Shyam Sunder Gupta

Creative Puzzles to Ignite Your Mind



Problem Books in Mathematics

Series Editor
Peter Winkler
Department of Mathematics
Dartmouth College
Hanover, NH
USA

Books in this series are devoted exclusively to problems - challenging, difficult, but accessible problems. They are intended to help at all levels - in college, in graduate school, and in the profession. Arthur Engels "Problem-Solving Strategies" is good for elementary students and Richard Guys "Unsolved Problems in Number Theory" is the classical advanced prototype. The series also features a number of successful titles that prepare students for problem-solving competitions.

Creative Puzzles to Ignite Your Mind



Shyam Sunder Gupta Indian Railways Jaipur, Raiasthan, India

ISSN 0941-3502 ISSN 2197-8506 (electronic)
Problem Books in Mathematics
ISBN 978-981-19-6564-7 ISBN 978-981-19-6565-4 (eBook)
https://doi.org/10.1007/978-981-19-6565-4

Mathematics Subject Classification: 00A08, 00A09, 00A07, 00A05, 97U40, 97N60

© The Editor(s) (if applicable) and The Author(s), under exclusive license to Springer Nature Singapore Pte Ltd. 2023, corrected publication 2024

This work is subject to copyright. All rights are solely and exclusively licensed by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors, and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd. The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

This book is dedicated with great respect to the memory of my mother late Smt. Shanti Devi, with the hope that it will keep alive her contribution in solving problems of all her near and dear ones.

Preface

The author takes great pleasure in presenting the book, *Creative Puzzles to Ignite Your Mind*, which is a great treasure for everybody who enjoys the beauty of the fascinating world of recreational mathematics. The book has been written keeping in view a large number of people who can be benefitted by reading the book. Apart from puzzle enthusiasts and math lovers, the book is considered of immense value for aspirants of Mathematics Olympiad, CAT/MBA and placement interviews of big companies like Google, Microsoft, Amazon, Apple, Facebook, Yahoo, NVidia, Oracle, Adobe, Morgan Stanley, Bloomberg and so on.

The book covers a large variety of intriguing puzzles with detailed ingenious solutions generally not found elsewhere. The puzzles can be browsed at random as these are not grouped in any orderly manner. The book is a result of collection and framing of puzzles to make them entertaining over a large period of thirty-five years, so references/sources of many puzzles are not available.

The book contains 25 short riddles and brainteasers in Chap. 1, which vary from simple but tricky to challenging ones. Chapter 2 provides solutions to all these short riddles and brainteasers given in Chap. 1.

Chapter 3 contains 125 creative puzzles of varying difficulty covering arithmetic and algebraic like clock, calendar, weight, age and digital puzzles, geometric puzzles, logical reasoning puzzles, combinatorial puzzles, match puzzles and game puzzles like who'll become a decamillionaire (kaun banega crorepati)?, new year winner, winning numbers, etc. Some famous and old puzzles like Cheryl's birthday, Bachets weight, liquid decanting, crossing bridge/river/desert, etc., have been included so that readers can find all types of puzzles at one place.

Only basic mathematics is required to solve these puzzles. But most of these puzzles are tricky and can be simplified by ingenious ideas. Chapter 4 provides detailed ingenious solutions to all 125 creative puzzles in a lucid manner along with special comments section which covers generalization of solutions, different techniques of attacking the same puzzle wherever possible. This will create interest in readers for further exploring the puzzle.

viii Preface

The title of puzzles has been suitably framed. Repetition of similar type of puzzles has been avoided to keep the book in concise form. However, important aspects of similar puzzles if any have been covered in comments section.

Like physical exercise is essential for a healthy body, mental exercise is equally essential for a healthy brain/mind. Puzzle solving is one of the brain exercises which can also train the mind to solve the real-world problems. So let us not waste time and start brain exercise as puzzles are eagerly waiting to ignite your mind.

Jaipur, India

Shyam Sunder Gupta

The original version of the book has been revised: The author provided belated corrections have been incorporated. A corrections to this book can be found at https://doi.org/10.1007/978-981-19-6565-4_5

Acknowledgements

I sincerely acknowledge and thank Dr. Akhilendra Bhusan Gupta and Mukesh Singhal for their comments after going through some of the puzzles in the book. I thank my sons Tarun Kumar and Amit Gupta for going through the manuscript and helping in making all the illustrations in the book.

