

History of Mathematics Education

Dirk De Bock *Editor*

# Modern Mathematics

An International Movement?

 Springer

# **History of Mathematics Education**

## **Series Editors**

Nerida F. Ellerton, Illinois State University, Normal, IL, USA

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History of Mathematics Education aims to make available to scholars and interested persons throughout the world the fruits of outstanding research into the history of mathematics education; provide historical syntheses of comparative research on important themes in mathematics education; and establish greater interest in the history of mathematics education.

Dirk De Bock

Editor

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Dirk De Bock  
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# Foreword

The curriculum of schools, universities, and other institutions has always been a matter of contestation. The history of the curriculum, and the areas and subjects within it, provide an important insight into the political and professional disputations that have been central to movements for change and reform. Teachers with the good fortune to spend a lifetime teaching their subject will know that curriculum development, implicitly and explicitly, is always on the agenda.

There are some who argue, for example Hussain and Yunus (2020), that recent swings to more populist governments have increased the level of political engagement with schooling and curriculum. There is plenty of contemporary evidence to support this view. Donald Trump planned to set up a commission to promote “Patriotic Education” that would address, in his words, the “twisted web of lies” being taught in American schools (Wise 2020). The British politician, Michael Gove, when Secretary of State for Education, attempted to change the school history curriculum to reflect a more patriotic perspective on British history. He railed against the demonization of the British Empire (Watson 2021). Controversial views such as these go back a long way in curriculum history.

In the late 1970s, I became interested in the way the reform of mathematics teaching in the 1960s, sometimes called the “New Maths” movement, had spread across Europe (Moon 1986). I went into the research with an open mind. What story would I find? What patterns emerged from those stories? I looked at reforms in England, France, and the then West Germany as well as Denmark and the Netherlands. In each context, there was a clear link between the heady, progressive social ideas of the 1960s and the reform of mathematics teaching. I pondered whether the impact of the Royaumont Seminar, well covered in this book, could have been achieved without the political climate of reform that characterized Western Europe in the 1960s. The French “Bourbakist”<sup>1</sup> movement, which was so successful in reforming the understanding of mathematics research in the university, moved almost seamlessly into an advocacy for reforming the mathematics curriculum in schools. Looking at the rhetoric and proclamations of the day, there is a clear social as well as epistemological purpose in these activities.

There were other indications that subject curriculum reform was inseparable from social and political trends. I found cartoons in German newspapers that parodied the hippy notion of Alef, the most well-known modern mathematics project. In England, the pushback against progressive ideas, represented by the series of Black Papers (Cox and Dyson 1969), made a great play of the mystique of modern mathematics. Curriculum and ideology were intertwined. And the momentum for reforming the mathematics curriculum was so strong that it bypassed traditional structures of curriculum control. Even in the formidably centralized French system, the schools were changing methods and textbooks well in advance of any reformulation of regulations.

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<sup>1</sup>Nicolas Bourbaki, a nineteenth-century French general, was the pseudonym used by a group of mathematicians who, from the mid-1930s on, advocated radical reform of the way mathematics was taught and researched within the university. In the 1950s, these “Bourbakist” ideas were seen by some as a model for mathematics in the school curriculum (see also Chap. 3 in this volume).

Schools, and to some extent universities, are buffeted by these ideological tremors. This works across the political spectrum. When I began teaching in London in the 1960s, I was involved in a notable British Curriculum initiative, The Humanities Curriculum Project, led by Lawrence Stenhouse. This acquired some notoriety, and in part foundered, because the treatment of race alienated all shades of political opinion (Elliott and Norris 2011). Today, in the wake of the Black Lives Matter movement, the controversies continue.

Lawrence Stenhouse subsequently went on to do more than anyone to establish a rapprochement between competing perspectives on curriculum theory (Lawton 1983). One of the key aspects of his contribution to curriculum research was the accessibility of his work. He was confident with the historical and sociological literature, and this showed in the clarity of his arguments. He also saw the importance of comparative data and ideas that feature so strongly in this book.

Curriculum historians have a vital role in describing and explaining the motives and rationale behind curriculum reform and change. Over the last half century, and more, significant progress has been made in appropriating the methodological and theoretical ideas of historians and social scientists to the study of curriculum history. These ideas are not static. Narrative history, the “longue durée” of Fernand Braudel and ongoing debates about how we do history, informs our task. As do the various schools of thought that have given sociology such an influence on the study of all aspects of education. The challenge to establish a strong cross-disciplinary base for curriculum history is ongoing. American authors such as Franklin (2006), Kliebard (2018), and Popkewitz (2013) have given greater rigor to these modes of enquiry.

