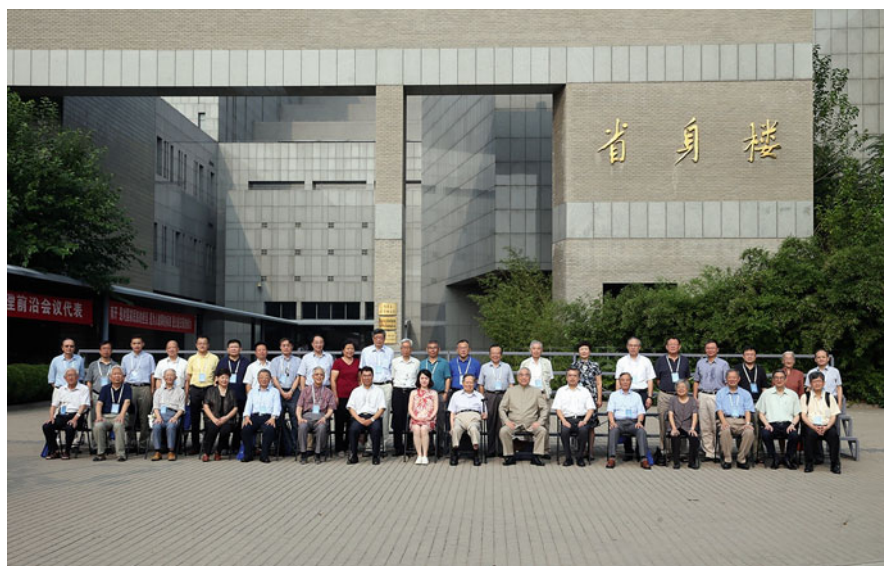
C.N. Yang's 95th birthday celebration at the Chern Institute of Mathematics, Nankai, circa 2017 (Private Collection)



At a conference in Nankai University, 1987, with signature (Private Collection)



Ceremony of C. N. Yang receiving Honorary Professorship from Nankai University, 1986 (Private Collection)



Frontier of Physics Conference at the Chern Institute, Nankai University, with various Academicians of the Chinese Academy of Science, 2012, celebrating C.N. Yang's 90th birthday; to the left (not pictured) is the grave of S. S. Chern (Private Collection)



C. N. Yang and M.-L. Ge climbing the Great Wall, ca. 1990 (Private Collection)



M.-L. Ge and C. N. Yang, at the Chern-Yang Mural, ca. 2015, Chern Institute foyer, in front of the mural of Chern and Yang in Dialogue (Private Collection)

Acknowledgements

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