Lecture Notes in Mechanical Engineering

Vijay Kumar Gupta
C. Amarnath
Puneet Tandon
M. Zahid Ansari *Editors*

Recent Advances in Machines and Mechanisms

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Preface

This book presents the select peer-reviewed proceeding of the 5th International and 20th National Conference on Machines and Mechanisms (iNaCoMM 2021) organised in the series of biennial conferences under the aegis of the Association for Machines and Mechanism (AMM), which is the Indian national affiliate of the International Federation for the Promotion of Mechanism and Machine Science (IFToMM). The main objective of AMM is to contribute to mechanical design at all levels starting from academic projects to industrial production, thus enhancing the quality and the reliability of indigenous machines. This conference brought together more than hundred researchers, industry experts and students working in various aspects of design and analysis of machines and mechanisms on a single platform for discussion and sharing of the knowledge and expertise.

The chapters included in this book represent broad topics including kinematics and dynamics of machines, compliant mechanisms; gear, cams and power transmission systems; mechanisms and machines for rural, agricultural and industrial applications; mechanisms for space applications; mechanisms for energy harvesting; robotics and automation; human centric robotics; soft robotics; man–machine system, mechatronics and micro–mechanisms; CAD and CAGD; control of machines; vibration of machines and rotor dynamics; acoustic and noise; tribology; condition monitoring and failure analysis; fault diagnosis and health monitoring; biomedical engineering; and composites and advanced materials. Given the broad contents, the book is expected to be a valuable resource for science and engineering students, researchers and industrialists interested in carrying research and development in advanced research areas of mechanical engineering and material science in general and in machines and mechanisms in particular.

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Jabalpur, India — Prof. Puneet Tandon
Jabalpur, India — Prof. Vijay Kumar Gupta
Jabalpur, India — Dr. M. Zahid Ansari

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Prof. C. Amarnath is a Mechanical Engineering Graduate of 1968. He earned his Ph.D. in the area of Mechanism Synthesis and Design in 1976 and joined IIT Bombay as a faculty member soon after. He has held several senior positions like Head of Department and Dean at IIT Bombay. He also headed SINE, the Technology Business Incubator of IIT Bombay. Prof. Amarnath was instrumental in founding and is the current President of the Association for Machines and Mechanisms, the national body affiliated with the International Federation of Mechanism and Machine Theory. He has been associated with various Government of India organizations like DST, DSIR, TIFAC and Chairs some of their committees. Prof. Amarnath is currently working as Emeritus Professor at IIT Bombay, Cummins Chair Professor of Mechanical Engineering at Cummins College of Engineering, Pune and Visiting Professor at IIT Dharwad.

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Synthesis of Mechanisms

Studies on Coupler Curves of a 4-Bar Mechanism with One Rolling Pair Adjacent to the Ground

Abhishek Kar and Dibakar Sen

Abstract This paper presents an expository study of the shapes of the coupler curve of a four-bar mechanism with one of its fixed pivots replaced with a rolling pair. Such rolling pairs provide the advantage of being friction and clearance free. In this paper, circle-on-circle and circle-on-line type of rolling pairs have been explored. This arrangement introduces three additional design variables to study the continuous change in the shape of the coupler curve of a conventional 4-bar mechanism. For a given input rotation at the crank, the mechanism does not have a closed-form solution for its configuration. However, providing input rotation at the rolling link allows easy derivation of a closed-form solution for both branches of the configuration. The tracing of the coupler curves is done for arbitrary radius ratios for the rolling link and choice of coupler point on the coupler link. A computer program has been written to study and visualize the coupler curves and its properties; the program not only finds the closure configurations but also identifies the situations of non-closure. Illustrative examples show variety and complexity of coupler curves which are not achieved in linkages. Although these coupler curves of mechanisms with rolling pair are transcendental in nature, the velocity states of the systems are easily derivable. Since the point of contact of the rolling pair is the instantaneous centre for the rolling link with respect to the ground, the velocity of the coupler point is determined using Kennedy's theorem. This helps in characterizing the occurrence of cusps in the coupler curves. The paper presents the geometric conditions for components, crunodes and cusps in the coupler curve with illustrative examples.

Keywords Coupler curves · Planar linkage · Kinematics

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1 Introduction

Variety of mechanisms used in machinery exploits the global and local geometric aspects of the coupler curves. Apparent complexity of form of the coupler curves of a simple 4-bar mechanism has intrigued mathematicians and kinematicians alike since the mid-nineteenth century [1–6]. A comprehensive review of the development of knowledge and application of coupler curves in design is available in [7–9]. Degree of the coupler curve has a strong bearing on the variety and distribution of features of interest in a coupler curve viz. a 4-bar coupler curve cannot have more than 3 cusps or crunodes because it is a 6-degree curve. 6-bar and higher chains offer larger variety [10–12] in terms of their degree, circuits, etc. Geared 5-bar mechanisms, although kinematically equivalent to a 6-bar Stephenson chain, produce many intriguing coupler curves depending upon the gear ratios chosen [13, 14]. Systematic generation of coupler curves with specific geometric properties, as commonly utilized in designs, is presented in [15]. The above studies were presented with an emphasis on their analytical formulation and computer implementation in the earlier days of the use of computer-graphics! Although numerous techniques of optimal synthesis of coupler point path for a given mechanism are available in literature, dedicated studies on the nature of coupler curves are not found in contemporary literature. This paper studies the variation in the *coupler curve of a 4-bar mechanism resulting from replacing one of its joints with a rolling pair*.

