Kazuhiko Nishijima

Masud Chaichian
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Quantum Field Theory

By Academician
Prof. Kazuhiko Nishijima

A Classic in Theoretical Physics

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ISBN 978-94-024-2189-7 ISBN 978-94-024-2190-3 (eBook)
https://doi.org/10.1007/978-94-024-2190-3

Originally published by Kinokuniya Company Ltd., Tokyo, Japan, 1987

Japanese original edition BA NO RIRON by Kazuhiko Nishijima published in 1987 by Kinokuniya Company Ltd., Tokyo

Back Cover: Photo of Prof. Nishijima, 2008 (courtesy of late Mrs. Hideko Nishijima)

This Springer imprint is published by the registered company Springer Nature B.V.
The registered company address is: Van Godewijckstraat 30, 3311 GX Dordrecht, The Netherlands

Foreword

In the Preface, Professors Masud Chaichian and Anca Tureanu describe in vivid detail how the English version of this book on quantum field theory by the great Japanese physicist Kazuhiko Nishijma evolved as a labor of love on their parts. Nishijima enjoyed visiting Helsinki and Finland (Who doesn't? I certainly do.) regularly and was often seen consulting a physics book in Japanese, of which he was actually the author. (I can tell you from personal experience that authors like to read their own books; I certainly find my books ridiculously clear and easy to understand.) Chaichian and Tureanu urged Nishijima to have the book translated into English. After Nishijima's death in 2009, they found a translator, had it proofread, improved the English, arranged for a publisher, and invited one of the most eminent theoretical physicists of our times, Professor Yoichiro Nambu, to write a Foreword.

After Nambu's untimely death in 2015, they asked me to step in, perhaps because I have also written a textbook on quantum field theory. Any reader qualified to read this book knows full well that I am far from the level of achievements attained by Yoichiro Nambu, for whom I have the greatest admiration as a person and as a physicist. Nevertheless, out of respect for Nishijma, Nambu, Chaichian, and Tureanu, I agreed to do so.

I believe that Dick Feynman said that learning physics is something like painting a house. (In case you have no experience painting houses, one coat is never enough. Only the most shoddy house painters would do that.) John Wheeler told me something similar: one should not imbibe physics like sipping water from a cup, but more like drinking from a fire hydrant. Anybody who tells you that he or she could learn quantum field theory by reading a single textbook is not only kidding you, but also himself or herself. During my four years in graduate school, I attended Julian Schwinger's course on quantum field theory four times, not because I was especially stupid, but because quantum field theory is a profound subject. In this spirit, I strongly urge students, and also physics professors who claim that they understand quantum field theory, to read this book in addition to whatever other books they have read or possibly in parallel with other books they are reading. I also find that this book treats some topics that are not discussed in detail, or omitted entirely, in more recent books on quantum field theory.

Santa Barbara, CA, USA Anthony Zee
July 2022

Preface to the English Edition

During several years of our collaboration with Prof. Kazuhiko Nishijma, we had noticed that always he was carrying a book in Japanese, which he consulted from time to time on different subjects. Prof. Nishijima used to visit us at the University of Helsinki regularly in the years 1985–2008. During one of his visits, he finally left a copy of that book with us in Helsinki to keep it for him for his future use. Not knowing which book it was, we asked about it and he explained that it was a book on Quantum Field Theory he had written. Upon our question whether it will be translated into English, Prof. Nishijima said that it was his wish to translate the book himself.

After Prof. Nishijima passed away in Tokyo on 15 February 2009, the book and Prof. Nishijima's wish sadly came to our mind and we reached a wishful thought of getting the book by all means translated into English as a tribute to Prof. Nishijima. Prof. Yoichiro Nambu, the 2008 laureate of the Nobel Prize in Physics, knew Prof. Nishijima very well and in the early 1960's was nominating him for the Nobel Prize for the discovery of the strangeness quantum number. Knowing this, we discussed the plan with Prof. Nambu, and he supported the idea with great enthusiasm, mentioning that he knew the book well and that he would like to write a foreword for its English edition. However, he also mentioned about the well-known difficulty of translating from Japanese to any other language of any group of English kind, and that it was for this reason that Nishijima wanted to translate his book himself.

We contacted Springer-Verlag about publishing the translated book and they consulted Prof. Nambu for a review. Prof. Nambu wrote a very strong evaluation, mentioning the transparency of the book and, in several parts, the originality of the presentations of the subject. The next task was to find a person who could translate the book, being a physicist with a good grasp of English, especially as regards the terminology used in the book. It was only thanks to the efforts of Prof. Nambu, who advertised the project in Japan, asking for volunteers, that a translator was found. Eventually, a young researcher in particle physics at Nagoya University, Dr. Yuki Sato, wrote to us mentioning that he would like to take the task merely as a sign of respect to Prof. Nishijima, one of Japan's greatest physicists who made significant contributions to theoretical physics.

The translation of the book by Dr. Yuki Sato, together with us the undersigned as editors going through it and checking the text and all the formulas again and again, as well as adding the reference list, took a few years and meanwhile Prof. Nambu passed away. Instead, Prof. Anthony Zee from University of California, Santa Barbara, most kindly accepted to write the foreword for the book.