A few years ago, Bernadette Baker wrote a review of curriculum history research (Baker 1996). She built her arguments around the questions “What is the field? And who gets to play on it?” This book makes an important contribution to answering those questions, not only for mathematics education but for curriculum history more generally. The French historian Marc Bloch wrote that history was about finding, and following, the tracks left by previous generations. And to quote Bloch, “misunderstanding of the present is the inevitable consequence of ignorance of the past” (Bloch 1954, p. 43). Curriculum historians need publications such as this, as do future generations of curriculum reformers.

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# Abstracts

## Chapter 1

We reflect on the non-parallel origins and development of modern mathematics, as an educational movement, and its American counterpart New Math. The 1959 Royaumont Seminar played a decisive role in bringing together American and European reformers, acted as a catalyst, but did not lead to substantial reform cooperation on either side of the Atlantic. We pay attention to the pluriform nature of the movement(s), shaped by national traditions, existing educational systems, and societies at large. Moreover, we characterize the modern mathematics reform movement and list some of its main features. Adri Treffers' and Hans Freudenthal's model of classifying different approaches to mathematics education into four *ideal* types proved helpful. We conclude with some reflections on the rapid demise of modern mathematics, which in our view should not be regarded as a total failure, but was a breeding ground for a thorough reflection on mathematics education, nationally and internationally, and was the basis for the emergence of *mathematics education* as an autonomous scientific discipline.

## Chapter 2

In the 1940s, the teaching of mathematics in the secondary schools of the United States began to recover from a long period of disrespect. This augmented prestige was due in part to an increased demand for mathematically trained workers arising from World War II and the Cold War. At the same time, undergraduate mathematics instruction was undergoing revision, bringing it more into line with the “modern” viewpoint of research mathematicians, focused on unifying concepts and “structures.” There was a sentiment among a significant segment of mathematics educators that school mathematics had become too estranged from these exciting new developments. This environment encouraged, in the 1950s, the development of innovative secondary school curriculum programs, featuring higher levels of abstraction and precision of language. The University of Illinois Committee on School Mathematics (UICSM) was an early, and notably radical, exemplar, while the School Mathematics Study Group (SMSG) was the largest and best-funded program. By the end of the 1950s optimism that the “New Math” would fundamentally and permanently change the school curriculum for the better was widespread, although far from universal.

## Chapter 3

The cradle of modern mathematics in Europe is likely to be traced to the founding meeting of the International Commission for the Study and Improvement of Mathematics Teaching in 1952 in La Rochette par Melun (France). The organizer of that meeting, Caleb Gattegno, had chosen *Mathematical and Mental Structures* as a theme and succeeded in bringing together several “big names” from the fields of psychology, epistemology, and mathematics, including Jean Piaget, Ferdinand Gonseth, and the “Bourbakists” Jean Dieudonné, Gustave Choquet, and André Lichnerowicz. A few outstanding

secondary school teachers of mathematics participated too, among them Lucienne Félix and Willy Servais. Dieudonné explained the architecture of modern mathematical science, based on set theory and on the so-called “mother structures” of mathematics. Piaget linked these fundamental structures of mathematics with the stages of early mathematical thinking, as revealed by psychology. The claim of alignment between Bourbaki’s mother structures and Piagetian theory provided a strong argument for a substantial reform of mathematics education.

#### **Chapter 4**

After a succinct description of the meeting opportunities for mathematics educators up to the 1950s, this chapter describes how, in the wake of the New Math/modern mathematics reform movement, meetings have become a fundamental tool for focusing on problems and potential of reform proposals. Bodies that have played the most relevant roles are ICMI, CIEAEM, OEEC/OECD, and UNESCO. In the conferences that followed the Royaumont Seminar, particular interest was turned to the search for new axioms for geometry, with many proposals and discussions. But modern mathematics was not just this; in other places, the attention was turned to more general questions of a methodological and social nature. This congress season has fostered the creation of new traditions such as the birth of journals specialized in mathematics education, and periodic conferences on mathematics education, as exemplified by the four-year ICMEs.