Frictionless mechanical devices such as rolamite and roller-band devices as mechanical embodiment of rolling between moving rigid bodies were invented long ago [16–18]. In [19], we can see the emergence of a band-constrained rolling pair and its application for a prosthetic knee. More recent applications of rolling joints are reported in [20–23]. However, a comprehensive kinematic treatment of mechanisms comprising of rolling pairs is available in [24]. The above studies emphasize the engineering advantages of roller joints. However, these studies do not emphasize the implication of roller pairs on the kinematics of the mechanism. The study presented in this paper focuses on the geometric implication on the coupler curve of a 4-bar mechanism with a rolling pair.

In any mechanism, except for the driving link, most other links have a limited kinematic range of motion; joints undergo varying loading due to the inherent nature of motion transmission in a linkage. These are not favourable conditions for conventional bearings. From a kinematics point of view the locus of a point on a link bearing a rolling pair is a transcendental curve viz. trochoid, involute, etc. Hence, the locus of points on other links is also likely to overcome the limitations on shape of algebraic curves produced by linkage mechanisms. The above observations motivated the authors to explore use of rolling pairs on the kinematic behaviour of a mechanism in terms of the coupler curve generated by it.

2 Kinematic analysis

Rolling pairs have been embodied in literature as rolamites (Fig. 1a) or tendons (Fig. 1b), both of which employ a set of presumably flexible and inextensible members for the purpose of articulating the rigid members forming the pair. In the present work, we propose to replace one of the fixed pivots of a 4-bar mechanism with a rolling joint as shown in Fig. 2. The dual tendon arrangement constraining the motion of two convex surfaces as shown in Fig. 1b ensures relative rolling, without slipping over each other. Since gearing ensures rolling motion of the associated pitch circles, *rolling pairs can also be implemented as gears.* The distance between B_0 and O, the centres of curvature of the moving and fixed profiles respectively is a constant when both the contacting profiles are circles. Hence, it is possible to consider B_0 and O as fixed pivots to obtain a direct kinematic inversion of the proposed mechanism; this inversion is nothing but the geared 5-bar mechanism studied in [13, 14]. In this inversion, comparing with Fig. 3, OB_0 and B_0 are fixed pivots and the locus of A_0 and B are circles (algebraic curves); therefore, the coupler curve of a point on link BB_0 is an algebraic curve. However, when link A_0O is fixed, instead of B_0B, although the locus of A is a circle, the locus of B is a trochoid which is a non-algebraic curve. Thus, the locus of C on AB is, theoretically, a non-algebraic curve. Thus, the analytical characteristics of coupler curves studied in the proposed mechanism is different from the ones available in literature, including those of the geared 5-bar mechanisms.

2.1 *Kinematics of the Rolling Pair*

Rolling phenomenon between two rigid bodies in point contact is said to happen when during any finite interval of motion, the length of the trace of the point of contact on both the bodies are equal; when the lengths of the trace are unequal, then the body containing the shorter trace is said to have skidded and the one with longer length slipped! Rolling in plane is characterized by location of the instantaneous centre of velocity being at the point of contact. Two rolling pairs under consideration are shown in Fig. 2 wherein (a) and (b) illustrates change of configuration when a circular profile of a rigid body rolls on a circular profile of another rigid body, and