After the translation of the book, another professional native English translator and a physicist by education, Mr. Stephen Lyle, went through the whole translated text and performed a more complete polishing of the text into a fine English language. We express our utmost thanks to Mr. Stephen Lyle for his devotion to perfection. We would like to thank Dr. Ramon Khanna, executive editor, and Ms. Christina Fehling, editorial assistant, at Springer Nature for their kind support and advice during the whole publication process, and also for their unlimited patience with several delays in our preparing the English edition of the book to send for print, with the aim that it will become a genuine tribute to Prof. Nishijima.

During the preparation of the English edition of the book we have received the encouragement of several physicists in Japan, to whom we are most grateful. Our special gratitude goes to Prof. Misao Sasaki from Kyoto University, and Prof. Kei-Ichi Kondo from Chiba University, for their constant support and much help during several years.

Helsinki, Finland
July 2022

Moshe M. Chaichian
Anca Tureanu

Preface of the Author

One of the important concepts in quantum mechanics is the dual particle-wave nature of matter. The same object is treated as a particle in one case and as a wave in another case, which is not the case in classical physics. This was one of the launch pads of quantum mechanics. Its typical example is light: in classical physics it was mainly described by the wave theory, although there existed also Newton's particle theory. However, once Einstein's photon hypothesis was proposed at the beginning of this century,[1] one could no longer claim that light is a wave or a particle. In this way quantum mechanics made its appearance. In its framework, the electron, which is thoroughly treated as a particle in the classical theory, is always described as a wave. With that the following problem comes up: How can we introduce the particle-wave duality in the same theory? Quantum field theory answers this question.

Quantum field theory was developed no later than the birth of quantum mechanics. It has experienced many changes up to the present. In the early stage, studies were entirely restricted to the area of electrodynamics, and turned out to be quite successful. Nevertheless, early quantum field theory reached a deadlock called the *problem of divergences*. However, with the advent of the renormalisation theory after the war, quantum electrodynamics based on perturbation theory reached completion. On the other hand, after pions were created by accelerators, the struggles of field theory started again. The reasons was that the perturbative approach can not be used for strong interactions, and one encountered the difficulty of extending the renormalisation theory to an approximation method beyond the perturbation theory. As a consequence, field theory was stagnant for a while; instead, the S-matrix theory based on dispersion relations became pre-eminent. In addition, after hundreds of "elementary" particles were created by accelerators, it became necessary to answer the question: What on earth are fundamental particles? And this is how the quark

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This is the only Preface written by Prof. Nishijima in Japanese to the 1st edition published in 1987 in Tokyo; the 8th edition with extended chapters was published in 1997 in Tokyo.

[1] "This century" means the twentieth century.

model appeared. On the other hand, from the studies of weak interactions, gauge theories were introduced. This book covers a survey of quantum field theory related to the developments above.

After the emergence of the gauge theory, there have been quite many developments toward new directions. For example, the study of monopoles, solitons, and instantons related to the topological nature of classical gauge theories; lattice gauge theory, in which gauge invariance is given the leading role to the detriment of Lorentz invariance; applications of grand unified theories to cosmology; more recently, supersymmetric theories, supergravity theories and superstring theories, and so on. These theories are developing; each of them is sufficiently vast already as to require an independent book. I myself do not yet understand them very well, so that I have omitted them in this book, considering that more adequate people will write about them.

Although I published “Fundamental Particles” (1963) [1] and “Fields and Particles” (1969) [2] with the W.A. Benjamin Corporation as text books for elementary particle theory and quantum field theory, the contents of those books are by now outdated, and the topics to be emphasized have also changed in time. However, as fas as the fundamentally invariable parts are concerned, this book is based on the two books above. Additionally, since I have had a reluctance to write a book in the Mathematical Library in ignorance of mathematics, I have decided to write the book based not on the mathematical logic but on the physical logic, on the advice of Professor Seizo Ito. It has been long time since the request to write this book, but Professor Ito has patiently waited, and I would like to express my appreciation for him.

In the 61 year of the Showa era (1986), at Rakuhoku,

Tokyo, Japan Kazuhiko Nishijima

Contents

Chapter 1
Elementary Particle Theory and Field Theory

In a broad sense, field theory can be defined as quantum mechanics with infinite degrees of freedom. This broad definition includes condensed matter physics, which treats non-relativistic many-body problems. However, in this textbook, we discuss the formalism of field theory in a narrow sense, namely, in the framework of relativistic quantum mechanics. The objects of field theory in this narrow sense are the so-called *elementary particles*, and the academic discipline that studies the properties of elementary particles is called *elementary particle theory*. In a way, we can thus say that field theory provides a "grammar" for describing the properties of elementary particles. Since the two theories were developed together, they cannot be clearly separated. Elementary particle theory is concerned with the types of elementary particles that can exist and the types of interactions that they undergo. In each case, there is a phenomenological level and a fundamental level. For instance, atoms, which had been thought to be indivisible, in fact consist of atomic nuclei and electrons, while the atomic nuclei themselves consist of protons and neutrons. Thus, the solution to the problem of what the truly fundamental particles are has changed over time. Correspondingly, the solution to the problem of what the fundamental interactions are has also changed. We do not know if there will ever be an end to the changing viewpoints. However, what has been established so far is that particles called *quarks* and *leptons* seem to be the fundamental particles, while the fundamental interactions are assumed to be gauge interactions. We will discuss later how concepts such as quarks and gauge interactions arised. For now, we shall just give an overview of the developments of elementary particle theory at the phenomenological level.