#### **Chapter 5**

The reform of mathematics teaching in France in the 1960s and 1970s was one of several reforms which affected the disciplines of primary and secondary education at the time, while the school system structures were also profoundly modified. It was, however, in its course, scope, successes, and difficulties, a reform different from the others, which was emblematic of the period. Considering the reform of modern mathematics within the global dynamics of redefinition of the curricula, I will place it at the crossroads of different ambitions and requirements of modernity to better grasp its characteristics and aims. Firstly, I will deal with the new aims assigned to the French education system after World War II within the framework of a project of cultural, social, and economic modernity of the country. Then, I will examine the reform movement of the teaching of mathematics in relation to that of *français* and then to that of science. Finally, I will focus on a particular moment of the reform of mathematics itself and will return to its ambition of modernity and its contradictions in the socio-economic context of France.

#### **Chapter 6**

Today’s German perception of the New Maths movement in West Germany is strongly shaped by the view that the movement was a transfer of American ideas to Western European countries, that it was solely a reform of elementary school teaching, and that it failed. To understand this perception better without getting bogged down in numerous details of the eventful 1950s, 1960s, and 1970s, we concentrate on seven beliefs—presented in subchapters—about the West German New Math reform. It turns out that this reform of mathematics instruction was to a far greater extent aimed at mathematics teaching in the German *Gymnasium*. To understand the peculiarities of West German New Math, it will be also essential to see it in the context of earlier reforms of mathematics education and institutional reforms of the post-war period. It turns out that for a better comprehension of the failure of the reform one should go back even further in time and include considerations about the unfinished Meraner Reform.

## Chapter 7

The 1950s in the United Kingdom were marked by social and political reforms, which also led to new conceptions of mathematics teaching and learning. For mathematics, the Association for Teaching Aids in Mathematics, founded in 1952 by Caleb Gattegno and like-minded people, and internationally fostered by the International Commission for the Study and Improvement of Mathematics Teaching, played an important role in this. The 1959 Royaumont Seminar served as a booster for curriculum change in the UK, bringing in influences from the continent as well as from the United States of America. In its wake, several projects with accompanying textbooks and in-service teacher training programs emerged in the early 1960s. Most influential were the School Mathematics Project for secondary education and the Nuffield Mathematics Project for primary education, projects that were also implemented, in part and/or adapted, in some countries outside the UK. From the 1970s onward, criticism of the reform reverberated more loudly and led to the fall of the new mathematics paradigm in the UK.

## Chapter 8

In this chapter, we first describe the Italian context when the proposals of reforms, generically indicated under the label of “new math” or “modern mathematics,” were developed all around the world. The current mathematics programs dated to 1945, the main topic was geometry, taught according to a rooted tradition based on Euclid. The conference of Bologna in 1961, which followed those of Royaumont, Aarhus, and Zagreb-Dubrovnik, stimulated the Italian mathematicians to consider also for their country reforms in the light of the proposals that emerged at the international level. With the collaboration of the Ministry of Education and under the aegis of OECD, they organized refresher courses on the new approaches suggested by modern mathematics, edited books, and supervised experiments in selected classes. The plan by the Ministry was not efficient; however, this ferment stimulated various meetings for developing new mathematics programs. These programs were never implemented and only a few notions of modern mathematics remained, but new ideas and new contacts began to circulate which slowly changed the Italian context.

## Chapter 9

The application of modern mathematics ideas in Portuguese schools took place from the 1960s to the end of the 1980s. A first experiment in the higher grades of secondary school that started in 1963 laid the ground for the ways in which the reform was developed later. The compartmentalized nature of the educational system led the several subsystems to develop distinct concretizations of the new ideas, from primary school to the higher grades of secondary school. We argue that modern mathematics in grades 5 and 6 became essentially a linguistic endeavor, contrasted with the reliance on logic as the backbone for the reform in grades 10 through 12. Reformers for the technical schools struggled to accommodate real-world applications into the abstract flavor of mathematics fostered by the new trends. In grades 7 through 9, curricular change meant the introduction of transformational geometry together with a rephrasing of old content into set theory. For each of these cases, program content, textbook implementation, and teacher formation are discussed.