I thank my wife Sushil Gupta for her encouragement and support without which it would not have been possible to start writing and complete this book.

I got interested in puzzles after getting inspired by the puzzles of world's greatest puzzle makers like Sam Loyd, Henry Dudeney, Boris Kordemsky, Martin Gardner, Raymond Smullyan and others. I am grateful and thank them all.

Since the book has been prepared based on material prepared and collected over a long period of time, it is practically not possible to make available the references. However, I am thankful to everybody associated directly or indirectly with the puzzles in this book.

Every effort has been made to make the book error-free; however, some errors and mistakes may always remain. Therefore, I shall be grateful for any suggestions and comments not only for rectifying the errors and mistakes but for improving the book also.

Shyam Sunder Gupta

Contents

I	Short Riddles and Brain Teasers	1				
2	Solutions of Short Riddles and Brain Teasers	7				
3	Creative Puzzles	17				
4	Solutions of Creative Puzzles	49				
Correction to: Creative Puzzles to Ignite Your Mind						

About the Author

Shyam Sunder Gupta is former Principal Chief Engineer, Indian Railways, Government of India. He is an Indian Railway Service of Engineers (IRSE) Officer of batch 1981. He has experience of more than 35 years in various managerial, administrative and technical positions such as Principal Chief Engineer, Executive Director, Divisional Railway Manager and Director/RDSO on Indian Railways. A recreational mathematician, who is actively involved in popularizing mathematics at the national and international levels, his major discoveries are equal product of reversible numbers (EPORNS), rare numbers, unique numbers, palindromic pseudoprimes, fifth-order prime polynomial, 17,350-digit memorable prime, etc.

His interest in number recreations dates back to 1978, when his first paper "Miracles of last digit" was published. Since then, his contributions have been published in *Science Reporter*, *Science Today*, *Math Education*, *The American Mathematical Monthly*, *The Mathematical Gazette*, *Mathematical Spectrum*, *Scientia Magna* and several other international books like *Unsolved Problems in Number Theory* (by R. K. Guy), *The Universal book of Mathematics* (by David J. Darling), *The Penguin Dictionary of Curious and Interesting Numbers* (by David Wells), *Prime Numbers: The Most Mysterious Figures in Math* (by David Wells), *Prime Curios!: The Dictionary of Prime Number Trivia* (by Chris K. Caldwell and G. L. Honaker) and *Those Fascinating Numbers* (by Jean-Marie De Koninck). He is co-author of the book, *Civil Engineering Through Objective Questions*.

List of Short Riddles and Brain Teasers

- 1. Health Monitoring
- 2. The Surface Area and Volume of a Cube
- 3. Missing Numbers in the Sequence
- 4. Weight of Rice Packets
- 5. Solving Equations
- 6. Pair of Balls
- 7. Coin Denominations
- 8. Buildings in the Colony
- 9. Partially Completed Grid
- 10. Finding Birthday
- 11. Allowance for Casting Vote
- 12. Counterfeit Cheque
- 13. Speed of an Old Car
- 14. Rate of Interest
- 15. Clock Digits
- 16. Unique Number
- 17. Largest Value and Average
- 18. Playing the Card Game
- 19. Candle Light Study
- 20. Apparent Miscalculation
- 21. Colliding Cars
- 22. Four Digit Squares
- 23. Vanishing Squares
- 24. Two Different Date Formats
- 25. Product of Three Consecutive Even Numbers

List of Creative Puzzles

- 1. The Economist and a Mathematician
- 2. Mobile Phones and Pen Drives
- 3. Inter-hostel Athletics Competition
- 4. Ages and Inequalities
- 5. Consecutive Double-Digit Numbers
- 6. Interchangeable Hands of a Clock
- 7. Largest Product of Two Numbers
- 8. Three Balls and Four Boxes
- 9. Worm on a Rubber Rope
- 10. Winning Numbers for 101
- 11. Keys to Open the Locker Room
- 12. Fastest Three Horses
- 13. Climbing the Stairs
- 14. Shopping During Partial Lockdown
- 15. Shaking Hands
- 16. Equal Distribution by Doubling
- 17. Identify the Apple Juice Bottle
- 18. Lockers Finally Remain Opened
- 19. Weights in Increasing Order
- 20. Grant Distribution Among Districts
- 21. Trucks to Transfer Payload
- 22. Inverting an Equilateral Triangle
- 23. Largest Product from Distinct Numbers
- 24. Gold Chain for Daily Payment
- 25. Age of Two Liars
- 26. Three Sundays on Prime Numbered Dates
- 27. Squares from Adjacent Numbers
- 28. Crossing the Bridge at Night
- 29. Distinct Digit Milometer Reading

