Mathematics Education in the Digital Era

Claus Michelsen · Astrid Beckmann ·
Viktor Freiman · Uffe Thomas Jankvist ·
Annie Savard *Editors*

Mathematics and Its Connections to the Arts and Sciences (MACAS)

15 Years of Interdisciplinary Mathematics Education

Springer

Mathematics Education in the Digital Era

Volume 19

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ISSN 2211-8136 ISSN 2211-8144 (electronic)
Mathematics Education in the Digital Era
ISBN 978-3-031-10517-3 ISBN 978-3-031-10518-0 (eBook)
https://doi.org/10.1007/978-3-031-10518-0

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

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Introduction: 15 Years of Mathematics and Its Connections to the Arts and Sciences

Claus Michelsen, Astrid Beckmann, Viktor Freiman, Uffe Thomas Jankvist, and Annie Savard

Mathematics is part of almost every aspect of everyday life, and society consumes a lot of mathematics. Across nations and continents, mathematics plays a central role in educational systems from kindergarten to lifelong learning. But although a growing number of disciplines include ingredients from mathematics, it is still difficult for both teachers of mathematics and teachers of other disciplines to see the use of mathematics in other disciplines. This paradox of relevance leads to the problem of isolation, which works to the disadvantage of both mathematics and other disciplines which could profit from a conscious inclusion of mathematical competences (Niss, 1994). The argument for an interdisciplinary approach to education stems from the fact that in order to understand the world—with all its complexity—teaching has to incorporate multiple topics and disciplines. The challenge is to replace the current monodisciplinary approach, where knowledge is presented as a series of static facts disassociated from time with an interdisciplinary approach combining various disciplines in a single curriculum.

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C. Michelsen et al. (eds.), *Mathematics and Its Connections to the Arts and Sciences (MACAS)*, Mathematics Education in the Digital Era 19,
https://doi.org/10.1007/978-3-031-10518-0_1

1 ICME Topic Study Group 21, Copenhagen 2004

The challenge of combining mathematics with other disciplines in a single curriculum was the overarching theme at Topic Study Group 21 "Relations between mathematics and other subjects of science or art" (TSG 21) at the 10th International Congress of Mathematics Education (ICME-10) held in Copenhagen in July 2004. ICME conferences are the largest international meetings in mathematics education with sessions related to the entire set of activities in the field. The Congress in Copenhagen attracted over 2,000 mathematics researchers and educators from the international community. Plenary sessions, major lectures, topic study groups, discussion groups, country showcases, and special sessions were organized to capture the breadth and depth of mathematics education across the world.

The TSG 21 gathered a group of congress participants addressing how to position mathematics in relation to other disciplines. A total of 12 abstracts were submitted to the topic study group. During the four sessions of the study group, there was one keynote lecture and ten paper presentations. The keynote lecture was held by Mona Wijers of the Freudenthal Institute, University of Utrecht, who spoke on "Connecting mathematics and other subjects." The main conception that was extracted from the TSG 21 work was a deep interrelation between mathematics, arts, and sciences. The arguments for this are founded in the historical influence of mathematics in arts and sciences, in the common aspects that can be found between creative work in mathematics and art, and in regarding mathematics as part of human culture and as part of mankind's struggle to understand the world. The educational implication of this is a need for interdisciplinary activities, to provide a sense of meaning to student's mathematical learning by relating mathematics to different aspects of human culture. But issues related to this topic are complex because they comprise at least two different components, an extra-mathematical and a mathematical one. There was a general agreement among the participants of TSG 21 that to approach this complexity a theoretical framework is needed to share a vision of unifying mathematics, science, and the arts in school and university curricula. The establishment of a network was suggested for exchange and communication between mathematics educators interested in the relations between mathematics and other subjects (Anaya & Michelsen, 2005, 2008).

The symposium series MACAS, Mathematics and its Connections to the Arts and Sciences, was founded as an outcome of a continued collaboration between three researchers participating in TSG 21, Astrid Beckman from the University of Education, Schwäbisch Gmünd in Germany, Bharath Sriraman from the University of Montana in the USA, and Claus Michelsen from the University of Southern Denmark in Denmark. The MACAS initiative was based on a vision of developing a humanistic approach to education which combines various disciplines in a single curriculum. According to this vision, the aim is to educate students by enabling them to pursue diverse fields of research while at the same time exploring the aesthetic and scientific connections between science and the arts. In light of the challenges of the twenty-century, a modern approach to education with a focus on multi- and interdisciplinarity is more important than ever. The field of mathematics assumes a key role in this approach as it is connected to all other disciplines and can serve as a bridge between them.