## Chapter 10

The modern mathematics movement in Belgium is inextricably linked to Georges Papy, a flamboyant and uncompromising professor of algebra at the Free University of Brussels. From the late 1950s, Papy reshaped the content of secondary school mathematics by basing it upon the unifying themes of sets, relations, and algebraic structures. Meanwhile, he innovated the pedagogy of mathematics by functionally interweaving his rigorous discourse with multi-colored arrow graphs, filmstrips as non-verbal proofs, and playful drawings, as manifested in his revolutionary textbook series *Mathématique*

*Moderne*. From 1961, his Belgian Centre for Mathematics Pedagogy coordinated various reform actions: Curriculum development, classroom experiments, and in-service teacher training. Although the Belgian mathematics education community was divided about Papy's agenda, *zeitgeist*, media propaganda, and political support made it possible for Papy to realize his reform almost entirely. After the generalized and compulsory introduction of modern mathematics in Belgian secondary schools in 1968–1969, the primary schools followed in the 1970s.

## Chapter 11

New Math in the Netherlands will be described from the perspective of the ideals that were held within the institutes, instrumental in shaping Dutch education. In line with Bob Moon, and contrasting the analysis of some of the people who played a role in this history, we will describe the rise of realistic mathematics education as a realization of New Math, rather than as a breach with the past. Trust in mathematics and mathematicians played a role in the realization of a curriculum which accompanied the introduction of a modern school system, in 1968—supplanting the nineteenth-century system. Pillarization of Dutch society resulted in (1) a large number of stakeholders who all had their own ideas and power bases; and (2) the necessity to find common ground, which was found in a focus on individual learning processes and stressing the need for developing individuality. This gave Dutch New Math its distinct flavor.

## Chapter 12

After a seminar on new thinking in school mathematics held in Royaumont in 1959, four Nordic countries, Denmark, Finland, Norway, and Sweden, agreed to cooperate on school mathematics reform. A joint committee, the Nordic Committee for the Modernization of School Mathematics, declared a need for revising aims and content. Concepts from set theory, and the function concept, as well as greater precision in presentation, could promote interest, insight, and understanding of the subject. Working teams for three school levels: Grades 1–6, 7–9, and 10–12, wrote directives for joint experimental texts and teacher guides. A total of 1310 classes in the four countries took part in experimental instruction. More than 180,000 copies of experimental texts were produced. The Nordic cooperation on modernizing mathematics teaching was a remarkable experiment on the cooperation of independent nations. Gradually, each nation went its own way at grades 1–9, where comprehensive nine-year compulsory education was underway in each country. The experiments initiated a long-needed discussion about curriculum, stagnated in certain routines and topics, and had an impact on curriculum development in the redefined school systems. At grades 10–12, steps were taken to create coherence between the gymnasias and university level.

## Chapter 13

Efforts to modernize Polish mathematics education began in the first half of the twentieth century. After 1957, the moving spirit of the Polish reforms was Zofia Krygowska; a description of her role in Poland and in international fora is augmented with explicit quotations from her books and articles. In 1967, under a strong influence of the French and Belgian versions of New Math, a radical reform of Polish secondary mathematics education was introduced, followed by an equally radical reform of primary education. Unfortunately, the implementation of the latter was combined with a fundamental change of the whole 12-year schooling system to an unclear 10-year system. In 1980, when the reform reached grade 4, the Solidarity movement forced the government to abandon it; yet, some changes were irreversible. In 2008, the last remains of New Math disappeared from the Polish core curriculum. In the concluding part of this chapter, unique phenomena of New Math reforms and its ideology are discussed.

## Chapter 14

In Hungary, a reform movement called Complex Mathematics Education was led by Tamás Varga from 1963 to 1978. The Complex Mathematics Education reform is clearly inscribed in the context of the international reform movements of the New Math period but also bears specificities which can be traced back to a local, Hungarian tradition of mathematics and mathematics education: a heuristic epistemology of mathematics, and an approach to the teaching of mathematics which can be described with the terms “guided discovery.” With his work on this reform curriculum and the related experiments, Tamás Varga himself contributed also to the development of international research on mathematics education, especially in the domains of logic, combinatorics, and probability. In this chapter, we present a brief chronology of the reform; an analysis of its historical context and the different influences which shaped it; the characteristics of the curriculum; and the related resources and expected teaching practices. We illustrate with some examples how the reform combined lessons from the New Math movement with local, Hungarian influence.

