Engineering Optimization: Methods and Applications

Shahin Jalili

Cultural Algorithms

Recent Advances

Springer

Engineering Optimization: Methods and Applications

Series Editors

Optimization carries great significance in both human affairs and the laws of nature. It refers to a positive and intrinsically human concept of minimization or maximization to achieve the best or most favorable outcome from a given situation. Besides, as the resources are becoming scarce there is a need to develop methods and techniques which will make the systems extract maximum from minimum use of these resources, i.e. maximum utilization of available resources with minimum investment or cost of any kind. The resources could be any, such as land, materials, machines, personnel, skills, time, etc. The disciplines such as mechanical, civil, electrical, chemical, computer engineering as well as the interdisciplinary streams such as automobile, structural, biomedical, industrial, environmental engineering, etc. involve in applying scientific approaches and techniques in designing and developing efficient systems to get the optimum and desired output. The multifaceted processes involved are designing, manufacturing, operations, inspection and testing, forecasting, scheduling, costing, networking, reliability enhancement, etc. There are several deterministic and approximation-based optimization methods that have been developed by the researchers, such as branch-and-bound techniques, simplex methods, approximation and Artificial Intelligence-based methods such as evolutionary methods, Swarm-based methods, physics-based methods, socio-inspired methods, etc. The associated examples are Genetic Algorithms, Differential Evolution, Ant Colony Optimization, Particle Swarm Optimization, Artificial Bee Colony, Grey Wolf Optimizer, Political Optimizer, Cohort Intelligence, League Championship Algorithm, etc. These techniques have certain advantages and limitations and their performance significantly varies when dealing with a certain class of problems including continuous, discrete, and combinatorial domains, hard and soft constrained problems, problems with static and dynamic in nature, optimal control, and different types of linear and nonlinear problems, etc. There are several problem-specific heuristic methods are also existing in the literature.

This series aims to provide a platform for a broad discussion on the development of novel optimization methods, modifications over the existing methods including hybridization of the existing methods as well as applying existing optimization methods for solving a variety of problems from engineering streams. This series publishes authored and edited books, monographs, and textbooks. The series will serve as an authoritative source for a broad audience of individuals involved in research and product development and will be of value to researchers and advanced undergraduate and graduate students in engineering optimization methods and associated applications.

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Preface

According to the biocultural evolutionary theory, genes and culture are two interacting forms of inheritance and overall human evolution can be viewed as the product of the changes in biological and cultural traits. Both genetic and cultural evolutionary processes include a form of information transmission. In genetic evolution, the genetic information embedded within the genes is being vertically transmitted from parents to offspring. In cultural evolution, the cultural variants such as beliefs, habits, skills, traditions, and preferences, pass from one generation to the next and they can be changed or even replaced by new ones over time due to changes in the cultural environment. In comparison to genetic evolution, the information transmission in cultural evolution is a much more complicated process. The information flow in cultural evolution includes both vertical and horizontal transmissions in which the cultural information is not only vertically inherited from parents to offspring, but also offspring have the opportunity to socially learn and acquire information from other members of the society.

Cultural algorithms (CAs) are meta-heuristic numerical optimisation algorithms inspired by the abovementioned biocultural evolutionary theory. CAs have some characteristic features that make them unique in comparison to other evolutionary algorithms (EAs). They model the biocultural evolutionary theory to perform the search process for global optima, in which both types of vertical and horizontal learning behaviours of individuals are modelled. Since their emergence, CAs have been extended and successfully employed to solve a wide variety of problems in different branches of science and technology.

The main aim of this book is to explore the recent advances in the algorithmic framework of CAs and their applications to the different problems in the literature. While the main approach of the book is to briefly discuss and explain the application studies and algorithmic details of CAs, the detailed mathematical formulations and algorithmic pseudo-codes are also discussed in each chapter to provide a clear explanations for the different concepts. The book is mainly aimed at postgraduate students and researchers in computer science and engineering subjects with research interests in optimisation and meta-heuristic algorithms. Throughout the book, it is assumed that the readers are familiar with the basic concepts of optimisation theory and meta-heuristic algorithms. However, the author tried to provide relevant references in each chapter to assist the readers in understanding the contents of the book.

