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James U. Gleaton
Worcester, MA, USA

David Han
Management of Science and Statistics
University of Texas at San Antonio
San Antonio, TX, USA

James D. Lynch
Department of Statistics
University of South Carolina
Columbia, SC, USA

Hon Keung Tony Ng
Department of Mathematical Sciences
Bentley University
Waltham, MA, USA

Fabrizio Ruggeri
Istituto di Matematica Applicata e
Tecnologie Informatiche
Consiglio Nazionale delle Ricerche
Milano, Italy

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Preface

The evolution of the book started during the 2019–2020 academic year when SAMSI (Statistical and Applied Mathematical Sciences Institute, in Durham, NC, USA) hosted a one-year program called “Games and Decisions in Risk and Reliability.” There, the authors organized a SAMSI program reliability working group on Load-Sharing Systems. This was the start of their three-year collaboration on the topic. The original working group plan was to write an overview paper, but the material that we collected and produced, far exceeded the usual size of a paper. We also realized that there was a need to produce something more structured and detailed for a wider audience. An important class of load-sharing systems are fiber bundle models, which have applications in the physical and material sciences. Many in the statistical community may not be that acquainted with the physical aspects of these applications. In addition, the failure of fiber bundles and chains of such bundles are based on stochastic reliability models and methods, which some in the physical and material science community may not be familiar with. Therefore, we thought a book which described both the physical and statistical modeling in a rigorous, but accessible, way to both communities, would be an important contribution.

We start the book by introducing the basic elements in probability and statistics about distributions, classical inference, and stochastic models, mostly related to reliability. This is followed by the two main parts of the book. In Part I, we discuss classical electrical circuits of ordinary capacitors, including circuit laws. This is followed by a discussion of the solid-state physics of thin-film dielectrics, including structure, conduction mechanisms, and dielectric breakdown for both silica and hafnia dielectrics, as well as cell models for thin-film dielectrics. In Part II, the statistical fiber bundle model is applied to the breakdown phenomenon, as well as to the failure of fibrous composite materials. The book closes with a summary and some suggestions for future research.

Worcester, MA, USA — James U. Gleaton
San Antonio, TX, USA — David Han
Columbia, SC, USA — James D. Lynch
Waltham, MA, USA — Hon Keung Tony Ng
Milano, Italy — Fabrizio Ruggeri
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Contents

Acronyms

AD	Anderson-Darling
AFR	Accelerate failure rate
AFT	Accelerated failure time
AFM	Atomic force microscopy
AIC	Akaike information criterion
BIC	Bayesian information criterion
BD	Breakdown
CAFM	Conductive atomic force microscopy
CBD	Current breakdown
cdf	Cumulative distribution function
CTBD	Cycle times to breakdown
EOT	Equivalent oxide thickness
FBM	Fiber bundle model
GB	Grain boundaries
GOF	Goodness-of-fit
HBD	Hard breakdown
i.i.d.	Independent and identically distributed
KM	Kaplan–Meier
MEV	Minimum extreme value
MLE	Maximum likelihood estimator
pdf	Probability density function
PE	Polyethylene
pmf	Probability mass function
r.v.	Random variable
RVE	Representative volume element
SBD	Soft breakdown
TBD	Time to breakdown
TAT	Trap-assisted tunneling
VBD	Voltage breakdown

Chapter 1
Introduction and Preliminaries

1.1 Overall Introduction

1.1.1 Early Origins of Fiber Bundles Model

Over the last sixty years, fiber bundle models (FBMs) have played an indispensable role in "Modelling Critical and Catastrophic Phenomena." The phrase in quotes is part of the title of a book on FBM (Bhattacharyya & Chakrabarti, 2006). This book consists of several tutorial introductory chapters, one of which is by Kun et al. (2006) entitled "Extensions of fibre bundle models," where they state that "The fibre bundle model is one of the most important theoretical approaches to investigate the fracture and breakdown (BD) of disordered media extensively used both by the engineering and physics community." The chapters after the introductory ones are specialized applications of the FBM in the geosciences.

A related reference that is an excellent introduction to FBM and accessible to non-physicists is Hansen et al. (2015)'s book entitled "The Fibre Bundle Model: Modeling Failure in Materials." Another is Bažant and Le (2017)'s book entitled "Probabilistic Mechanics of Quasibrittle Structures—Strength, Lifetime, and Size Effect."

The point of the current book is to present a friendly introduction of this important topic to those statisticians that are not familiar with it and an introduction to statistical methods for FBM for non-statisticians. This is accomplished by concentrating on both the physical and statistical aspects of a specific load-sharing example, the BD for circuits of capacitors, and related dielectrics. By concentrating on this specific situation, the presentation can be done in an axiomatic framework that is more comfortable to statisticians and probabilists; e.g., the load-sharing rule can be derived from first principles, and the physical aspects of dielectric breakdown are discussed at an elementary level. On the other hand, material scientists also might find the overview enlightening; the statistical aspects presentation is self-contained.