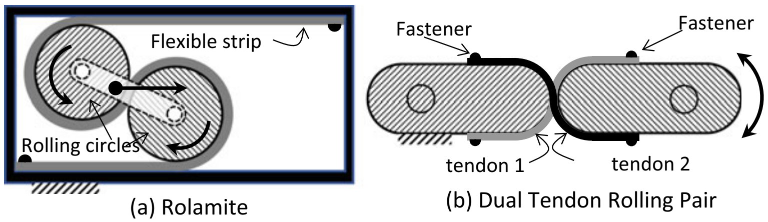


Fig. 1 Embodiments of two rolling pairs

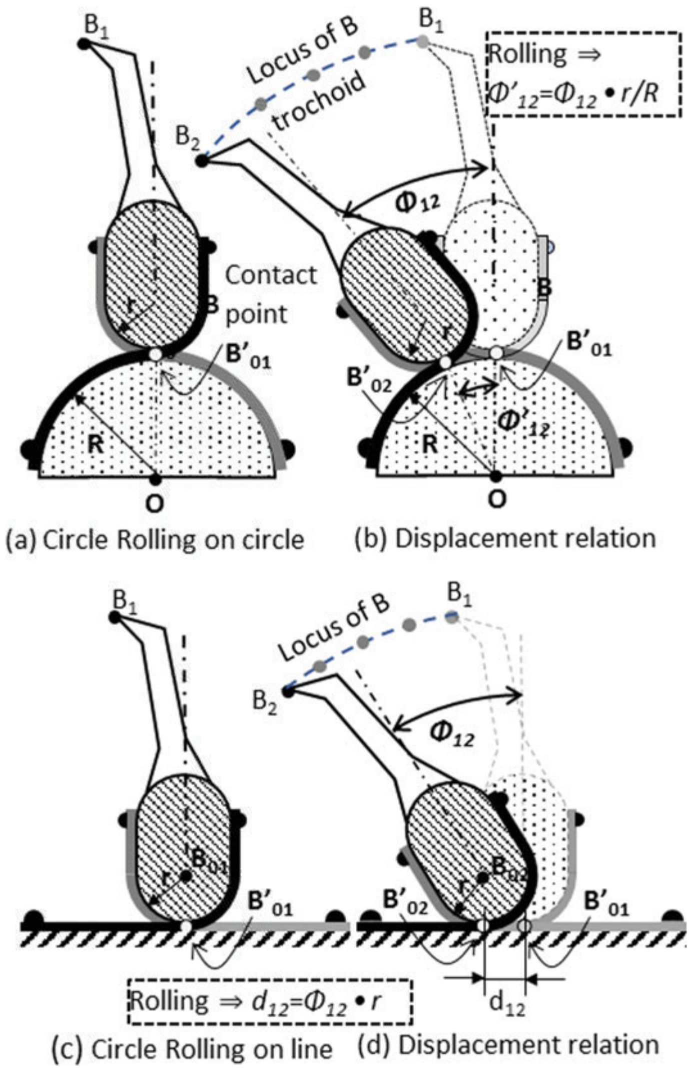


Fig. 2 Configuration of a circle-based rolling pair

(c) and (d) illustrates the situation for a circle rolling over a straight profile. Actual embodiment of the constraints to ensure rolling is unimportant in the analysis. The condition of equal length of the trace between the points of contact B'_{01} and B'_{02} relates the change of orientation, ϕ_{12} of the moving body with respect to the fixed body. The displacement of the point of contact is given by ϕ'_{12}, in case the fixed body is a circle, and d_{12} for the straight one. If the radii of the moving and fixed circles are r and R, respectively, it is easy to see that,

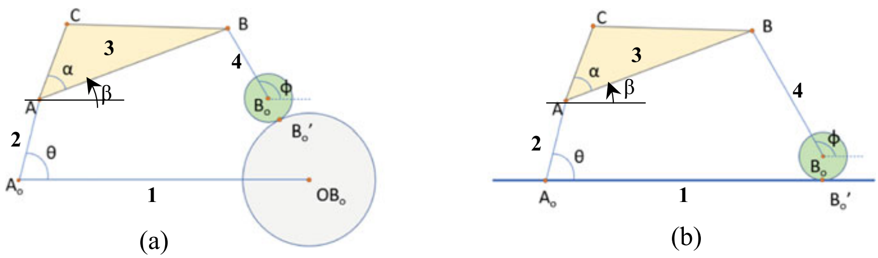


Fig. 3 Coupler point in the 4-bar with a rolling pair

$$\phi_{12}^{'} = \phi_{12} \cdot \frac{r}{R} \tag{1}$$

and,

$$d_{12} = \phi_{12} \cdot r \tag{2}$$

The locus of any point B on the moving body is a trochoid. The instantaneous centre of velocity of the moving body is given by the instantaneous point of contact. These two observations are important for the kinematic analysis of the mechanisms containing these rolling pairs.

2.2 Position Analysis of the Mechanism

In this work, we are interested in the two mechanisms shown in Fig. 3a, b which are obtained by adding a dyad, A_0AB, to the rolling pairs shown in Fig. 2a, b, respectively. In conventional 4-bar mechanisms, A_0 and B_0 is fixed; hence, the locus of A and B are circles. Input angle θ gives A directly; B is located at the intersection of two circles centred at A and B_0, and radii AB and B_0B, respectively. The result is obtained easily either geometrically or algebraically by solving a quadratic equation.

In Fig. 3 locus of B is a trochoid therefore, considering input at A_0 requires one to find the intersection point between a circle and a trochoid, as shown in Fig. 4a. This problem cannot be solved in closed form. Hence, we consider an inversion, wherein the output angle, ϕ, is assumed to be known and the corresponding input angle, θ, is determined. Link-4 is rolling over a fixed circle with centre at O and radius R. The change in configuration for link-4 has both rotation (ϕ_{12}) and translation ($B_{01}B_{02}$) components, which determine the location of B on the trochoid. For a known location of B (say B_1), locations of A (A_1 and A'_1) are obtained at the intersections of the two circles as shown in Fig. 4b. This formulation has a closed-form solution, as it involves circles alone.

Let A_0 be the origin and the centre of the fixed profile O be given. Let AA_0 be l_2, AB be l_3, BB_0 be l_4, radius of fixed profile be R and radius of rolling profile be r.