The book comprises nine different chapters divided into three parts. Part I contains the basic concepts of standard CAs and their theoretical background. The first part explains how the basic concepts in biocultural evolutionary theory have been employed to develop standard CAs. Part II discusses the applications of CAs to a wide range of real-world problems and presents their detailed mathematical formulations, including decision variables, objective functions, and constraints. Part III investigates the different variants of CAs developed in literature and their algorithmic details. The third part includes a comprehensive survey and detailed pseudo-codes of different extended, hybrid, and multi-population versions of CAs. The last chapter of the book presents the application study of CAs to the real-world structural optimisation problems.

Although a significant effort has been made to minimise the errors and typos in the book, the author warmly welcomes receiving feedback, suggestions, and comments from readers on the contents of the book.

The author would like to thank the series editors, Prof. Amir H. Gandomi and Dr. Anand J. Kulkarni, as well as the publishing editor, Ms. Kamiya Khatter, and her colleagues in Springer Nature for their efforts and supports during the production process of this book.

Aberdeen, Scotland, UK Shahin Jalili
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About the Author

Shahin Jalili is a Research Fellow in the School of Engineering at the University of Aberdeen working on renewable energies. His research interests focus on problems that link engineering, computer and mathematical sciences, with particular emphasis on optimization methods and numerical algorithms. He has developed a series of numerical approaches for various optimization problems in different branches of engineering science, ranging from the optimum design of skeletal and composite structures to optimum scheduling of offshore wind operations and performance optimization of transportation networks. He has also worked on the mathematical aspects of finite element analysis and developed a set of new structural and sensitivity reanalysis formulations based on the polynomial vector extrapolation methods to reduce the computational complexity of structural optimisation.

Part I
Foundations

Chapter 1
Introduction to Stochastic Optimisation

For since the fabric of the universe is most perfect and the work of a most wise Creator, nothing at all takes place in the universe in which some rule of maximum or minimum does not appear.
—Leonhard Euler

Abstract Modern stochastic heuristic and meta-heuristic optimisation methods are efficient tools to deal with the "black-box" problems in which the objective and constraint functions cannot be expressed as explicit functions of decision variables. This chapter presents a brief introduction to available conventional and stochastic optimisation approaches in the literature and discusses their applicability in dealing with real-world problems in science and technology. The chapter also provides a taxonomy of meta-heuristics based on their sources of inspiration.

Keywords Optimisation · Heuristic · Meta-heuristic · Hyper-heuristic · Evolutionary algorithms

1.1 Introduction

The optimisation is everywhere. The universe minimises the efforts during its evolution. The principle of minimum energy states that the internal energy of a closed physical system, with constant external parameters and entropy, always tends to approach a minimum value to satisfy equilibrium conditions. Fermat's principle in optical physics, which is also referred to as the principle of least time, states that a ray always chooses the path with minimum travel time between two points. The human evolutionary process can be viewed as an optimisation process in which human biology tries to enhance human adaptability to harsh environmental conditions over different generations. Ants deposit pheromone as they travel and indirectly communicate with each other to find the shortest path between their nest and food resources.

In our daily life, we find ourselves in a position in which we have to make decisions to achieve certain goals. We consciously or unconsciously try to make the best possible judgement in all circumstances and optimise our decisions to achieve

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the most favourable outcomes. This tendency for optimisation takes different forms of behaviours in literature, art, business, etc. For example, poets make an effort to adopt the best combination of words with different rhythms and styles to deliver their thoughts, ideas, emotions, and messages to the readers. In the music improvisation process, the musicians try different combinations of music pitches to achieve the best possible harmony. In stock markets, the shareholders compete to take efficient investment strategies in their trades, minimise their losses, and maximise their profits. The manufacturing companies always plan to minimise their production costs and maximise their profit margin and market share. Numerous additional examples can be found in which the human plays the role of the optimiser. Despite our innate tendency for optimising our decisions, our capabilities in taking the right decisions with the most promising outcomes are limited. In some cases, the available time for making the judgements is quite short, and we are not able to react on time in response to dynamic environmental conditions. On the other hand, there is a huge number of complex situations in which human is unable to make the best possible judgements, no matter how long it would take. Nowadays, the complexity of problems in science and technology further highlights the fact that we desperately need powerful decision-making technologies to achieve different goals with minimum effort and energy.