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The starting point of FBM originated with Daniels (1945) seminal work on the distribution of breaking strength of a bundle of threads. Here, in equilibrium, Hooke's law is $\sigma = Y\varepsilon$, where σ, ε, and Y are, respectively, *stress*, *strain*, and *Young's modulus*. For homogeneous bundles, where all the threads have the same Young's modulus, Hooke's law leads to the equal load-sharing rule when all the threads have the same cross-sectional area (homogeneous case) and proportional load-sharing if they have different areas (inhomogeneous case), where the proportions depend on the cross-sectional areas.

These are equilibrium rules that are abruptly violated when a thread fails under increasing load. A thread breakage initiates a violent process, not easy to model, that can cause a cascade of thread failures. After this cascade, once the bundle is again in equilibrium, the surviving threads share the load equally (homogeneous case) or proportionally (inhomogeneous case).

1.1.2 Organization of This Book

In Part I, we consider series circuits of capacitors to illustrate FBM. Much like stressed threads that store potential energy, capacitors are electrical devices that store electrical energy, and series circuits of capacitors behave like bundles of threads. In addition, the capacitor law for a given capacitor, $V = C^{-1}Q$, where V, C, and Q are, respectively, the voltage, capacitance , and charge for that capacitor, is analogous to Hooke's law. Here, V, C^{-1}, and Q, respectively, play the roles of stress, Young's modulus, and strain leading to the equal load-sharing rule if all the capacitances are the same in the circuit. Besides series circuits of ordinary capacitors, we also discuss the electrical breakdown of thin dielectrics and cell models that have been used to model them. In particular, we discuss the load-sharing cell model where the thin dielectric is modeled as a parallel circuit of cells and where the cell consists of a series circuit of nanocapacitors subject to the electrical laws for ordinary capacitors. This conceptualism leads to a weakest link chain-of-bundles/cell model where, for an infinite chain, extreme value asymptotics leads to a Weibull distribution for the BD distribution for the dielectric. Lack of fit for the Weibull is considered a size effect and leads to consideration of the finite weakest link model to account for this.

In Part II, we consider the statistical aspects of fibers and fibrous composites and of circuits of ordinary capacitors and thin dielectrics. Statistical analyses of these materials and electric circuits are given, and related size effects are illustrated and discussed. We give a critical overview of the model assumptions and propose modifications.

We close the overall introduction with a discussion of further background and other applications of FBM. In addition to the physics-based reliability analysis of semiconductor dielectrics, there exist limitless applications of the FBM, many found in the fields of material science, mechanical and structural engineering, and nanotechnology. Its application to understand and explain the physical failure process has a long and rich history. During World War II, it was used to analyze the sudden and unexpected failure of the American Liberty cargo ships. They were the first ships built with hulls that were welded rather than riveted, and some of them broke in half without warning. Another catastrophic example is the hull failure of a Boeing 737 airplane during flight in April 1988 in Hawaii. An explosive decompression occurred, and the airplane fuselage was ripped away mid-air. After the investigation based on the FBM, it was realized that the failure process had started long before as a small crack near a rivet due to metal fatigue initiated by crevice corrosion. The crack grew due to the cyclic pressure loading from flying and being on the ground.

Recently, Mishnaevsky (2013) used FBM for the micromechanics of wind turbine blade composites. The strength, stiffness, and fatigue life of composite materials were predicted, and the microstructural effects and suitability of different groups of materials were analyzed for applications in wind turbine blades. Pugno (2014) reviewed the mechanics of nanotubes, graphene, and related fibers. For designing super-strong carbon nanotube, graphene fibers, and composites, FBMs were applied to quantify the effect of thermodynamically unavoidable atomistic defects on the fracture strength. Using FBM, Orgéas et al. (2015) discussed the rheology of highly concentrated fiber suspensions. Polymer composites reinforced with fibers or fiber bundles are suitable for many aeronautic, automotive, shipbuilding, electrical, electronic, health, and sports applications. Among these materials, sheet molding compounds, bulk molding compounds, glass mat thermoplastics, and carbon mat thermoplastics are the subject of several ongoing research, and their structural properties with respect to the material reliability are understood using the FBM. More recently, Boufass et al. (2020) studied the composite material energy for the FBM when the fibers in the composite are randomly oriented. Also, Leckey et al. (2020) described the construction of prediction intervals for the time that a given number of components fail in a load-sharing system. Their interest was in the successive failure of tension wires (the components) in prestressed concrete beams.

Although unconventional, FBM could also be applied to understand natural phenomena in which rapid mass movements are triggered. Reiweger et al. (2009) used a FBM to describe a slab snow avalanche, the most dangerous snow avalanches, accounting for 99% of fatal avalanches in Canada during the period 1972–1991. The slab snow avalanche presupposes the existence of a weak layer below the surface, which triggers a complete sheet of snow to slide. Further down the slope, the slab may break up into smaller pieces.

Among different types and causes of landslides, some landslide models involve fiber bundles. Like the slab snow avalanches, a buried weak layer may cause a shallow landslide to occur. The weak layer is usually caused by infiltration of water, by rapid snow melting, or by heavy rainfall. This results in reduction of the soil strength. The water-induced weakening is modeled by making the strength distribution of the fibers in the fiber bundle depend on the water content. To estimate the time to failure, Lehmann and Or (2012) modeled the time-dependent water infiltration for a given rainfall. As roots have a stabilizing effect in soil and may inhibit landslides, Cohen et al. (2011) developed FBM for shallow landslides by treating roots as fibers.