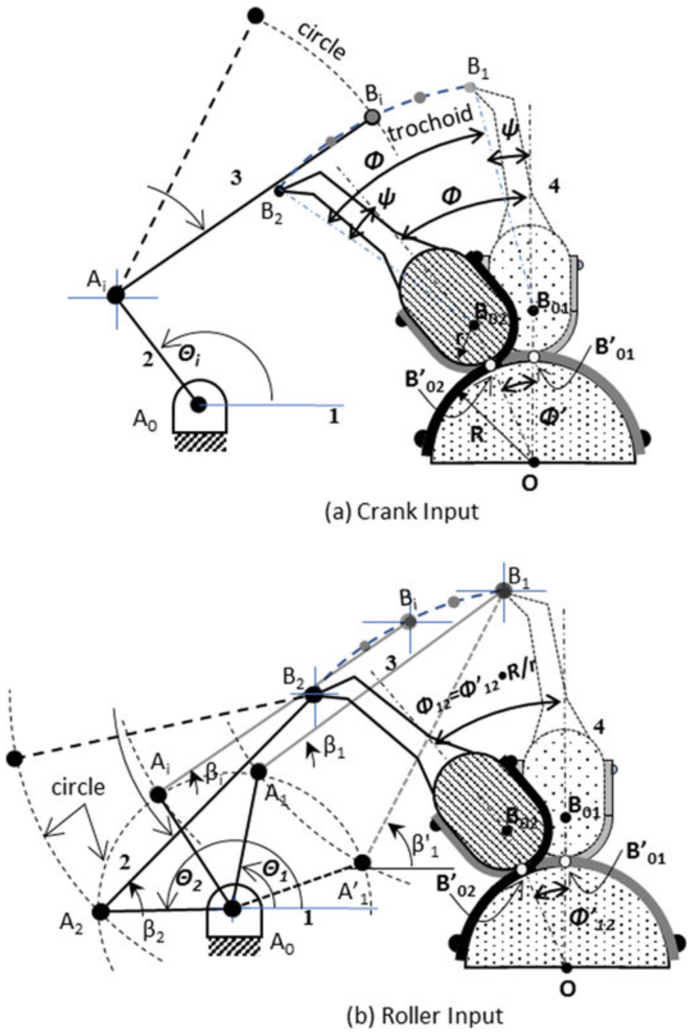


Fig. 4 Displacement analysis of a 4-bar with a rolling pair

Note that the contact point, B'_0, and the centres of the two contacting profiles, O and B_0, are always collinear. A_0, O and the initial location of contact, B'_{01} are given. Let BB_0 make an angle ψ with $B_0B'_0$ and $R = \left\|B'_{01} - \mathrm{O}\right\|$.

The homogeneous transformation matrices $R(\alpha, P)$ and $T(t)$ for rotation, α, about a given point, P (x, y), and translation by a vector $t(t_x, t_y)$, respectively, are given as below.

$$\mathbb{R}(\alpha, P) = \begin{bmatrix} \cos\alpha & -\sin\alpha & x\cdot\cos\alpha - y\cdot\sin\alpha - x \\ \sin\alpha & \cos\alpha & x\cdot\sin\alpha + y\cdot\cos\alpha - y \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\boldsymbol{T}(t) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$\text{Let,} \quad \hat{v} = \frac{1}{R}\left(B_{01}^{'} - O\right) \tag{5}$$

$$\text{Then,} \quad B_{01} = O + (R + r)\hat{v} \tag{6}$$

$$B_1^{'} = O + (R + r + l_4)\hat{v} \tag{7}$$

$$\text{and} \quad B_1 = \mathbb{R}(\psi, B_{01})B_1^{'} \tag{8}$$

Let link-4 be rotated by an angle φ. Let us use then subscript 1 for current locations and 2 for new locations for the points reached because of rotating link-4 by φ.

$$\text{Let,} \quad \hat{k} = \left[\, 0\ 0\ 1 \,\right]^T \tag{9}$$

$$\text{Forthecircle}-\text{on}-\text{circlecase,} \quad B_{0_2}^{'} = \mathbb{R}\left(\varphi\cdot r\big/R, O\right)B_{0_1}' \tag{10}$$

$$\text{And,} \quad B_{0_2} = \mathbb{R}\left(\varphi\cdot r\big/R, O\right)B_{0_1} \tag{11}$$

$$\text{Forthecircle} - \text{on} - \text{linecase,} \quad B_{0_2}' = \boldsymbol{T}(\varphi\cdot r)B_{0_1}' \tag{12}$$

$$\text{And,}\ B_{0_2} = \boldsymbol{T}(\varphi\cdot r)B_{0_1} \tag{13}$$

$$\text{Then,} \quad B_2 = \boldsymbol{T}\left(B_{0_2} - B_{0_1}\right)\mathbb{R}\left(\varphi, B_{0_1}\right)B_1 \tag{14}$$

$$\text{Let,} \quad D = B_2 - A_0 \tag{15}$$

$$\text{Wherein,} \quad d = \|D\|, \hat{D} = D/d \tag{16}$$

$$\text{Define,} \quad e = \left(l_2^2 - l_3^2 + d^2\right)/(2d) \tag{17}$$

$$\text{Then, usingChace′sequation,} \quad \mathrm{A}_2 = \pm\sqrt{\mathrm{l}_2^2 - \mathrm{e}^2}\left(\hat{\mathrm{D}} \times \hat{\mathrm{k}}\right) + \mathrm{e}\hat{\mathrm{D}} + \mathrm{A}_0 \tag{18}$$