## Chapter 15

Yugoslavia was a young country when the Royaumont Seminar took place in 1959, a seminar that emerged from the *New Thinking in School Mathematics* initiative. This chapter, on Yugoslavia, seeks to illuminate that part of the mathematics education history of the country by looking at the three mathematicians who all contributed to a specific view of mathematics, mathematics education, and its practical manifestations. The contributions of Đuro Kurepa, Miloš Radojčić, and Judita Cofman are explored, and their lasting impact on mathematics education in the framework of the New Math in Yugoslavia and further away, on a global scale. Kurepa’s influence and prominence are explored through his role with the ICMI and his global networks; Cofman’s influence through her work in Europe: Germany and England—as much as her work in Yugoslavia; and Radojčić’s work reached Africa through the non-aligned movement. Their contributions to mathematics education were diverse. Kurepa concentrated mostly on delineating the principles of teaching and learning and had leading roles in many institutions, nationally and internationally. Radojčić’s contribution to the philosophy of mathematics education was deeply colored by his epistemology based on anthroposophical principles. Finally, and perhaps the most enduring, was Cofman’s contribution to mathematics education in Serbia, England, and Germany, on structuring the learning of mathematics around problem solving.

## Chapter 16

In the Soviet Union, a reform movement in mathematics education was triggered by Andrey Kolmogorov in the 1970s, and it was followed by a counter-reform. This movement was rooted in the very different socioeconomic conditions of that time and place and followed a strategy with significant contrasts to similar programs in the United States, England, and France. This provides an interesting case study that may illuminate the way such movements arise and succeed or fail, and, at the social level, certain fundamental commonalities of constraints as well as significant differences according to local conditions. We shall show that the principal reasons for the failure of the Kolmogorov reform were political: (a) The reform ignored the reality of the socioeconomic conditions of the country; (b) The human factor was ignored, and very little attention was given to professional development and retraining of, and methodological help to, the whole army of teachers; and (c) An attempt to transfer mathematical content and methods from the highly successful advanced extension stream for mathematically strong and highly engaged children to mainstream education was an especially grievous error.

## Chapter 17

The influence of the 1959 Royaumont Seminar on US mathematics education is described. Five periods are discussed: The 1960s New Math, the post-New Math of the 1970s and 1980s, the 1990s reform, the 2000s standards-based era, and the 2010s Common Core State Standards. The landmarks are initially described by tracing the teaching of geometry, but then other key Royaumont themes are developed: The use of set theory, logic, and mathematical structure; the use of problem solving and inquiry; and the role of research mathematicians. The views of three European mathematicians, Jean Dieudonné, René Thom, and Hans Freudenthal, whose writings in US publications stimulated important discussions, provide a second lens for examining the streams of thought that emerged. Examples from US mathematics education research, state policy documents, and instructional materials are given to illustrate the impact of the Royaumont meeting on the USA.

## Chapter 18

Not often cited as one of the countries making major contributions to new/modern math movements in school mathematics, the case(s) in Canada may seem of lesser interest to the international community. However, in this chapter, we show how the multiple forces acting on school mathematics curricula across the country led to a slow brew of quiet change. In addition to surveying existing writings on various Canadian contexts, we also examine some book and article publications written in the early 1960s that shed some light on the intricacies of this quiet change. We also highlight some of the significant reasons for Canada's slow pace.

## Chapter 19

The “modern mathematics” reform (New Math) in Latin America is described, with an emphasis on its main international agents on the continent: The Inter-American Committee of Mathematics Education (CIAEM) and the conferences that this organization nurtured. A distinction between the first four conferences that tried to propagate the reform and the fifth one is documented. In the latter, a separation from the New Math was evidenced, and it began a new stage in the evolution of these agents. The particular experience of the reform in Costa Rica is included not only to provide details of a special case but to highlight characteristics that to some extent were also present in other countries of the region. The reform in Costa Rica will be contrasted with a new and very ambitious mathematical reform that the country launched in the second decade of the twenty-first century. It can be seen as a “tale of two reforms.” The concluding remarks summarize some results that New Math (ideas and developments, or reactions toward them) provoked so far as the teaching of mathematics was concerned. Finally, comments on some elements of the current situation of CIAEM are offered.