The concept of field is a generic one used to refer to dynamical variables defined at each point in space. Examples of fields in macroscopic physics are the temperature and the product of the density and the stream velocity of a fluid. In microphysics, only very few such dynamical variables actually survive at the classical level, and perhaps the only candidate is the electromagnetic field. After the birth of quantum mechanics a crucial question was how to apply quantum

K. Nishijima, *Quantum Field Theory*,
https://doi.org/10.1007/978-94-024-2190-3_1

mechanics to the electromagnetic field. As mentioned in the foreword, although light is described classically as an electromagnetic wave, at the microscopic level, it also exhibits particle-like features, as expressed in Einstein's photon hypothesis. Therefore, it is essential that the quantum mechanical description of the electromagnetic field should contain such particle-like characteristics. This line of research was pursued by Dirac in [3]. In 1928, Dirac [4] also rewrote the non-relativistic Schrödinger equation as a relativistic equation. This led to the development of quantum electrodynamics. In 1928, Heisenberg and Pauli [5, 6] formulated quantum electrodynamics in a systematic way, and this provided the starting point for field theory.

Dirac's relativistic theory of electrons predicted the existence of a positron, the antiparticle of the electron. Its existence was confirmed by Anderson in 1932 [7]. Similarly, the existence of the anti-proton, the antiparticle of the proton, was also expected. However, we had to wait until 1955 for confirmation of its existence [8]. On the other hand, again in nuclear physics, the discovery of the neutron by Chadwick in [9, 10] was of fundamental importance, leading to the picture of atomic nuclei consisting of protons and neutrons. Yet, the *birth of elementary particle theory* originated in Yukawa's meson theory.

1.1 Classification of Interactions and Yukawa's Theory

Current theories such as grand unified theories or the theory of supergravity start by recognizing that there exist a variety of interactions with different strengths. Among these interactions, the best known, apart from gravity, is the electromagnetic interaction which, as mentioned before, has been studied since the birth of quantum mechanics. However, from the early days it was known that there were lots of phenomena that could not be explained by the electromagnetic interaction alone. Beta-decay is an example. The lifetime of a radioactive atomic nucleus is longer than that of states excited electromagnetically. This implies that the interaction causing beta-decay is much weaker than the electromagnetic interaction. In fact, in order to explain the continuous energy spectrum of electrons emitted in beta-decay, Pauli introduced the neutrino hypothesis [11, 12]. Pauli had a very convenient excuse for proposing such a new particle, namely, saving the conservation laws of energy, momentum, and angular momentum.

Fermi substantiated Pauli's suggestion by proposing the *four-fermion interaction* [13] to describe beta-decay in field theory. This was the second interaction discovered after the electromagnetic interaction. It meant that field theory now had to deal with electromagnetic and weak interactions.

After the detection of the neutron in 1932 [9, 10], it gradually became clear that an atomic nucleus consists of protons and neutrons. It was now essential to understand the nature of the so-called *nuclear force* binding protons and neutrons to form atomic nuclei. Since the neutron is electrically neutral, the nuclear force cannot

Table 1.1 Elementary particles and their interactions

Elementary particles	Interactions
ν	Weak
e	Weak + electromagnetic
n, p	Weak + electromagnetic + strong
γ	Electromagnetic (carrier)

be the electromagnetic force. Just after the proposal of Fermi's theory, Ivanenko and Tamm calculated the nuclear force by assuming that it was produced by exchange of an electron–neutrino pair via Fermi's interaction [14–16]. It turned out that the calculated force was too weak to be at the origin of the nuclear force.

Consequently, the nuclear force had to be of an entirely different kind from the electromagnetic and the weak interactions. It was also understood that various particles have their own roles and take part only in certain interactions. This is shown in Table 1.1. The symbols for the fundamental particles are ν (neutrino), e (electron), n (neutron), p (proton), and γ (photon), while "weak", "electromagnetic", and "strong" stand for the weak interaction, the electromagnetic interaction, and the strong interaction (the nuclear force), respectively. The proton and the neutron are collectively dubbed *nucleons* in the sense of elements of an atomic nucleus.

In 1935, Yukawa put forward the *meson theory*, reflecting the principle that *all forces are mediated by fields* [17]. Since the electromagnetic interaction is mediated by the electromagnetic field, the strong interaction and the weak interaction must be mediated by fields, too. Hence, Yukawa introduced the *meson field* as the mediator of the strong interaction. Just as quantizing the electromagnetic field yields the light quantum (photon), it turns out that quantizing any field yields corresponding quanta describing their particle-like aspects. The quantum corresponding to the meson field is an elementary particle called a *meson*. One of Yukawa's most important contributions is that he clarified the relationship between the mass of a quantum and the range of action of a force. In general, the range of a force mediated by some field is given by the Compton wavelength of its quantum. For instance, the range of the electromagnetic interaction mediated by a massless photon is infinite. Since the quantum of the meson field mediates the nuclear force with a range of around 10^{-13} cm, the meson mass must be about 200 times heavier than the electron mass. This mass is midway between the nucleon and electron masses, hence the name meson. Moreover, from a phenomenological standpoint, the quantum of the field mediating the weak interaction described by Fermi's model at a single point must be very massive. Yukawa thought that both the strong nuclear force and the weak interaction might be mediated by the meson field. Shortly afterwards, Klein suggested that a different field should be introduced for the weak interaction [18]. In any case, at that time, such ideas were mere hypotheses. In order to prove them, mesons had to be produced. However, since no accelerator had enough energy to create them, the only way to detect them was by studying cosmic rays. Historically, Yukawa's particle was discovered after some bumps and detours.