A_1 and A'_1 in Fig. 4b correspond to the two solutions from the '$\pm$' sign in Eq. (10). For the coupler point, C, let, $AC = l_c$, $\angle BAC = \alpha$ and C_2 be the new location of C

$$\text{Let,} \quad \beta_2 = \arg(B_2 - A_2) \tag{19}$$

$$\text{Then,} \quad C_2 = T(A_2)R(\alpha + \beta_2, A_2)\begin{bmatrix} 0 \; l_c \; 0 \end{bmatrix}^T \tag{20}$$

$$\text{Similarly,} \quad \beta_2^{'} = \arg\left(B_2 - A_2^{'}\right) \tag{21}$$

$$\text{And} \quad C_2^{'} = T(A_2)R\left(\alpha + \beta_2^{'}, A_2^{'}\right)\begin{bmatrix} 0 \; l_c \; 0 \end{bmatrix}^T \tag{22}$$

Thus, the location for the coupler point is determined for both the branches for an incremental input of $d\varphi$ at the roller pair from an arbitrary initial configuration. A representative set of coupler curves generated for the mechanism is shown in Fig. 5

3 Study of Geometry of the Coupler Curves

It can be observed in the representative map of the coupler curves (Fig. 5) that the shape is varied and sometimes complex. Hence, it is important to study the composition in detail (Fig. 6).

3.1 Components or Circuits of Coupler Curves

The coupler curves of 4-bar mechanisms are classified as bicursal and unicursal wherein the coupler curve has two closed components and a single component, respectively. In bicursal coupler curves, a coupler point is topologically constrained to trace one of the components, even if the components have mutual intersections. Hence, understanding the segmentation of components of coupler curves is important.

It may be noted that Eq. (10) has a solution only when the distance $A_0B > (AB + A_0A)$ and $A_0B < (AB - A_0A)$. If we consider an annular region formed by two concentric circles, Ω_{max} and Ω_{min}, with centre at A_0 and radii $r_{max} = (AB + A_0A)$ and $r_{min} = \|AB\text{-}A_0A\|$, respectively, the mechanism will have a closure configuration only when the trochoidal locus of B lies within it. At the intersection of the trochoid with Ω_{max}, the dyad A_0AB is fully stretched out, and at that with Ω_{min}, it is fully folded. Hence, these are ***singular configurations*** of the mechanism. In these two configurations $A_2 = A'_2$, and therefore, $C_2 = C'_2$; i.e. in these two configurations, the two branches of the coupler curve merge to give a ***dichromatic coupler curve*** wherein one segment of the closed curve is traced by one branch of solution while the