Finally, a Markov chain random walk on a graph is basic to well-defining local load-sharing rules since these rules do not fully describe the load-sharing over all possible configurations of component failures. In addition, it also gives a way to obtain the "equilibrium" joint distribution of the states of the components of the related FBM. To do this, let the nodes in the Markov chain graph correspond to the components in the FBM where the edges indicate how the load is transferred locally. The local load-sharing rules define the one-step transition probabilities of the chain. Consider a set of surviving components in the FBM and their corresponding nodes that are now considered as absorbing states of the chain. The absorption probabilities are used to extend the local load-sharing rule to this set of components.

The above is a generic model for the failure of a FBM network based on the Markov chain graph. As the load increases, components/nodes fail, one considers the BD of the network. This with the components/nodes' BD distributions can be used to construct the Gibbs measure for the state of the network (Sect. 6.4). The Gibbs measure indicates what routes are available, if any, between two components.

This approach was used to determine the shape of a bundle in Li et al. (2019) where a bundle failed when there was a route/crack across bundle. This network model may also have implications for the reliability of certain types of nano-sensors. Ebrahimi et al. (2013a, 2013b) used a lattice structure for the nano-sensor where the sensor fails when there is no conductive route across the lattice, but they use a percolation model to produce a conductive route. We do not elaborate on this abstraction in the sequel except for discussing it as an area for future research in Sect. 11.2.

1.2 Preliminaries

1.2.1 Elements of Probability

1.2.1.1 Sample Space and Events

In the book, we consider random phenomena (or experiments) whose individual outcomes are uncertain, although we know all the possible realizations. The set of all possible outcomes is called the *sample space* of the experiment, and we denote it by S. Any outcome s of the experiment is called an *elementary event*, and more generally, any subset A of the sample space S is called an *event*. Therefore, an event is an outcome or a set of outcomes of the random phenomenon.

As an example, if we roll a dice once, the sample space is made of all the possible outcomes, i.e., $S = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}\}$, whereas an event is any set of the outcomes, e.g., the set of even outcomes is $A = \{2, 4, 6\}$.

For any two events A and B, we define the new event $A \cup B$, called the union of A and B, to consist of all outcomes that are either only in A or B or in both A and

B. We define the event AB (or $A \cap B$), called the intersection of A and B, to consist of all outcomes that are in both A and B. The definitions can be generalized to more than two events. Therefore, given the events $A_1, \ldots, A_n$, their union, denoted by $\cup_{i=1}^{n} A_i$, is defined to consist of all outcomes that are in any of the A_i, whereas their intersection, denoted by $\cap_{i=1}^{n} A_i$, is defined to consist of all outcomes that are in all of the A_i.

For any event A, the event A^C, referred to as the complement of A, consists of all outcomes in the sample space S that are not in A. Therefore, A^C occurs if and only if A does not. Since the outcome of the experiment must lie in the sample space S, it follows that S^C contains no outcome and thus cannot occur. We call S^C the null set and designate it by $\emptyset$. If $A \cap B = \emptyset$, so that A and B cannot both occur, we say that A and B are mutually exclusive, and the events A and B are disjoint.

1.2.1.2 Axioms of Probability

For each event A of a random phenomenon having sample space S, we consider a number, denoted by $\Pr(A)$, which is called the *probability* of the event A. A more formal definition, based on measure theory, requires a measurable space $(S, \mathcal{S})$, where $\mathcal{S}$ is a σ-algebra over S (e.g., all the subsets of S if S has finite cardinality). The probability Pr is defined as a function over $\mathcal{S}$ and taking values in the interval [0, 1]. The probability is characterized by the following three axioms:

- **Axiom 1**: $0 \leq \Pr(A) \leq 1$, for any $A \in \mathcal{S}$.
- **Axiom 2**: $\Pr(S) = 1$.
- **Axiom 3**: For any sequence of mutually exclusive events $A_1, A_2, \ldots \in \mathcal{S}$,

$$\Pr\left(\cup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \Pr(A_i),\ n = 1, 2, \ldots, \infty.$$

These axioms can be used to prove a variety of results about probabilities, like:

- $\Pr(A^C) = 1 - \Pr(A)$, for any $A \in \mathcal{S}$ (complement rule).
- $\Pr(\emptyset) = 0$.
- $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$, for any $A, B \in \mathcal{S}$.

1.2.1.3 Conditional Probability and Independence

We denote by $\Pr(A|B)$ the *conditional probability* of A given that B has occurred. If the event B occurs, then, in order for A to occur, it is necessary that the actual occurrence is a point in both A and B, i.e., it must be in $A \cap B$. Since we know that B has occurred, it follows that B becomes our new sample space , and hence, the probability that the event $A \cap B$ occurs will equal the probability of $A \cap B$ relative to the probability of B, i.e.,