With recent rapid technological development, numerous complex optimisation problems have emerged in different branches of science and technology. Early real-world optimisation problems were relatively easy to solve and handle. Hence, the classical techniques in applied mathematics were almost capable of dealing with these problems in a relatively efficient way. However, the complexity of human artefacts has been rapidly and consistently increased over time. Today's optimisation problems in different branches of science and technology are categorised as highly nonlinear problems with discrete and continuous variables under numerous equality and inequality constraints.

The main goal of this chapter is to present a brief introduction to available optimisation approaches in the literature. In Sect. 1.2, the conventional optimisation techniques are briefly reviewed, and their limitations in dealing with complex problems are discussed. Section 1.3 provides a brief review of stochastic heuristic and meta-heuristic algorithms as well as their taxonomy. This section categorises the meta-heuristics based on their sources of inspiration. As the main focus of this book is on CAs, the chapter also discusses the position of CAs in the taxonomy of meta-heuristic algorithms. In this chapter, it is assumed that the readers are familiar with the basic optimisation terminology. However, the readers are referred to the relevant references for more details.

1.2 Conventional Optimisation Methods

Conventional mathematical approaches are efficient tools to solve different types of optimisation problems. Linear programming deals with the problems in which the objective and constraints are linear functions of decision variables. Dantzig's simplex algorithm is one of the popular linear programming approaches (Dantzig 2016). The linear integer programming techniques have been developed for problems in which some or all of the decisions variables can only take integer values. There are a wide variety of problems in which mixed types of variables, including discrete and continuous, should be optimised, such as scheduling and production planning problems.

The dynamic programming method developed by Bellman in the 1950s is another popular optimisation approach in literature (Bellman 1966). The main idea of the approach is to break down the original difficult problem into several sub-problems which are easy to handle and deal with. The dynamic programming recursively calculates the optimal solutions for these sub-problems and then uses acquired information to obtain the global optimal solution for the original problem.

Many problems in science and technology are highly nonlinear. Nonlinear programming is a branch of mathematical science that focuses on optimisation problems with nonlinear objective and constraint functions. Although these techniques are capable of solving a set of practical problems, most real-world optimisation problems are much more difficult to be formulated and addressed by the nonlinear programming approaches (Foulds 2012). There are a series of outstanding monographs in the literature on the conventional optimisation methods, such as those by Luenberger and Ye (2021), Denardo (2012), and Foulds (2012).

The conventional optimisation approaches use gradient information of objective and constraint functions to search and locate the optimum solution for the problem at hand. Despite their remarkable performance in simple problems, their applicability to complicated problems is challenging from different perspectives. The conventional approaches perform the search process for optimal solutions based on the information acquired from derivations of objective and constraint functions. However, the calculation of gradient information is not always an easy task. Most of the modern optimisation problems belong to the category of "black-box" problems. In these problems, the objective and constraint functions cannot be expressed as explicit functions of decision variables, which make it difficult or sometimes even impossible to calculate their sensitivities for a given variable. The type of decision variables is also another source of difficulties in conventional techniques. Calculation of gradient information for discrete variables may be possible; however, it could be difficult or time-consuming in some cases. Another challenge in the application of conventional techniques is that they have been developed for the problems with single optimum solutions. While most of the modern optimisation problems are highly non-convex and nonlinear with multiple local optimum points. In such problems, the performance of classical techniques is highly sensitive to the initial solution, and they can easily get stuck into local optimums without efficient exploration of search space.

The abovementioned challenges intensified the efforts to develop alternative tools to deal with complex nonlinear problems. In the past decades, stochastic heuristic and meta-heuristic algorithms have emerged which are capable of providing optimal or near-optimal solutions for real-world problems within a reasonable time. Compared to the conventional approaches, heuristics and meta-heuristics are simple and easy to implement, and more importantly, they are independent of gradient information. These characteristic features make them attractive tools for “black-box” problems. By taking the advantage of their stochastic nature, “hopefully”, they can escape from the local optimum points in the search space and enhance their chance of converging to the global optimum solutions. The main goal of the next section is to provide a brief literature survey on the historical development of heuristic and meta-heuristic algorithms as well as their taxonomy.