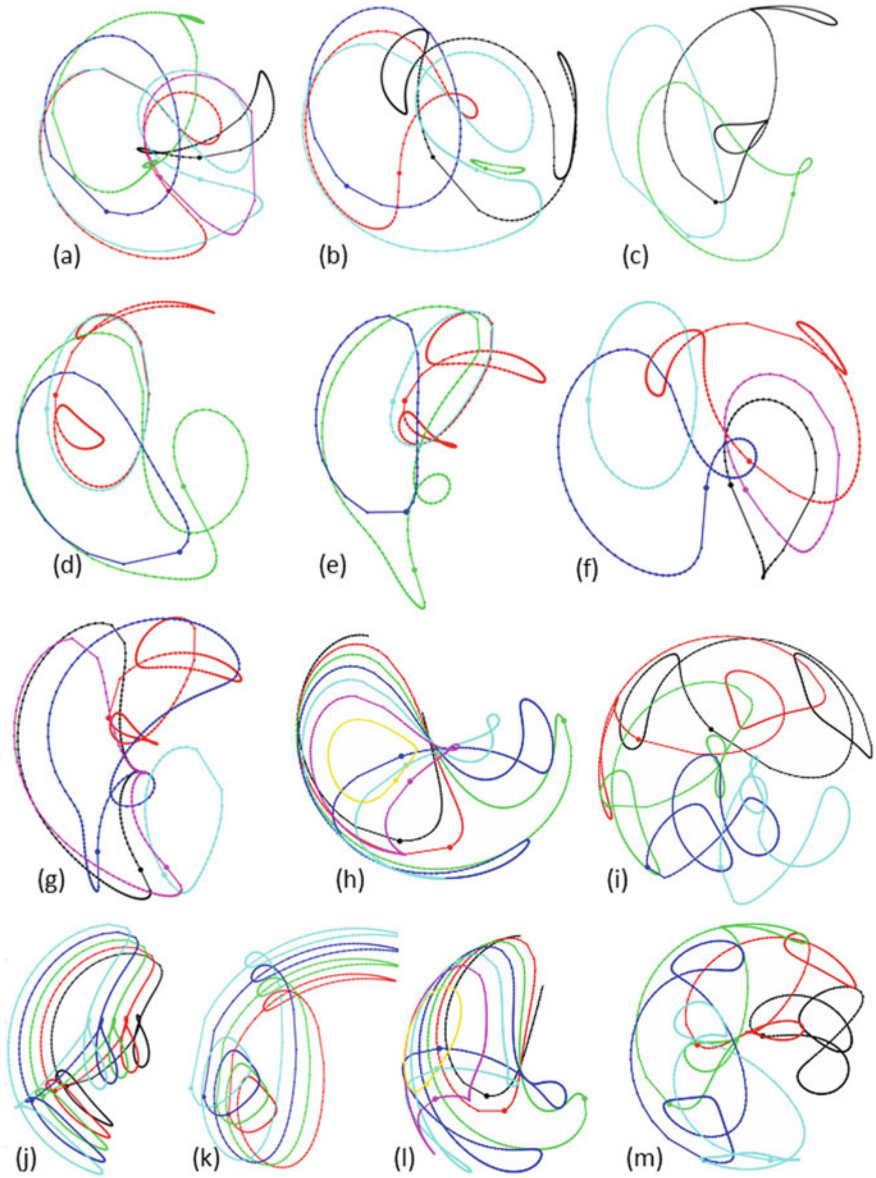
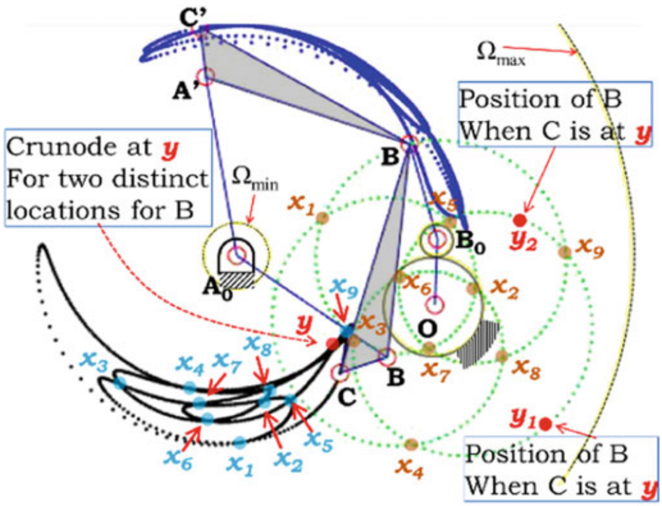
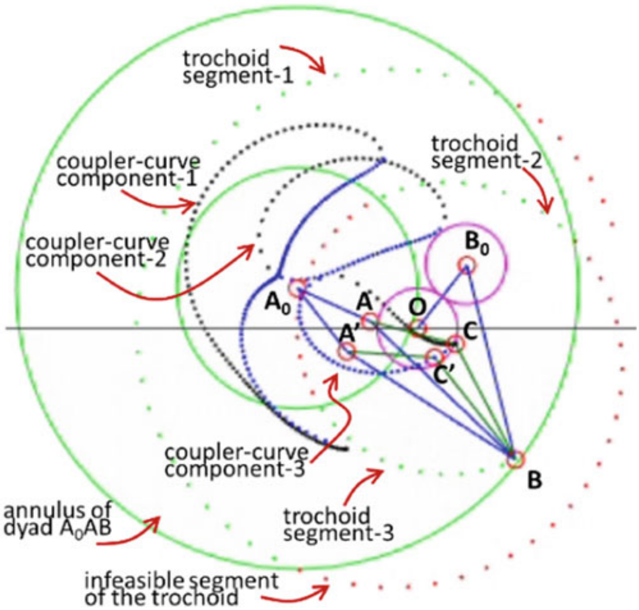


Fig. 5 An assortment of coupler curves of the mechanism



(a) monochromatic bicursal coupler curve with maps of crunodes



(b) dichromatic multicursal components of coupler curve

Fig. 6 Dyadic annulus and nature of components of coupler curve complementary segment is simultaneously traced by the alternate branch of solution of the coupler point. When all the points on a coupler curve are traced by the same branch of solution, we call it a ***monochromatic coupler curve***.

The trochoidal locus of B as a point on the rolling link is complex although it is symmetric about O; it potentially intersects $\Omega_{\min}$, $\Omega_{\max}$ and fixed circle of the rolling pair multiple times. If the trochoid is such that it does not intersect with $\Omega_{\max}$ or $\Omega_{\max}$ over the full range of motion, then the two branches of solution does not merge, and we get *two distinct monochromatic coupler curves* which are traditionally referred to as ***bicursal coupler curves.*** In the circle-on-circle case, the trochoid is bounded by an annulus whose centre is at O and radii are $(OB_0 + BB_0)$ and $\|OB_0 - BB_0\|$. *We get bicursal coupler curves only when the annulus of the trochoid is completely contained in the annulus of the dyad A_0AB.*

Let us consider a segment of the trochoid intercepted by the circles $\Omega_{\min}$ and $\Omega_{\max}$. Since at these configurations the branches merge, it will generate a dichromatic coupler curve. We get as many closed curves as the number of segments of the trochoid intercepted in the annulus of the dyad A_0AB by the circles $\Omega_{\min}$ and $\Omega_{\max}$. Since the number of component coupler curves is potentially more than two (unlike in the case of a conventional 4-bar coupler curve), we call it ***multi-cursal coupler curves.*** Each component is dichromatic because there is merging of the two branches at singularities.