## Chapter 20

This chapter discusses Brazil's participation in international movements on the mathematics curriculum and how it shaped local proposals for mathematics teaching. We focus on three fundamental moments in the history of mathematics education in the twentieth century. The first was the creation of the International Commission on Mathematical Instruction in the early twentieth century and Felix Klein's proposal of merging the different mathematical branches in the school curriculum. The second, known as the modern mathematics movement, resonated between the 1960s and 1980s. The third was the international mobilization launched by the World Conference on Education for All in 1990, which promoted the reorganization of the school curriculum around competencies development. By taking a look at different eras, we seek to understand how national debates and practices in mathematics teaching embraced international thinking before, during, and after the modern mathematics movement. The *international* becomes *national*, with different justifications. It appears that modernization

is a recurring theme in curriculum reforms in Brazil. At different times, for changing reasons, different actors have advocated local reforms to bring mathematics teaching up to date with the most recent international trends.

## Chapter 21

Australia's physical isolation from Europe and North America during the second half of the twentieth century meant that the main "New Mathematics" messages which emerged from the Royaumont Seminar of 1959 were slow to reach its shores. Nevertheless, in the 1950s and 1960s, primary school teachers in Australian schools were challenged to make greater use of structured aids such as Cuisenaire rods and Multibase arithmetic blocks, and the leaders of that movement were two European reformers—Caleb Gattegno and Zoltán Dienes—who both established groups of followers in some of the Australian states. At that same time, the ideas of Jean Piaget also became better known. But a lack of national organization and participation in international forums meant that any changes tended to be local, and a long-established "colonial echo" of British traditions continued to hold sway. The chapter closes with an outline of the work of the Schools Mathematics Research Foundation (SMRF) in Victoria, which, in keeping with the Royaumont themes, emphasized the importance of the language of sets, of structure, and of functions in upper-secondary school mathematics. Another change was the move away from the canonical school mathematics curriculum, with its separate "subjects" of arithmetic, algebra, geometry, trigonometry, and calculus, toward more unified intended and implemented curricula.

## Chapter 22

Hong Kong joined the global trend of the New Math movement in the mid-1960s, yet wrote its own story. Started first as a teaching experiment, the reform could not be contained on a small scale owing to the stakes involved. Yet Hong Kong managed to sail through, and eventually settle down with some harvest. It resulted in the localization of the mathematics curriculum and the growth of a professional community. In addition, Hong Kong strived to tackle teacher preparation, which is the heart of all educational reforms, through a long-term subtle change in its teaching culture. The notion of curriculum reform was conceived from an unconventional perspective. Teachers were gradually induced into curriculum initiatives in which a curriculum document was a summary of these changes rather than a blueprint for teachers to implement. Such an experience not only opened up a new horizon to future curriculum development but may also have shed light on curriculum as well as educational reform in general.

## Chapter 23

The implementation of modern mathematics in Morocco occurred in three main phases, from the early 1960s through the mid-1970s. In the first phase, the implementation mainly concerned the introduction of new vocabulary and symbols of set theory and algebraic structures into upper secondary education. In the second phase, which began in 1968, the latter notions were introduced in lower secondary education and were reinforced in upper secondary education by vector, affine, and analytical geometries. In the third phase, which began in 1971, there was a strengthening of modern mathematics by a restructuring of the content of the programs and by removing classical concepts of geometry from them. But, in 1975, the teaching of mathematics—which elicited a negative reaction from users of mathematics—was the subject of a national conference whose recommendations advocated changes in the mathematics programs taught in secondary schools so that they would be beneficial to the majority of students. Then, in 1978, the Minister of Education created a commission to discuss mathematics programs and made the necessary changes. Modern mathematics was abandoned gradually from 1983 to 1989. In this chapter, we describe the different stages and the peculiarities of implementation of modern mathematics in Morocco.



## Chapter 24

In this chapter, we examine the nature of the mathematics curriculum experienced in Ghanaian schools in the pre- and post-independence eras. In the early 1960s, when the school mathematics curriculum reforms reached Africa, there was a desire for change in all spheres of life, including education. There was also optimism that governments would initiate policies to change the educational systems to transform the nation's youth into a completely literate working population for the rapid economic development of their country. The chapter discusses the nature of changes to school mathematics brought by the UK-led initiatives (grounded in the British School Mathematics Project tradition) and those that came with the USA-led initiatives (grounded in the American School Mathematics Study Group tradition). The chapter also examines the differences between the USA- and UK-led curriculum development approaches and their emphasis on content. Finally, factors which delayed the full implementation of the curriculum reforms, the criticisms of the reforms, particularities, and how the reforms have influenced the school mathematics curriculum in Ghana in the past five decades are discussed.