1.3 Modern Stochastic Methods

One of the characteristic features of conventional optimisation approaches is their deterministic nature. This means that the same outputs are obtained for a given initial solution or starting point over different independent runs. Due to the limited capabilities of conventional approaches and the difficulties in their applications to real-world problems, stochastic heuristic and meta-heuristic optimisation methods have been developed to solve difficult problems in science and technology. They are suitable tools to find near-optimal, or hopefully optimal, solutions for complex problems within a reasonable time. These techniques do not necessarily guarantee the achievement of global optimum for different problems. Rather, they are expected to provide near-optimal solutions for a certain range of problems, for which there is no available algorithm to find the optimal solution in polynomial time. The literature survey reveals the absence of general agreement on the exact definitions for heuristic and meta-heuristic algorithms. However, this chapter distinguishes between heuristics and meta-heuristics. This section provides a brief review of heuristics and meta-heuristics.

1.3.1 Heuristics

The heuristic[1] is a Greek word that means “find” or “discover”. The main aim of heuristic optimisation methods is to discover optimum or near-optimum solutions for complex problems with relatively less computation effort. They are capable of finding near-optimum solutions for NP-hard problems in which the conventional approaches fail to provide meaningful results. Nowadays, they are the only available options to deal with a range of complex real-world problems. The heuristics work in a stochastic trial-and-error manner. They include a set of rules that are employed in a stochastic manner to discover useful clues about the optimum solution in the search space. The heuristics have a simple framework that makes them easy to understand and implement. In comparison to conventional approaches, they do not need gradient information of objective and constraint functions, and they are less sensitive to the

[1] εὑρίσκω.

initial solutions. By taking advantage of their stochastic behaviour, they can provide an efficient exploration of search space in highly nonlinear problems.

To decide whether a heuristic-based approach should be used for a given problem, a set of trade-off criteria is usually considered, including optimality, completeness, accuracy and precision, and execution time (Pearl 1984). The optimality criterion raises the question that whether the global optimum solution is needed for the problem at hand. In some problems, the global optimum exists, and it is necessary to discover it. While in others, finding an exact global optimum solution is not vital and only an approximation of the optimum solution is needed. Due to the stochastic nature of heuristic methods, they do not always guarantee the achievement of the global optimum solution. The completeness criterion discusses whether multiple global optimums exist in the problem. Although heuristics could potentially converge to different final solutions in different runs, they are usually designed in a way to provide a single output solution that can limit their applications to the problems with multiple global optimums. Accuracy and precision is another important criterion that highlights the question that whether the heuristics will be able to provide the final solution with an acceptable level of accuracy. This is particularly important in engineering optimisation problems. In most cases, a certain level of error can be tolerated. However, the errors in some cases could be irrationally large. The last criterion, i.e., execution time, considers whether the selected heuristic exhibits a faster convergence rate in comparison to the conventional methods. Some of the heuristics may not provide a clear advantage over conventional approaches in terms of convergence speed and required computational effort. Hence, the selection of the most promising heuristic for the problem at hand is vital.

As Rothlauf (2011) argues, heuristics can be categorised into construction and improvement heuristics. The construction heuristics start with an uncomplete solution and gradually construct the full solution for the problem over multiple steps. In these approaches, the search process is terminated as soon as a complete solution is constructed for the problem. On the other hand, the improvement heuristics start with a complete solution and gradually modify the value of each variable to achieve better results based on a given performance measure. As was mentioned earlier, heuristics are problem-specific tools. A wide variety of heuristics can be found in the literature that has been developed for different types of problems. One of the popular problems in computing science is the Travelling Salesman Problem (TSP) for which numerous heuristic techniques have been developed. The nearest neighbour, nearest insertion, cheapest insertion, and furthest insertion are examples of construction heuristics designed for TSP (Rosenkrantz et al. 1977; Rothlauf 2011). The two-opt, *k*-opt, and Lin–Kernighan heuristics belong to the category of improvement heuristics developed for the TSP problem (Rothlauf 2011).