xviii List of Creative Puzzles

- 30. Three Hands of a Clock
- 31. Review Meeting Days
- 32. Curious Olympic Rings
- 33. Maximum Area of Plot
- 34. Building Location for a Senior Citizen
- 35. Roll Number Dilemma
- 36. Relative Motion on Circular Track
- 37. Race Competition
- 38. A Gem from Bhaskaracharya II's Work
- 39. Five Children and 517 Gold Coins
- 40. Sum = Product
- 41. Investment and Returns
- 42. The Grazing Cow's
- 43. Labour on a Construction Site
- 44. The Biggest Number Using Three Identical Digits
- 45. Five Jealous Husbands and Their Wives
- 46. The Look and Say Sequence
- 47. Thirty-Six Inch Scale with Minimum Markings
- 48. Number of Triangles
- 49. The Ants on a Stick
- 50. Winner of the Year
- 51. Careless Bank Cashier
- 52. Five Working Days a Week
- 53. Defective Coin Among 24 Coins
- 54. Two Trains and a Bird
- 55. Basket and Eggs
- 56. Ranking Large Numbers
- 57. A Unique Polydivisible Number
- 58. Handless Clock Chimer
- 59. Mango Sharing
- 60. Five Sundays in February
- 61. Employment During Covid-19 Crisis
- 62. Camel and Banana Transportation
- 63. Distance Between Two Colonies
- 64. Bachet's Weights
- 65. Match Stick and Squares
- 66. Insufficient Pounds
- 67. Mixing Water and Milk
- 68. The Three Jug-Decanting Liquid
- 69. Crack the 3-Digit Lock Code
- 70. Gold and Silver Bricks
- 71. Three Men and Their Wives
- 72. Hundred Balloons in Hundred Rupees

List of Creative Puzzles xix

- 73. Counterfeit Coin
- 74. Effect of Paper Folding
- 75. Seven Security Posts
- 76. Rotating Balls
- 77. Holidays During Covid-19 Crisis
- 78. Making the Chain from Cut Pieces
- 79. Four Operations on Two Numbers
- 80. Who'll Become a Decamillionaire (Kaun Banega Crorepati)?
- 81. India's Independence Day
- 82. Palindromic Mileage
- 83. Time Measurement with Hourglass
- 84. Smallest Product of Two Numbers
- 85. Squares and Rectangles on a Chessboard
- 86. New Year Party and the Poisoned Drink
- 87. Black and White Balls
- 88. Missing Page Numbers
- 89. How Many Byes
- 90. Hourly Consumption of Lemon Juice
- 91. Unique Four Digit Square
- 92. The Age of Three Children
- 93. Pairs of Friendly Rectangles
- 94. When RIGHT Becomes LEFT
- 95. Seventh Root
- 96. Motion on a Straight Road
- 97. Unlucky Thirteen
- 98. Doubly True Alphametic
- 99. Tickets for the New Railway Stations
- 100. Egg Dropping
- 101. How Many Squares
- 102. Identify Gold Coin Box
- 103. Taxi Charges from Jaipur to Delhi
- 104. Cheryl's Birthday
- 105. Twelve Dots and Connecting Lines
- 106. Equalize Liquid Concentration
- 107. Two Boats and Width of a River
- 108. First Day of a New Century
- 109. Largest Product from Given Sum
- 110. Ten Digit Self-descriptive Number
- 111. Upward Journey of a Ball
- 112. Mathematician on the Escalator
- 113. Incorrectly Labeled Boxes
- 114. How Long Professor Walked
- 115. Ratio of a Number to its Digit Sum

xx List of Creative Puzzles

- 116. Financial Assistance to Villagers
- 117. Gold Coins and Culprit Jeweller
- 118. Pricing of a Fruit Set
- 119. Perfect Powerful Number
- 120. Minimum Number of Houses
- 121. Locker and Coins
- 122. Sum of Digits of Numbers
- 123. Number of Diagonals in a Polygon
- 124. Peculiar Pages in a Book
- 125. Coin Distribution