## Epilogue

The New Math reform was a global phenomenon, which can be found in almost every country of the world. It may be seen as a manifestation of the universal character of mathematics. But equally, the uniformity of the New Math movement may be questioned. It could point to the dependency of non-Western nations from ideas developed in some leading centers. The success of the New Math movement may even be linked to the longevity of colonial traditions. On the other hand, historians may look at the appropriation of the New Math ideology by local communities of scholars. The agency of local mathematicians and mathematics teachers may even be broadened to include a professed belief in the universality of mathematics, even if this does not fit local educational challenges.

## Preface to the Series

Books in Springer's series on the history of mathematics education comprise scholarly works on a wide variety of themes, prepared by authors from around the world. We expect that authors contributing to the series will go beyond top-down approaches to history, so that emphasis will be placed on the learning, teaching, assessment, and wider cultural and societal issues associated with schools (at all levels), with adults, and, more generally, with the roles of mathematics within various societies.

In addition to generating texts on the history of mathematics education written by authors in various nations, an important aim of the series will be to develop and report syntheses of historical research that has already been carried out in different parts of the world with respect to important themes in mathematics education—like, for example, “Historical Perspectives on how Language Factors Influence Mathematics Teaching and Learning” and “Historically Important Theories Which Have Influenced the Learning and Teaching of Mathematics.”

The mission for the series can be summarized as:

- To make available to scholars and interested persons around the world the fruits of outstanding research into the history of mathematics education
- To provide historical syntheses of comparative research on important themes in mathematics education
- To establish greater interest in the history of mathematics education

The present book is an important addition to the series. Chapter authors tell the story of worldwide developments before, during, and after what has become known as the period of the “New Math” or “modern mathematics.” Early chapters raise the question whether developments in the United States of America and Europe need to be seen as having different origins and aims. Many of the authors of chapters focus on how a seminar held in Royaumont (France) in 1959 brought together US and European mathematicians. Many of the leading participants at the seminar were wedded to Bourbakist views that school and college mathematics curricula needed to be brought into line with major mathematical developments of the past 100 years, especially those relating to algebraic structures, mathematical terminologies and notations, and geometry. Most of the chapters offer penetrating analyses regarding the spread around the world of ideas expressed at the Royaumont Seminar.

Chapter authors in this book make reference to extensive supporting literatures. Readers are shown how thinking about modern mathematics in various countries differed, depending on the perceptions and involvement of mathematicians, mathematics educators, mathematics teachers, education administrators, politicians, and captains of industry. Of special interest, however, are comments on between-country effects of events and interactions related to questions like: “Given Nation  $X$ 's unique cultural and historical background, what should be the intended mathematics curricula in its schools?” “Should the intended curricula be the same for all learners?” And, “Who should be responsible for bringing about changes to school mathematics curricula?” There is a common theme—how did different nations respond to the push for “modern mathematics,” especially during the period 1945–1980?

We congratulate Dirk De Bock for his achievement in bringing this work to publication. A huge amount of very scholarly work was involved. Thank you, too, to all the contributing authors. We are proud to have this book as part of Springer's History of Mathematics Education series.

We hope that the series will continue to provide a multilayered canvas portraying rich details of mathematics education from the past, while at the same time presenting historical insights that can support the future. This is a canvas which can never be complete, for today's mathematics education becomes history for tomorrow. A single snapshot of mathematics education today is, by contrast with this canvas, flat and unidimensional—a mere pixel in a detailed image. We encourage readers both to explore and to contribute to the detailed image which is beginning to take shape on the canvas for this series.

Any scholar contemplating the preparation of a book for the series is invited to contact Nerida Ellerton ([ellerton@ilstu.edu](mailto:ellerton@ilstu.edu)), in the Department of Mathematics at Illinois State University, or Melissa James, at the Springer New York office.