1.3.2 Meta-Heuristics

Meta-heuristics are a new generation of stochastic optimisation techniques. A meta-heuristic can be described as an upper-level strategy that simultaneously takes the advantage of different heuristics to efficiently explore the search space and find optimum solutions. Several definitions for meta-heuristics are available in the literature. For example, Voß et al. (2012) state the following definition for meta-heuristics:

> "A metaheuristic is an iterative master process that guides and modifies the operations of subordinate heuristics to efficiently produce high quality solutions. It may manipulate a complete (or incomplete) single solution or a collection of solutions at each iteration. The subordinate heuristics may be high (or low) level procedures, or a simple local search, or just a construction method."

There are basic differences between heuristics and meta-heuristics. Contrary to heuristic approaches, which have been developed for certain problems, meta-heuristics are not problem-specific tools, and they can be applied to a large set of problems in science and technology. Meta-heuristics try to keep the balance between the exploitation (or intensification) and exploration (or diversification) mechanisms. The former refers to the capability of a search method in further improving the quality of the best solution, while the latter is related to the ability of the algorithm to search different regions of search space and escape from local optimum points.

1.3.3 No Free Lunch Theorems

As was discussed in previous sections, heuristics and meta-heuristics do not necessarily guarantee the consistent achievement of the global optimum solution for all kinds of problems. The reality is that their performance is very sensitive to the nature of the problem. They may provide highly satisfiable results for a given class of problems, while they may fail to exhibit an efficient performance in others. In the 1990s, Wolpert and Macready (1997) provided a set of theorems, called no free lunch theorems, to explain that there is no unique heuristic or meta-heuristic approach capable of exhibiting equally better performance than all others for all problem types. According to these theorems, although a specific heuristic or meta-heuristic algorithm can perform better than others for a given problem, there are other classes of problems in which the same algorithm might show weaker performance than other algorithms. Generally speaking, these theorems state that the average performance of all heuristic and meta-heuristic algorithms for different static and dynamic optimisation problems is almost the same. Hence, the main aim of algorithm designers should be the design of a heuristic or meta-heuristic approach that is capable of solving most types of problems in an efficient way, rather than all types of problems. This was the motivation for the researchers to develop a significant number of meta-heuristics to solve different types of problems in recent decades.