1.2 The Muon as the First Member of the Second Generation

Theorists predict the existence of experimentally unknown particles, while experimentalists detect sometimes particles which theorists have not predicted. The relation between Yukawa's particle and the muon can be described in this way. Using a modern term, we can say that the world surrounding us consists of elementary particles of the first generation. This is because elementary particles of the second generation are in general unstable, except for the neutrino. Thus, the discoveries of elementary particles of the second generation always gave rise to confusion. Elementary particles such as the muon, the strange quark, and the charm quark are in the second generation. The muon was discovered first.

In 1937, a particle with mass about 100 MeV, the muon, was detected in cosmic rays [19]. For almost a decade afterwards, this particle was thought to be Yukawa's particle.

In 1945, Conversi, Pancini, and Piccioni found that the interaction between the muon and an atomic nucleus was almost as strong as the interaction involved in the muon's weak decay [20], whence they were of a similar (weak) nature. One consequence of this discovery was that the muon and Yukawa's particle, the pion, had to be different particles. One year later, Powell's group observed the following two-step decay process [21] on a photographic plate:

$$\pi^+ \to \mu^+ + \nu \,, \quad \mu^+ \to e^+ + \nu + \nu \,.$$

Here π, μ, and ν stand for the pion, the muon, and the neutrino, respectively. Although there are several different neutrinos, we do not distinguish them here.

A two-meson theory which admits the existence of two different mesons, the pion and the muon, was developed by Sakata, Inoue, and Tanigawa in [22, 23]. One year later, it was also proposed in the United States by Marshak and Bethe [24].

1.3 Quantum Electrodynamics

After the Second World War, quantum electrodynamics made amazing progress. Quantum electrodynamics, or QED, is the most typical field theory. By introducing a new weapon called *renormalization*, it became possible to calculate, within the framework of QED, the Lamb shift in a hydrogen atom, i.e., the energy shift between the $2s$ and $2p$ states, and the anomalous magnetic moment of the electron. The formulation of the covariant perturbative theory by Tomonaga, Schwinger, Feynman, Dyson, and others was instrumental in this success. At this point, QED achieved the position of being the most successful discipline in exact science. Moreover, the notion of renormalizability was closely connected to subsequent developments in field theory, and in particular to the discovery of gauge theories, as the most important guiding principle in field theories.

1.4 The Road from Pions to Hadrons

In 1948, in Berkeley, pions were created artificially for the first time [25]. Furthermore, in 1952, Fermi's group found a resonance state of the pion–nucleon system, which they named Δ [26]. This resonance appears in the reaction

$$\pi + \mathrm{N} \to \Delta \to \mathrm{N} + \pi \ ,$$

where N stands for a nucleon. The existence of resonances is characteristic of strong interactions of the pion–nucleon system at low energies.

It soon became clear that the covariant perturbative theory which was such a useful tool in QED had almost no utility for understanding strong interactions. This made theorists suspicious of the usefulness of field theory for strong interactions. In fact, determinations of the pion's spin or parity were based on a general invariance principle, and the covariant perturbative theory was completely useless for this purpose. Of course, a non-perturbative method was developed in field theory, but it was not successful because the prescriptions proposed by renormalization were intimately connected to the perturbation theory.

This situation meant that many preferred to investigate the pion–nucleon system without considering the details of strong interactions, but considering this system in its own right. One thing that came up was the idea of *isospin*. The concept of isospin invariance was introduced by Kemmer in 1938 to make a connection between the charge independence of the nuclear force and Yukawa's theory [27]. The isospin transformation is a symmetry in a virtual isospin space. It became an active field of study again when pions could be created in the laboratory. This led to the first two examples of a modern term, *flavour*.

Another idea that came up was the dispersion relation, which is deeply connected to causality in field theory. This gives the relation between the absorptive (imaginary) part of a scattering amplitude and its dispersive (real) part. The form of the dispersion relation does not depend on the details of the interactions. The important factors for dispersion relations are just the representations of spacetime and internal symmetries carried by the particle states, together with the type of mass spectrum the dynamical system possesses. However, the dispersion relations leave slight signatures of the original interactions. For instance, although the best known dispersion relation for the forward scattering of the pion–nucleon system includes terms with first-order poles with respect to some Lorentz invariant variables, the poles can be described by the renormalized coupling constant of Yukawa's interaction. In addition, the dispersion relation for electromagnetic form factors is given by the charges of particles adjusted by so-called *subtraction constants*. In each case, these constants are renormalized coupling constants in field theory.