Chapter 1 **Short Riddles and Brain Teasers**



1. Health Monitoring

A survey was conducted in a town during the Covid-19 epidemic. It was found that 55% of the population is suffering from diabetes, 70% from heart disease and 80% from depression. Find the minimum possible percentage of the population suffering from all three diseases i.e. diabetes, heart disease and depression.

2. The Surface Area and Volume of a Cube

The surface area and volume of a cube are both five-digit integers. The volume of the cube is an integer multiple of the surface area. Find the side of the cube. Assume the units of side, surface area and volume of the cube as mm, mm² and mm³ respectively.

3. Missing Numbers in the Sequence

Find the missing numbers X and Y in the following sequence: 1100100, 10201, 1210, 400, 244, 202, X, Y, 100

4. Weight of Rice Packets

There are five packets containing rice. Instead of weighing these five packets individually, they were weighed in all possible combinations of two packets i.e., packet 1 and 2, packet 1 and 3, packet 2 and 3 etc. The weights in kilograms of all these combinations in ascending order are 6.7, 7.2, 7.3, 8.1, 8.3, 8.4, 8.9, 9.2, 9.7 and 9.8. It is not known which weight pertains to which pair of packets. Find the weight of each packet.

5. Solving Equations

Solve $(x^2 - 9x + 19)^{(x^2 - 15x + 56)} = 1$ and find all possible integer values of x.

[©] The Author(s), under exclusive license to Springer Nature Singapore Pte Ltd. 2023, corrected publication 2024

6. Pair of Balls

A box containing 5 red, 11 green and 17 white balls are kept in a room. At least two balls of the same colour are to be taken out of the box, without looking at it. Find the minimum number of balls required to be taken out of the box in order to be sure of getting at least two balls of the same colour.

7. Coin Denominations

In a certain Kingdom, three gold coins of different denominations in tolas were in vogue. Anupama had four gold coins worth 20 tolas. Akshara had five gold coins worth 15 tolas. If both Anupama and Akshara had at least one gold coin of each denomination, find the value of the denomination (an integer number) of each of the three different gold coins.

8. Buildings in the Colony

The Buildings in a small colony are numbered using different positive integers. The sum of the building numbers is 101. If the maximum possible number of buildings exists in the colony, find the building numbers.

9. Partially Completed Grid

There are 16 squares in the grid shown below. Ten squares are filled with the letters A to J and six squares are filled with whole numbers. Replace the letters A to J in ten squares with numbers from 1 to 10, using each number only once, such that the six squares containing numbers become the product of numbers in their adjacent squares filled with letters.

			А				
		В	378	C			
	D	144	Е	90	F		
G	160	Н	40	Ι	60	J	

10. Finding Birthday

In 1940, a man stated that he was 2n years old in the year n^2 . He also said that if his age in the year n^2 is added to the month number of his birth, the result obtained gives the square of the day of the month of his birth. Assume that n is a positive integer. Find the day, month and year of his birth.

11. Allowance for Casting Vote

During elections, it was decided by the administration to pay an allowance of Rs. 250 to every male and Rs. 200 to every female senior citizen who cast their votes. As per the census of the area, there were a total of 10000 senior citizens including both males and females. After the end of the voting day, it was noted that 60% of the male and 75% of the female senior citizens cast their votes. Find the total amount paid to senior citizens on the voting day.

12. Counterfeit Cheque

A person X bought an Antique Clock at a cost of \$500 and sold it on the same day to person Y for \$870. Person Y not having the change gave a \$1000 cheque to person X. Person X not having cash at hand, requested another person Z, who was his friend for help. So, person Z paid in cash \$130 to person Y in lieu of the \$1000 cheque. Before paying the balance amount of \$870 to person X, he (person Z) went to the bank to encash the \$1000 cheque. However, to his surprise, the \$1000 cheque was bounced and found to be counterfeit. So, person Z informed person X about the counterfeit cheque. Person X tried to contact Y but he was not traceable. So, person X repaid in cash \$130 to person Z. Find the amount actually lost by person X in the above transactions?