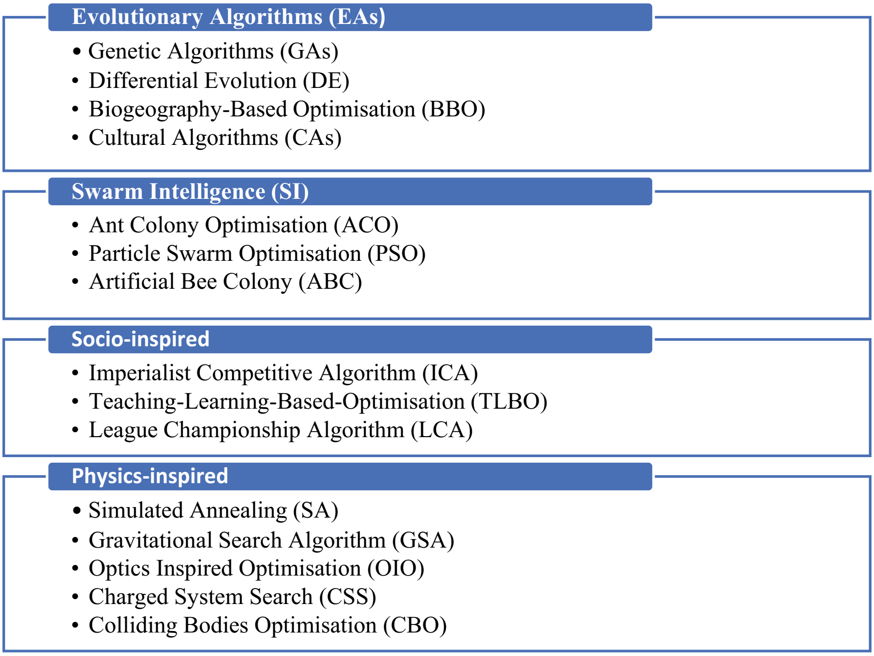

Fig. 1.1 Taxonomy of meta-heuristics based on their sources of inspiration

The emergence of meta-heuristic algorithms goes back to the 1960s, when Holland (1975) and his fellow researchers at the University of Michigan developed Genetic Algorithms (GAs) (Goldberg 1986) based on the principle of natural selection in Darwin's evolutionary theory of biological species. Following the successful application of GAs, a significant number of new meta-heuristics inspired by different phenomena in nature have been developed, such as Simulated Annealing (SA), Tabu Search (TS), Ant Colony Optimisation (ACO), Differential Evolution (DE), Particle Swarm Optimisation (PSO), Harmony Search (HS), Biogeography-based Optimisation (BBO), and Teaching–Learning-Based Optimisation (TLBO). Meta-heuristics can be categorised based on their sources of inspiration. In the subsequent sections, a taxonomy of meta-heuristics are presented based on their sources of inspiration as shown in Fig. 1.1.

1.3.4 Evolutionary Algorithms (EAs)

EAs have been developed mainly based on Darwin's evolutionary theory of biological species. GAs developed by Holland (1975) is the pioneering and most prominent EA. The GAs adapt the biological evolutionary concepts, such as mutation, cross-over, and selection, to stochastically perform the search process for optimal solutions. In

GAs, the solution for the problem is encoded as a chromosome consisting of a set of genes that are evolved over different generations. According to the principle of natural selection in the theory of evolution, the genes of individuals with better adaptation ability are more likely to be transmitted to the next generations. Mutation refers to the errors and damages that can happen in genes during the transmission process, which can potentially lead to random changes in genes and genetic diversity in the population (Newson et al. 2007). GAs model these processes to gradually evolve a population of solutions. These algorithms have found a vast amount of applications in real-world problems (Katoch et al. 2021).

The DE algorithm proposed by Storn and Price (1997a) is another EA. Storn and Price developed DE when they were trying to modify the Genetic Annealing algorithm (Price et al. 2006a). The modifications of the Genetic Annealing algorithm resulted in a new mutation equation on which DE has been developed. The DE uses three main operators, including mutation, cross-over, and selection, in which the evolutionary process is modelled based on the weighted differences between the individual vectors in the search space.

BBO introduced by Simon (2008) is another EA that was developed based on the probabilistic mathematical models in biogeography science (Wilson and MacArthur 1967). The algorithm assumes the solution candidates as a set of habitats in which the species perform the emigration and immigration processes. In BBO terminology, the position of each habitat is represented by Suitability Index Variables (SIVs), and the corresponding fitness is indicated by Habitat Suitability Index (HSI). BBO simulates the emigration and immigration processes based on the migration and mutation operators. The migration and mutation operators use a set of emigration and immigration rates which are obtained based on migration models from biogeography science. A recent literature survey performed by Ma et al. (2017) reveals that BBO has attracted relatively remarkable attention from different research communities over the past decade.

CAs developed by Reynolds and his colleagues can be categorised as an EA, which have been developed based on the biocultural evolution theory (Reynolds and Rolnick 1995a, b; Ostrowski and Reynolds 1999). According to this theory, genes and culture can be viewed as two interacting forms of inheritance that form the overall evolution of the human species. This means that human behaviour is a product of two different and interacting evolutionary processes, genetic and cultural evolutions. CAs have some features that make them unique in comparison to other EAs. The conventional EAs work on the population level in which vertical genetic inheritance mechanism is modelled. Contrary to conventional EAs, CAs employ a dual inheritance system that is consisted of two parallel spaces, population and belief spaces. The population space models the genetic evolution and represents the micro-evolutionary level. While the belief space simulates the cultural evolution process of individuals in the population space and can be viewed as a macro-evolutionary level. The belief space includes a network of cultural knowledge that can be used by individuals during the decision-making process to find better results for the problem. During the solution-finding process, the population and belief spaces exchange information based on communication protocols. Chapter 2 will present the theoretical background on cultural evolution, and Chap. 3 will explain the full details of CAs.