Much can be learned from the dispersion relation method. First, comparing with perturbative calculations in field theory, we can know what type of renormalization is carried out preturbatively. Considering a given *S*-matrix element (a scattering amplitude, for example) at some order in perturbation theory, its absorption part can be expressed in terms of lower order S-matrix elements because of the unitarity of the S-matrix. In this evaluation, integrals have to be carried out in a finite phase-space, but divergences do not appear here. Next, we obtain the dispersion part of the S-matrix element from its absorption part using the dispersion formula. This corresponds to carrying out a *Hilbert transformation*, and this integral sometimes diverges. Such a divergence just corresponds to the one in field theory. If a divergence exists, we can remove it using a dispersion formula in which a subtraction has been made. These subtractions correspond to the renormalization in field theory, and the subtraction constant corresponds to the renormalized coupling constant. Thus in the dispersion theory, the divergence appears only in the dispersion part, not in the absorption part. Furthermore, the validity of the subtraction is not restricted to a perturbative approach.

We may ask ourselves whether the dynamics of elementary particles can be formulated in terms of the S-matrix, that is, whether or not we can determine the S-matrix by combining the unitarity and the dispersion relations of the S-matrix as mentioned above. In this case, the renormalization is automatically processed by introducing the dispersion formula in which subtractions have been made, thereby realizing a renormalization procedure which is independent of perturbation theory. In this formalism, stable particles which cause poles to occur in scattering amplitudes give the same dispersion relations no matter whether they are elementary particles or composite particles. In other words, what is important in the S-matrix theory is only the stability of particles, so it does not matter whether the particles are elementary or composite. This ended up having a great significance on subsequent developments in the elementary particle theory. The theory proposed by Chew and Low in 1956 is a typical example of the non-relativistic S-matrix theory [28]. The idea was developed further by the discovery of the Mandelstam representation in 1958 [29]. In this representation, scattering amplitudes can be written as double dispersion integrations, and they have crossing symmetries (we shall not touch on such symmetries here, but they will be explained later).

Note, however, that the dynamical S-matrix theory encounters two difficulties. The first is the impossibility or extreme difficulty in finding the complete set of dispersion formulas. The second difficulty is that the solution cannot be determined unambiguously, even under the approximation where we neglect many-particle states. This is related to the fact that we cannot tell the difference between elementary particles and composite particles in the S-matrix theory, as mentioned before. For example, for a given pair of dispersion formulas we can adopt the Lagrangian of Yukawa's theory or the Lagrangian of quantum chromodynamics (QCD).

The S-matrix theory itself, however, went in a different direction. From the study of the high-energy behaviour of scattering amplitudes with crossing symmetries, the Regge pole theory, or Regge trajectory theory, was formulated in 1960s [30]. In the 1950s, following the discovery of the antiproton, many hadrons were detected at the Bevatron using bubble chambers, and the Regge trajectory was applied to classify those hadrons. "Hadron" is the generic name for elementary particles participating in strong interactions. When we consider a two-body bound state, the

Regge trajectory is obtained by plotting the relation between the angular momentum J extended to a continuous variable and the mass squared of the state, M^2. In this method, allowing many hadrons with different quantum numbers except for their spins to follow the same trajectory clarifies the dynamical family relationships among them, and this of course can help to classify them. In particular, Chew and Frautschi succeeded in classifying many hadrons by considering that all the Regge trajectories are continuous lines with universal tangents [31]. As mentioned before, the Regge trajectory corresponds to a bound state, so the success of the Chew–Frautschi plot implied that all hadrons were composite rather than fundamental. The way we interpret this fact is a problem of fundamental importance for elementary particle theory, and we will discuss it in great depth later.

1.5 Strange Particles as Members of the Second Generation

Elementary particles which obey Fermi statistics are classified into hadrons, which engage in strong interactions, and leptons, which do not. The muon, which belongs to the second generation, is a lepton. We have already mentioned its detection and the confusion this created. Similarly, when second generation hadrons were detected, they caused the same confusion. For this reason, hadrons belonging to the second generation were first called *strange particles*. In connection with the detection of the muon, we mentioned that experimentalists sometimes detect particles that have not been predicted by theorists, and this is indeed what happened in the case of strange particles.

In 1947, Rochester and Butler detected two V-particles in a cloud chamber [32]. The V-particle was named after the shape of the track in the cloud chamber, indicating a particle decay. Although the observations continued for two more years, no new particle was discovered. In such a case, two things can happen: either we find nothing when we carry out additional tests or we suddenly detect many cases by improving experimental methods. In the case of V-particles or strange particles, the latter happened. The improved method was to detect V-particles from cosmic rays by climbing high mountains. In the early 1950s, experimental groups in Pasadena and Manchester observed dozens of V-particles on the White Mountains [33] and at the Pic du Midi in the Pyrenees [34], respectively. The biggest problem with the V-particle was to reconcile its frequent detections with its relatively long lifetime of about 10^{-10} seconds (long for an elementary particle). This problem was somewhat similar to the muon problem. With hindsight, this was because both particles belonged to the second generation. Many new particles were subsequently detected in cloud chambers and on photographic plates. For example, *hyperons* like Λ^0, $\Sigma^\pm$, and Θ^-, which are hadrons satisfying Fermi statistics, heavy K-mesons (also called θ- or τ-mesons), and so on.