13. Speed of an Old Car

My son used to go to the school by cycle rickshaw. One day he left home at 7 AM but forgot his homework notebook. When I came to know about it, I started by my old car at 7:20 AM and after handing over the homework notebook to my son on the way, I returned home at 7:40 AM. Find the speed of my old car, if the speed of the cycle rickshaw is 20 km/h. Assume uniform speeds of cycle rickshaw and car in both directions. Ignore any time lost in handing over the notebook and turning back by my old car.

14. Rate of Interest

On a certain amount of money, the compound interest earned during the 6th and 7th year is 1000 rupees and 1060 rupees respectively. Find the annual rate of interest if compounding is done annually.

15. Clock Digits

A regular 12-hour digital clock displays hours, minutes and seconds like **08:37:56**

At what time in each of the 12-hour periods, do all six digits of the clock change simultaneously.

16. Unique Number

Find the number N such that N^3 and N^4 together contain each of the ten digits i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 exactly once only.

17. Largest Value and Average

If the average of 20 different positive integers is 20, find the largest possible value that any one of the integers can have.

18. Playing the Card Game

Three persons namely A, B and C before starting to play the card game among themselves had balances as under:

A and B combined had five times as much money as C had, while B and C combined had four times as much money as A had. At the end of the play, their balances become as under:

A and B combined had four times as much money as C had, while B and C combined had three times as much money as A had.

If one person lost Rs. 100, find the amount each person had before starting the card game.

19. Candle Light Study

A student was preparing for his examinations. The light went off. The student lights two candles of equal length, one of which was thicker than the other candle. The thick candle was designed to last for five hours while the thin candle was designed to last for three hours. After completing his study, when the student went to sleep, the thick candle was three times as long as the thin candle. How long did the student study in the candlelight?

20. Apparent Miscalculation

This is an interesting ancient puzzle, where two fruit sellers used to sell apples every day in the market. One day, one fruit seller sold all his 30 apples at the rate of 3 apples per dollar, whereas, another fruit seller sold all his 30 apples at the rate of 2 apples per dollar. Thus, the total amount received by two fruit sellers is 25 dollars. It may be assumed that the apples of both fruit sellers are not distinguishable visibly, though the apples of the second fruit seller are of superior quality.

The next day, one fruit seller falls sick, so he requested the other fruit seller to sell his 30 apples also at yesterday's rate. Accordingly, the fruit seller went to the market to sell his 30 apples along with 30 apples from the other fruit seller at yesterday's rate. On the way, all apples got mixed up, so he decided to sell all the apples at the rate of 5 apples for 2 dollars. This appeared logical to him as three apples for one dollar plus two apples for one dollar are the same as five apples for two dollars. Thus, the total amount received is 24 dollars, one dollar less than yesterday.

How do you explain the difference of one dollar?

21. Colliding Cars

Two cars that are 60 km apart are travelling towards each other. The speed of cars is 108 km/h and 144 km/h. Find the distance between two cars just 1 second before they collide.

22. Four Digit Squares

If each digit of a four-digit perfect square is increased by 1, another perfect square is obtained, find all such four-digit perfect squares.

23. Vanishing Squares

Forty match sticks each 1 unit in length are placed in square form as given below:



A total of 30 squares (1 of 4×4 , 4 of 3×3 , 9 of 2×2 and 16 of 1×1) can be seen in this arrangement. Find the minimum number of match sticks which on removing, vanishes all 30 squares.

24. Two Different Date Formats

Two different date formats i.e., DD/MM/YY and MM/DD/YY are prevalent in most of the countries. For example, 6th September 1960 is written as 6/9/60 in DD/MM/YY format and as 9/6/60 in MM/DD/YY format. Many such dates create confusion unless we know, which date format is used.

How many dates in a year can be written as two different valid dates in two different date formats which can create confusion?

25. Product of Three Consecutive Even Numbers

The product of three consecutive even numbers consists of seven digits. If the first and last digit of this product is 8 and 2 respectively, find the remaining five digits of this product and three consecutive even numbers.