1.3.5 Swarm Intelligence (SI)

SI refers to the family of mate-heuristic algorithms inspired by the collective behaviour of a group or swarm of species in biological systems, such as ants and birds. The basic characteristic features of these cooperation models observed in biological systems are their decentralised and self-organised nature. In these systems, a set of agents interact with each other and their environment. Their behaviours are not controlled by any centralised forces, and they are free to locally and globally interact within the swarm in a relatively random manner. Despite the simple individual behaviour of agents, their communication and collective interactions lead to the emergence of a global complex behaviour that is much more complicated. SI meta-heuristic techniques stimulate this intelligent behaviour to model the search process for the global optima in optimisation problems.

ACO developed by Dorigo et al. (2006) is a SI technique inspired by the foraging behaviour of ants in finding the shortest path between their nest and food sources. The ants indirectly communicate with each other through a chemical process, called pheromone deposition. They can deposit pheromone on the ground to inform other ants about potential danger or trail the paths between their nest and food sources. In ACO, the solution candidates are modelled as ants which deposit pheromone on parts of the search that could potentially result in better fitness values.

PSO originally introduced by Kennedy and Eberhart (1995) is another popular SI algorithm. The algorithm imitates the collective behaviour of a bird flock or fish school in finding food sources. In PSO, each solution candidate is represented as a particle in the search space that moves with a variable velocity. PSO considers the personal experience gained by every single particle as well as global experience acquired by the whole swarm to update the positions and velocities of particles in each iteration. The algorithm has been a quite popular approach for solving a wide variety of problems and, occasionally, its different variants have been developed by researchers (Parsopoulos and Vrahatis 2010; Mirjalili et al. 2020).

1.3.6 Socio-inspired Algorithms

The socio-inspired meta-heuristic algorithms adopt the social learning behaviours observed in real human societies to perform the search process for global optima. Imperialist Competitive Algorithm (ICA), TLBO, and League Championship Algorithm (LCA) are examples of this category of algorithms. ICA introduced by

Atashpaz-Gargari et al. (2007) imitates the imperialist competition process in political science, TLBO developed by Rao et al. (2011) simulates the interactions between teacher and students in a class, and LCA presented by Kashan (2009) models the competition of teams in a sports league to achieve most favourable outcomes. The socio-inspired algorithms have been employed to solve various problems in literature (Hosseini and al Khaled 2014; Rao 2016; Jalili et al. 2017; Husseinzadeh Kashan et al. 2018).

1.3.7 Physics Inspired Algorithms

Several meta-heuristics model a given physical phenomenon observed in nature. The algorithms in this category adopt the governing equations in physics to build the search operators. Examples include, but are not limited to, SA, Gravitational Search Algorithm (GSA), Optics Inspired Optimisation (OIO), Charged System Search (CSS), and Colliding Bodies Optimisation (CBO).

SA (van Laarhoven and Aarts 1987) is a single point meta-heuristic algorithm inspired by the annealing process in metallurgy in which a material is being heated and gradually cooled down to improve its physical and chemical properties. GSA introduced by Rashedi et al. (2009) metaphorically models Newtonian gravity and the laws of motion into the searching process, in which a set of particles interact with each other based on their masses. In GSA, the particles with given masses calculated based on the fitness values represent a set of solutions for the problem. In a similar approach, Kaveh and Talatahari (2010) adopted the Coulomb law from electrostatics and the Newtonian laws of mechanics to develop a new meta-heuristic, called CSS. In CSS, a set of charged particles are solution candidates for the problem that impose electrical forces on each other according to Coulomb's law. CBO is another physics-inspired algorithm presented by Kaveh and Mahdavi (2014), in which the governing equations in the one-dimensional collision process of moving bodies are adopted to design a new meta-heuristic.

OIO is a population-based algorithm that models the optical phenomena observed in spherical mirrors (Kashan 2015; Jalili and Husseinzadeh Kashan 2018, 2019). The algorithm uses the governing equations in optical physics which explain how the images are formed in concave and convex mirrors. OIO treats the surface of the objective function as a wavey reflecting surface consisting of concave and convex parts on which the artificial images are formed. The algorithm models each solution candidate as an artificial light point that forms an artificial image on the function surface which represents a new solution for the problem.