In order to explain the discrepancy between the frequent detections and the long lifetime, it was suggested that V-particles or strange particles might be created in pairs. Such theories were proposed in 1951 by Nambu, Yamaguchi, and the author

[35, 36], by Oneda [37], and by Miyazawa [38], then by Pais in [39]. However, the pair-production theories were hard to demonstrate from observations of cosmic rays. What these models had in common was that strange particles were always produced in pairs through strong interactions, and the isolated strange particles decayed through weak interactions. This explained both the frequent detections and the long lifetime. However, some phenomena could not be explained by this hypothesis alone. For example, it was not understood why the decay processes $\Sigma^- \rightarrow \Lambda^0 + \pi^-$ and $\Sigma^- \rightarrow n + \pi^-$ did not occur through strong interactions. Nor indeed why the heavy mesons with positive charges were more common then those with negative charges both on plates and in cloud chambers. These questions could not be answered using the above hypothesis.

In 1953, the pair production of strange particles that had not been confirmed by cosmic rays was verified experimentally using an accelerator, the Cosmotron, at Brookhaven [40]. The concept of *strangeness* was introduced by Nakano and the author [41], and independently by Gell-Mann [42]. This concept is closely related to isospin. All the members of a given isospin multiplet have the same value of the strangeness. Further, strangeness is an additive quantum number. It is conserved in strong and electromagnetic interactions, but not in weak interactions. As we shall see later, parity is a quantum number with the same features. Moreover, the K-meson came to play an extremely important role in elementary particle theory. In the history of modern physics, we may say that the K-meson has played the most important role after the hydrogen atom. The concept of strangeness is closely related to the fact that the neutral K-meson, denoted by K^0, differs from its antiparticle $\bar{K}^0$. Furthermore, the non-conservation of parity in weak interactions was discovered through the diversity of K-meson decay patterns. The fact that the CP transformation, where C stands for charge conjugation and P for parity, is not conserved in weak interactions was discovered through the observation of certain decay patterns of the neutral K-mesons. As can be seen from this, the K-meson continued to provide many new ideas. To introduce a modern term, strangeness is a third example of flavour and it is carried by hadrons of the second generation. We shall see later that the symmetry group $SU(3)$ was proposed as a way of combining the isospin symmetry group $SU(2)$ with strangeness.

1.6 Non-conservation of Parity

The strange K-meson was given different names according to its decay patterns:

$$\theta \rightarrow 2\pi \,, \quad \tau \rightarrow 3\pi \,. \tag{1.1}$$

The question was whether θ and τ were the same particle or not. The improvement in the accuracy of measurements revealed that the masses and lifetimes of θ and τ coincided. In addition, it became clear that the two production rates were constant, and quite independent of the production process. This suggested that θ and τ were the same type of particle. However, from the analyses by Dalitz [43] and Fabri [44], this implied the non-conservation of parity in decay processes. The discrepancy between the identification of the particles and the conservation of parity was called the θ–τ puzzle. Under the circumstances, given that the non-conservation of parity might occur, not only in K-meson decays, but in all weak interactions, Lee and Yang proposed various methods of experimental validation [45]. The ^{60}Co experiment by Wu's group was the first to confirm the non-conservation of parity in beta decays [46]. In particular, it was shown that parity was maximally broken in decay processes which involved neutrinos. This led to the two-component theory of the neutrino put forward by Lee and Yang [47]. Since parity is a space-time symmetry, in contrast to strangeness, many physicists were shocked by the non-conservation of parity.

After this discovery, theoretical research was very active. Here we mention the most important development. Assuming that all forces are mediated by fields, we can determine the characteristics of the fields that mediate the weak interactions by identifying the specific types of phenomenological Fermi interaction. An important first step in this direction was the $V - A$ theory proposed in 1958 [48–50]. This theory treated the fields mediating weak interactions as vector fields. The quanta of these vector fields were called W bosons, where W here stands for "weak".

In this connection, in the late 1940s, the conserved vector current (CVC) hypothesis was put forward [49], based on the universality of the Fermi interaction. The idea was proposed by Marshak and Sudarshan [48], by Sakurai [50], and by Gerstein and Zeldovich [51], and further promoted by Gell-Mann and Feynman [49]. According to experiments, the coupling constants for vector couplings of Fermi interactions for various processes are the same, irrespective of the processes, in spite of corrections from strong interactions. This can be explained by assuming that a four-vector density expressing hadronic parts of the Fermi interaction is proportional to an isospin four-current density. This is the *CVC hypothesis*, which will be explained in more detail in the text. When describing phenomenological Fermi interactions as products of two Yukawa interactions mediated by W bosons, each Yukawa interaction is described as the product of the conserved isospin current density and the W field. It is suggestive to compare with the fact that the electromagnetic interaction is the product of the conserved current density and the four-potential of the electromagnetic field.