The number of rotations of the rolling circle that gives a closed trochoid is the lowest common multiple (LCM) of the perimeters of the closed rolling profiles; for circular profiles, it the LCM of the two radii. Thus, B_0 may go around O multiple times before the trochoid closes. This has a bearing on the number of components in the multi-cursal map of the coupler curves. When the number of components is more, naturally the number of intersections a straight line would make with the set of coupler curves is more. Hence, unlike the coupler curves of linkage mechansisms (e.g. 4-bar, 6-bar, etc.) *there is no upper-bound on the number of intersections a straight line can make* with the coupler curve of a 4-bar mechanism with one rolling pair.

When the ratio of the radii of the rolling circle and the fixed circle is an irrational number, then the trochoid is theoretically an open curve of infinite length that would fill the area of the annulus defined by $(OB_0 + BB_0)$ and $\|OB_0 - BB_0\|$. Correspondingly, the coupler curve is also of infinite length that would fill an area. This filled-area coupler curve will have two components if the condition of bicursal curve is satisfied; otherwise, it will be a single area composed of infinite number of closed dichromatic coupler curves.

3.2 Crunodes in Coupler Curves

A crunode or self-intersection of a curve in plane is easy to perceive visually. From analytical point of view, although a coupler curve is a continuous map of the input

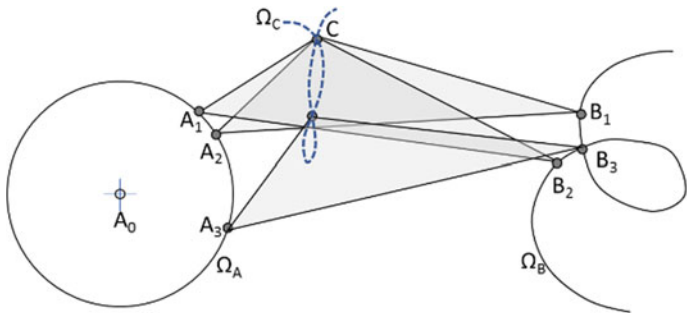


Fig. 7 Conditions of self-intersection

parameter in the Cartesian space through a 1-DoF mechanism, this map is not one-to-one. Each branch of configuration of the mechanism generates distinct curves which we are referring to as components. Since in each embodiment of a mechanism, only one of the branches is operational, only one of the components is generally available for a design. Thus, intersection among the components is not of much significance. There are theories available for self- intersection of a component coupler curve of a 4-bar mechanism. Since no point on link-4 is fixed in the present mechanism, those theories are not directly applicable. Figure 7 shows the situations when the point C will be at a crunode, wherein Ω_A, Ω_B, and Ω_C are the loci of the points A, B and C of the mechanism in Fig. 3, respectively. Ω_C is the coupler curve. Since Ω_B is a trochoid, geometric characterization of the situation is not possible.

In a 4-bar mechanism with a rolling pair as shown in Fig. 3, the location of the coupler point, C, will be same when the location of B is same. B is a point on the trochoid which is a potentially self-intersecting curve. The point of intersection is associated with distinct values of the parameter (ϕ) that generates the trochoid. Thus, ***for every point of self-intersection of the trochoid, there would be a self-intersection of the coupler curve, for each branch $\boldsymbol{\Omega_C}$ and $\boldsymbol{\Omega_{C'}}$, simultaneously***. Figure 6a maps all the crunodes of trochoid and coupler curve. This situation does not occur in conventional linkage mechanisms. It also shows a crunode on the coupler curve without a corresponding crunode on the trochoid.

Since crunodes on a trochoid does not have closed-form expression, a geometric characterization of this situation is also not possible. However, since the locus of B is a trochoid defined by r, R and l_4, it exhibits multiple crunodes. The number of crunodes in a coupler curve, therefore, is not restricted, unlike the algebraic coupler curves of a pure linkage mechanism.