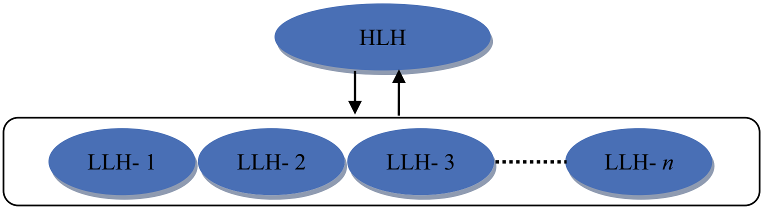

Fig. 1.2 General framework of hyper-heuristics

1.3.8 Hyper-Heuristics

According to the no free lunch theorems explained in Sect. 1.3.3, there is no single meta-heuristic algorithm capable of exhibiting equally efficient performance for different problems with different features in the search space. During the past decade, this has been intensified the efforts to develop more general and efficient algorithms that can be applied to most types of problems with a relatively stable performance. The emergence of hyper-heuristics is an example of such efforts. The logic behind the hyper-heuristics is to perform the search process in the heuristic space, rather than in the solution space (Burke et al. 2010). In the hyper-heuristic framework, a set of Low-Level Heuristics (LLHs) perform the search process in the solution space which are controlled by a High-Level Hyper-heuristic (HLH) strategy as shown in Fig. 1.2. In the high-level strategy, the algorithm decides which heuristic in the lower level should be applied to perform the search process at a given time in the solution space. The high-level strategy provides the learning capability for the algorithm to apply the most efficient heuristic strategy depending on the feedback received from the application of different LLHs in the past. The learning process in hyper-heuristics can be online or offline (Burke et al. 2013a). It is also possible to construct a hyper-heuristic without learning capability, in which the LLH is selected in a purely random manner. The learning process in hyper-heuristics can potentially make them capable of dealing with different types of problems. The LLH can be any constructive or improvement heuristics. In literature, various HLHs have been developed, such as choice function, greedy selection, Multi-Armed Bandit (MAB), Hidden Markov Model (HMM), and Monte Carlo Tree Search (MCTS) methods (Choong et al. 2018). Although most of the hyper-heuristics in literature employ heuristics in the lower level, population-based meta-heuristics can also be used in the lower level to perform the search within the solution space.

1.4 Summary

This chapter provided a brief review of deterministic and stochastic optimisation approaches. The chapter started with a short overview of conventional optimisation approaches available from applied mathematics. The conventional approaches use gradient information of objective and constraint functions to search and locate the optimum solutions. Although the conventional approaches are efficient and primary options to solve relatively simple problems, their application to highly nonlinear problems is challenging from different perspectives.

Modern stochastic heuristic and meta-heuristics optimisation methods are efficient tools to deal with the "black-box" problems in which the objective and constraint functions cannot be expressed as explicit functions of decision variables. These approaches do not necessarily guarantee the achievement of global optimum for different problems. Rather, they are expected to provide near-optimal solutions for a certain range of problems, for which there is no available algorithm to find the optimal solution in polynomial time. The chapter discussed the no free lunch theorems in stochastic optimisation which state that although a specific heuristic or meta-heuristic algorithm can perform better than others for a given problem, there are other classes of problems in which the same algorithm might show weaker performance than other algorithms. According to these theorems, the main aim of algorithm designers should be the design of a heuristic or meta-heuristic approach that is capable of solving most types of problems in an equally efficient manner, rather than all types of problems.

During the past decades, a significant number of meta-heuristic algorithms have been developed to solve optimisation problems in different branches of science and technology. Based on their sources of inspiration, the chapter categorised meta-heuristics into EAs, SI, socio-inspired, and physics-inspired algorithms. It was discussed that CAs belong to the category of EAs that model the biocultural evolution theory into the searching process for global optima. In the eyes of biocultural evolution theory, genes and culture are two interacting forms of inheritance that form the overall evolution of the human species. The CAs employ a dual inheritance system that makes them unique in comparison to other EAs. As the main focus of this book is on CAs, the theoretical background on biocultural evolutionary theory will be discussed in Chap. 2, and the full algorithmic details of CAs will be presented in Chap. 3.

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