The idea of including the axial vector current, corresponding to A in $V - A$, developed into the *current algebra method* [52, 53], inspired by the fact that in quantum mechanics a set of conserved quantities forms an algebra. In Sect. 1.4, we presented the methods based on the dispersion formulas and on symmetries for dealing with the strong interactions, although without going into the details. The current algebra method is an extension of the latter. The three components of isospin satisfy the same commutation relations as the components of angular momentum, and they generate the $SU(2)$ algebra. Although this is the algebra formed by just the spatial integral of the time component of the V part of $V - A$, i.e., the algebra satisfied by isospin, if we take $V - A$ itself, we also get the $SU(2)$ algebra. Since the $V - A$ current corresponds to the left-handed current, which will be explained later, we write this algebra as $SU(2)_L$. Introducing the strangeness-changing current, this algebra is extended to $SU(3)_L$. Furthermore, introducing $SU(3)_R$ in the context of the $V - A$ theory, the current algebra is extended to $SU(3)_L \times SU(3)_R$. The first successful application of this algebra was the Adler–Weisberger formula derived in 1965 [54–56]. Then, in 1963, Cabbibo extended the concept of the universal Fermi interaction to the strangeness-changing interaction [57], and this was an important step on the path to the quark model.

1.7 Second Generation Neutrinos

The neutrino was first introduced by Pauli in 1933 [11, 12], and immediately used by Fermi to formulate the field theory of beta-decay [13]. The existence of this neutrino was demonstrated by Cowan, Reines, et al. in [58]. This was the first generation neutrino.

Research on several processes involving weak interactions suggested that there might be a lepton number conservation law and a corresponding selection rule. However, two different definitions were given for the lepton number. One taking e^-, μ^+, and ν as leptons was given by Konopinski and Mahmoud in [59], and another taking e^-, μ^-, and the left-handed (anti)neutrino as leptons was given by Lee and Yang in [47]. It seemed that neither definition was in conflict with experiment. Then, in 1957, assuming a four-component neutrino and putting the case that each conservation law holds, the author set out to investigate which theories were consistent [60]. Combining the above two types of conservation law in the right way, it turned out that the electron family and the muon family could be attributed separate conservation laws. In the electron family, the difference between the numbers of e^- and ν_L and the numbers of e^+ and $\bar{\nu}_R$ was conserved, and likewise in the muon family for the difference between the numbers of μ^- and $\bar{\nu}_L$ and the numbers of μ^+ and ν_R.

In short, there were two types of lepton number conservation law: one concerned the lepton number of the first generation and the other concerned the lepton number of the second generation. The subscripts L and R stand for left-handed and the right-handed, respectively, and both are two-component neutrinos. There are thus two types of two-component neutrino: one belongs to the electron family in the first generation and the other to the muon family in the second generation. A similar idea was also introduced by Schwinger [61].

The second generation neutrino hypothesis was confirmed by a group at Columbia University in 1962 using the AGS accelerator in Brookhaven [62].

1.8 Democratic and Aristocratic Hadrons—The Quark Model

In 1948, Fermi and Yang constructed a theory treating the pion as a bound state of a nucleon and an anti-nucleon [63]. They were motivated by the idea that, since there were so many so-called elementary particles, it would be better to consider some of them as composite particles. In 1955, including the strange Λ particle with a pair of fundamental particles, the proton and the neutron, Sakata extended the Fermi–Yang model for hadrons [64]. He put forward the idea that all hadrons were bound states of protons, neutrons, the Λ particles, and their antiparticles. Given that there might exist a symmetry between p, n, and Λ, Ikeda, Ogawa, and Ohnuki introduced the symmetry group $SU(3)$, and studied the types of representations the hadrons belonged to [65]. For mesons and the hadrons satisfying Bose statistics, Sakata's model achieved many successful results. The η meson found in 1961 [66] was one successful example of Sakata's predictions. However, his model could not give a good explanation for baryons. In any case, this was just one attempt to interpret hundreds of hadrons as bound states of the most fundamental particles.

In 1961, Gell-Mann [52, 67] and Ne′eman [68] independently extracted the concept of $SU(3)$ symmetry from Sakata's model. In Sakata's model, p, n, and Λ were assigned to the three-dimensional representation of $SU(3)$. On the other hand, including Σ and Θ with the particles in Sakata's model, they were able to assign these particles to the eight-dimensional representation. This was subsequently called the *eightfold way*. For mesons, there is no qualitative difference between Sakata's model and the eightfold way, because mesons constitute the eight-dimensional representation in both models.

The $SU(3)$ symmetry is not exact, but explicitly broken. This led to the relation called the *Gell-Mann–Okubo formula* [69, 70] among the masses of particles in one irreducible representation. The highlight of this development was the observation of Ω^- with a mass of 1972 MeV, while the mass predicted by the mass formula was 1983 MeV [71].

Let us return to the question of how to interpret the large number of hadrons discussed in Sect. 1.4. From the phenomenological point of view, hadrons can be categorized by the Chew–Frautschi plot [72] and the $SU(3)$ group. The success of this classification method implied that all the observed hadrons were composite particles. There were two possible ways to interpret this outcome: one was that no fundamental particle existed other than the already observed hadrons, and the other was that more fundamental particles existed, but that they had not yet been observed. These were referred to as *democratic hadrons* and *aristocratic hadrons*, respectively.

The former idea was put forward in Berkeley around 1960, especially by Chew's group [73]. In modern physics, there is a tendency always to seek more fundamental levels, such as molecules → atoms → nucleons → elementary particles, but Chew's group suggested cutting off this sequence. According to this idea, all hadrons should be treated on an equal footing. In other words, since no hadron is more fundamental than the others, each hadron from the lightest pion to the heaviest nucleon should be thought of as a bound state of other hadrons. In this case, each hadron's mass and spin should be determined by a self-consistent method. In this approach, there is no idea of a most fundamental field, so field theory is not suitable to describe it. In this sense, it is a pure S-matrix theory. In particular, the dispersion formula is suitable for this purpose because it holds whether the relevant particles are elementary or composite. However, a framework describing the complete dynamics in a mathematically rigorous way was never provided. This was the idea of a democracy of hadrons.