Normal, IL, USA

Nerida F. Ellerton  
M. A. (Ken) Clements

## Preface to the Book

Modern mathematics or New Math, as an educational phenomenon, originated in the 1950s in the United States of America and in Francophone Europe, in the euphoria of revival after World War II, and in the background of the Cold War. The American and European emerging movements, which evolved largely independently of each other in the 1950s, met at the legendary Royaumont Seminar (1959), the impetus for a global rollout, from the Americas to the whole European continent, from the Soviet Union to Ghana and Australia. It became a worldwide reform movement, perhaps the most radical that school mathematics had ever seen. Curricula converged from computational techniques and Euclidean geometry to a more abstract approach based on set theory, algebraic structures, and topology. Not only secondary education became involved; in quite a few countries modern mathematics also affected primary education and even kindergarten. It all occurred in no more than two decades. This could only be realized by a widely orchestrated and mediatized action engaging the diverse stakeholders: Mathematicians, mathematics teachers, educational officials, school boards, and publishing houses.

Although in retrospect, many regarded modern mathematics as a fascinating yet failed experiment, no one will deny the strong influence it had exerted on the way school mathematics was taught and perceived. Despite that, the historiography of the movement is still in its infancy. Until the end of the twentieth century, only a limited number of authors had mapped the modern mathematics movement in their country, usually with only national objectives. A first attempt at an international overview, albeit on a European scale and focusing on primary education, was undertaken by Bob Moon in his oft-quoted book *The “New Maths” Curriculum Controversy: An International Story* (1986). In the first decades of the twenty-first century, however, interest in the history of mathematics education in general, and in the phenomenon of modern mathematics in particular, increased considerably. Witness, among others, the various scientific conferences at which studies on the history of teaching mathematics can be presented—in particular, the specialized series of biennial *International Conferences on the History of Mathematics Education*, launched in Iceland in 2009. In these circles, scholarly interest in the historical phenomenon of modern mathematics became common, with several studies on different aspects of national reforms in the 1960s and 1970s, especially but not exclusively related to countries that had played a pioneering role in the movement (the United States of America, France, Belgium, etc.). At these specialized meetings, the idea also arose to unite forces and confront and integrate research findings on this key movement in the history of mathematics education in the second half of the last century, resulting in this edited volume.

This volume basically consists of two parts. The first part documents the origins of modern mathematics on both sides of the Atlantic. In short, it can be concluded that the European debates were mainly related to a structural Bourbaki view on mathematics, while the American reform movement was stronger rooted in socioeconomic and political motives and from the start driven by the government. The European and American positions met briefly at the Royaumont Seminar (1959) and subsequent international meetings (organized by international bodies such as OECD, ICMI, and UNESCO), yet will largely go their separate ways afterward. The second and most extensive part of

this volume analyzes how different countries or groups of countries incorporated the reform into their own educational systems and culture. It paints a picture of a movement with shared principles and beliefs, but which were implemented in very different ways. The authors of this volume have diverse scientific backgrounds; they are experts in the history of mathematics education, research mathematicians, or mathematics educators, and their relationship to the subject also varies (from “actor,” in some way or another, to neutral observer-researcher). Authors’ different backgrounds are also reflected in the nature of the different approaches to the subject. Some chapters offer a detailed historical account of events in their socioeconomic and political context; others are more reflective in nature, making comparisons with other historical reform movements, and/or providing insights that can support future developments.

As the editor of this volume, I am indebted to several individuals and bodies: My colleagues and friends in the field of the history of mathematics education whom I regularly met at international conferences and who encouraged me to take the lead in this challenging publication project; the authors of the different chapters who responded enthusiastically to my call for collaboration (and were willing to revise their initial submissions); the editors of *Springer’s* History of Mathematics Education series who immediately favored the project; and the international scientific publisher *Springer*, who offered me this publication opportunity, and with care and flexibility molded this volume into its final form. Last but not least, I want to thank the board of the Faculty of Economics and Business of the KU Leuven, my employer, who provided me with the necessary time and resources to work on this project.

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Dirk De Bock

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**Danny Beckers** studied mathematics and cultural history at Nijmegen University. In 2003, he earned his PhD on a thesis that delved into the reasons for mathematics becoming a mandatory course in Dutch education, in the early nineteenth century. He published on various aspects of Dutch (mathematics) education, from late medieval until modern times. Currently, he teaches courses in history of mathematics, history of computer science, and history of AI to students at the Vrije Universiteit Amsterdam. His research projects are in history of education. He is writing a double biography of the Freudenthal couple and a history of mathematics education in the Netherlands, with a specific focus on educational practices and ideals.