3.3 Cusps in Coupler Curves

Although cusps are generally recognized as a sharp point in an otherwise smooth curve, its kinematic characteristic is that at this point on the coupler curve, the velocity

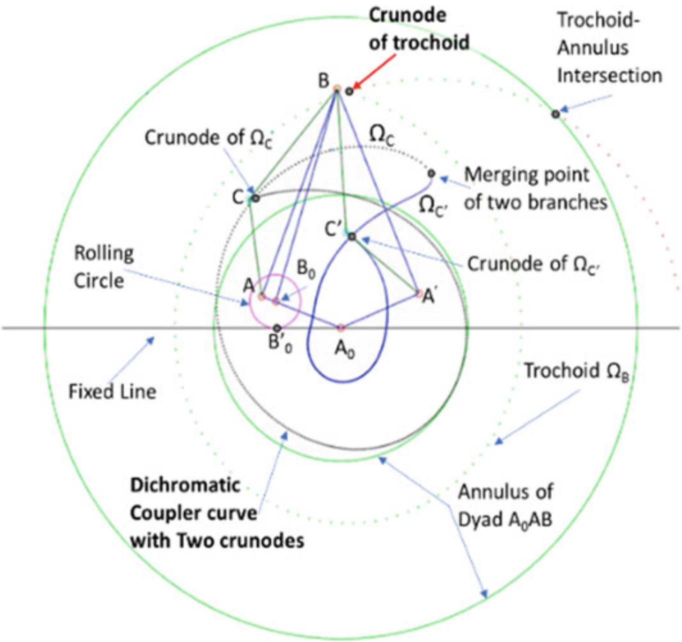
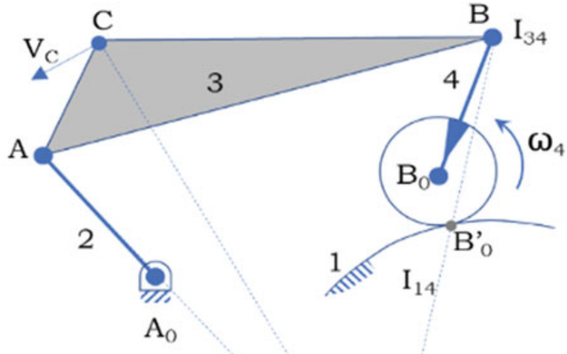


Fig. 8 Crunodes in a coupler curve

of the coupler point is zero for a non-zero input-velocity for the mechanism. Using Arnold-Kennedy's theorem of three instantaneous centres, we get (Fig. 9),

Fig. 9 Velocity of coupler point



$$\frac{\omega_3}{\omega_4} = \frac{I_{14}I_{34}}{I_{13}I_{34}} = \frac{BB'_0}{BI_{13}}$$

$$V_C = CI_{13} \cdot \omega_3$$

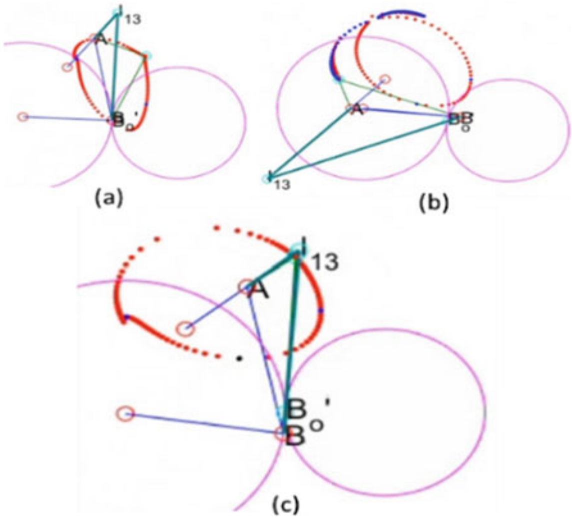


Fig. 10 Sharp and smooth cusps on Ω_C for $l_4 = r$

$$\frac{\omega_3}{\omega_4} = \frac{I_{14}I_{34}}{I_{13}I_{34}} = \frac{BB_0^{'}}{BI_{13}}$$

$$\text{And,} \quad V_C = CI_{13} \cdot \omega_3$$

$$\text{Thus,} \quad V_C = 0 \Rightarrow \text{either } CI_{13} = 0 or, \omega_3 = 0$$

$CI_{13} = 0 \Rightarrow$ Coupler point is coincident with instantaneous centre (Fig. 11),

And, $\omega_3 = 0 \Rightarrow BB_0^{'} = 0$ which is possible only when $l_4 =$ r (Fig. 10).

Note that a cusp occurs at a phase in a mechanism where $l_4 = r$ when the joint B coincides with the point of contact as the moving circle rolls over the fixed pivot; then the velocity of the coupler point is zero because the velocity of both ends of the dyad, A_0 and B, are zero. The situation is independent of the selection of the coupler point on link-3; hence, the *cusp will occur for the coupler curves of* ***all points in all branches*** *of the mechanism and as many times B touches the fixed profile* (circle or line), ***simultaneously***. This situation is unique for mechanisms with roller pairs and does not occur in conventional 4-bar mechanisms.

In an infinitesimal neighbourhood of a cusp, velocity vector of the coupler point typically has opposite directions. Motion in a mechanical system is continuous. Hence, it is understandable that the magnitudes of velocities of points in the neighbourhood of a cusp will be small. But, there is no causal relation between the directions of velocities of points in the neighbourhood of a cusp; when cusp occurs due to $CI_{13} = 0$, the direction of the tangent to the coupler curve is undefined; but, in case of $l_4 = r$, direction of the tangent is still given by the line perpendicular to CI_{13}. Figure 11c shows a special case when both the conditions are simultaneously satisfied giving a ***smooth cusp*** wherein the coupler point slows down as it approaches