The other idea is that detectable hadrons are composite particles of more fundamental particles. We can say that this is the aristocracy of the hadron world, treating the more fundamental particles as aristocrats. In 1964, Gell-Mann and Zweig proposed the quark model for hadrons based on the $SU(3)$ group [74, 75]. In this model, quarks became the most fundamental particles. This can be considered as a refinement of Sakata's model. Rather than p, n, and Λ used in Sakata's model, the quark model introduces quarks with three different flavours, namely u, d, and s. Mesons are considered to be bound states of a quark and an antiquark, which is similar to the construction in Sakata's model, while baryons are considered to be bound states of three quarks. The possible representations are:

$$\text{mesons} \quad \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8}\,,$$

$$\text{baryons} \quad \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}\,.$$

One merit of this model is that all the representations appearing on the right-hand side are actually realized in nature. Furthermore, this model succeeded in explaining intrinsic features of hadrons such as the mass and the magnetic moment. However, Gell-Mann [67, 68] and Han and Nambu [76, 77] showed that, when this model is applied to the interpretation of real hadrons, in order to avoid a contradiction with the exclusion principle, an until then unknown degree of freedom had to be introduced. This degree of freedom was called *colour* to distinguish it from flavour.

Unlike the democratic theory of hadrons, the quark model can be easily included in field theory. Quarks and leptons are described by fundamental fields, and hadrons are considered to be composite particles. Further, the left-handed or right-handed current densities in the current algebras can be expressed in terms of very simple bilinear forms using the quark model. In other words, when writing weak interactions in Yukawa form, the hadron parts can be written as products of bilinear forms and W fields. This was a great advantage, recalling from above that expressing the left-handed and right-handed current densities as hadron fields required highly non-linear representations. Thus the quark model was highly promising from the standpoint of including weak interactions.

The charges of the three types of quark can be determined through the fact that the baryon consists of three quarks. The charge of u is $2e/3$, and the charges of d and s are $-e/3$, where e is the positron charge. It is a feature of the quark model that quarks must have fractional charges. However, although experimentalists have tried to detect such fractional charges, they have not been found yet. Therefore, the hypothesis was put forward that independent quarks could not be observed in principle, for some reason. This is the hypothesis known as *quark confinement*. If we accept this hypothesis, there is a sense in which Chew's idea of a democracy of hadrons can still be realized. This is because the S-matrix theory treats only observable hadrons. The dispersion relation is given for the S-matrix elements relevant to the processes involving only hadrons, while the unitarity condition is also expressed by states consisting only of hadrons, which means that the quark states do not appear.

However, the dynamics of hadrons is not determined within the framework of the S-matrix theory. To do this, we need to know the dynamics of gluons mediating strong interactions and quarks. We can say that such an application of the quark model to both strong and weak interactions has paved the way for theories of gauge fields mediating strong, electromagnetic, and weak interactions. We will discuss the theories of gauge fields in the latter half of this book. In this chapter, we went through the story of trial and error that eventually led to the theories of gauge fields. We have used many terms without giving strict definitions, but these will be explained step by step in the following chapters.

Chapter 2
Canonical Formalism and Quantum Mechanics

Quantum field theory is the quantum mechanics of systems with an infinite number of degrees of freedom, and it is constructed as the limit of systems with a finite number of degrees of freedom. After reviewing general features of quantum mechanics for systems with a finite number of degrees of freedom, we will attempt to extend the formalism.

2.1 Schrödinger's Picture and Heisenberg's Picture

In general, in quantum mechanics, the equations of motion are either Schrödinger's or Heisenberg's. Given the Hamiltonian of a system as a function of the set of canonically conjugate variables, $q_1, q_2, \ldots, q_f$ and $p_1, p_2, \ldots, p_f$, Schrödinger's equation is

$$\mathrm{i}\frac{\partial}{\partial t}\Psi(t) = H\left(q_r, \frac{1}{\mathrm{i}}\frac{\partial}{\partial q_r}\right)\Psi(t) \ . \tag{2.1}$$

$\Psi(t)$ is called a *probability amplitude* or *state vector*. This equation holds in the so-called *Schrödinger picture*, in which an operator $\mathscr{O}$ representing a physical quantity is explicitly independent of time. The expectation value of the dynamical quantity $\mathscr{O}$ at time t is

$$\langle \mathscr{O} \rangle = \int \Psi^*(t)\mathscr{O}\Psi(t)\mathrm{d}q \ , \quad \mathrm{d}q = \rho \mathrm{d}q_1 \mathrm{d}q_2 \ldots \mathrm{d}q_f \ , \tag{2.2}$$

where ρ stands for an appropriate density.

In this book, we use natural units, such that $\hbar = c = 1$, where h and c stand for Planck's constant and the speed of light, respectively. From the point of view of relativity, it is natural to take the same unit for time and space; the unit of time is

K. Nishijima, *Quantum Field Theory*,
https://doi.org/10.1007/978-94-024-2190-3_2