2 MACAS 2005, Schwäbisch Gmünd

The first MACAS symposium was held in 2005 at the University of Education, Schwäbsich Gmünd, Germany from May 19 to May 21. Thirty-one researchers and educators from 13 countries in Africa, Asia, Europe, and North America participated in the symposium, and a total of 18 abstracts were submitted. There were three plenary lectures by distinguished international researchers:

- Dietmar Guderian, Germany: Mathematics in Contemporary Arts and Culture
- Bharath Sriraman, USA: Philosophy as a Bridge between the Arts, Mathematics and Sciences: Historic and Contemporary Connections
- Richard Lesh, USA: John Dewey Revisited—Making Science Practical VERSUS Making Practice Scientific

Guderian argued in his plenary lecture that although numerous examples of the integration of mathematics with other disciplines are found in the classical arts they convey a clichéd view of the connections between mathematics and the arts and culture. Guderian presented numerous approaches in the arts which often unconsciously and unintentionally visualize mathematical thinking and express mathematical thoughts which are difficult to comprehend. In his plenary lecture, Sriraman focused on philosophy as a bridge between mathematics, the arts, and sciences. Sriraman demonstrated how shifts occurring in perspectives in arts and sciences at the time often resulted in new paradigms for science and creation of new mathematics. The plenary lecture by Lesh put forth the models and modeling perspective of learning. Lesh pointed out that unlike good mathematics education theories which do not change the practice of mathematics education, the models and modeling perspective are adaptable, driven by reality and practice and do not fit neatly into any single theory analogous to how reality functions.

The participants who presented papers at the symposium were invited to submit their papers. All the papers were refereed and published together with papers by the plenary lecturers in the proceedings of the symposium. The themes of the papers are very diverse in nature, reflecting the fact that impacts of mathematics can have considerable spread in many cases. Despite the diversity and different viewpoints, there are clear thematic trends within which the papers and the proceedings are arranged: (i) Mathematics and Arts, (ii) Mathematics and Literature, (iii) Mathematics and Didactical Issues, (iv) Mathematics and Music, and (v) Mathematics and Physics (Beckmann, Michelsen & Sriraman, 2005).

3 MACAS 2007, Odense

The symposium in Schwäbisch Gmünd brought together researchers and educators who shared a vision of unifying the arts, mathematics, and the sciences in school and university curricula, and a network of people with common research interests emerged. Giving this success, the three initiators of the symposium decided to make the symposium a biannual event. The second MACAS Symposium was held at the University of Southern Denmark in Odense, Denmark from May 29 to May 31, 2007. The symposium had 44 participants from 15 countries in Asia, Australia, Europe, and North America, including math, science, and art educators, physicists, historians, and philosophers of mathematics and science, as well as some local classroom teachers. A total of 23 papers were submitted for the symposium, and there were four invited plenary lecturers:

- Lyn English, Australia: Mathematical Modelling: Linking mathematics, science, and the arts in the primary curriculum
- Michiel Doorman, the Netherlands: Learning mathematics through applications by emergent modeling: The case of slope and velocity
- Swapna Mukhopadhyay, USA: The decorative impulse: Ethnomathematics and tlingit baskery.
- Norma Presmeg, USA: Mathematics education research embracing arts and sciences

In her plenary lecture, English presented an approach to incorporating interdisciplinary experiences in primary school mathematics curricula through the creation of realistic mathematical modeling problems drawing on contexts and data from other disciplines. In her concluding remarks, English pointed out that consideration should be made not only for the mathematical ideas to be embedded in the modeling problems but also for the content of other disciplines. Models and modeling were also on the agenda in the lecture by Doorman. With the Dutch Realistic Mathematics Education as a theoretical framework, Doorman presented a reinvention approach to velocity–time and distance-time graphs aimed at a process in which mathematics relates to their understanding of the physical properties of motion and emerges from the modeling activities of the students. Doorman concluded that the integration of mathematics and science topics is important for education, and therefore, investigation into the constraints and possibilities for the integration of mathematics and science is needed. In her lecture, Mukhopadhyay shared her perception of ethnomathematics as the way mathematics is embedded in people's lives, particularly those who do not fall within the Western mainstream norm. This was illustrated by the Tlingit Indians' baskets, which had purely utilitarian purposes but were perceived differently for their aesthetic values after contact with European traders. With the starting point that mathematics education, unlike mathematics, is a young and emergent field drawing upon the established knowledge bases and methodologies of other fields, Presmeg argued in her lecture, using examples and a reference to Habermas' knowledge-constitutive interests, that both the sciences and the arts are implicated in mathematics education, whose research also requires the full gamut of methodologies available in the arts and sciences.

The proceedings of the symposium in Odense included the extended and peer-reviewed versions of 26 paper presentations. As was the case with the first MACAS proceedings, the papers are very diverse in nature, addressing theoretical as well as practical musings on the relationship between mathematics, science, and art. However, clear themes can be identified: (i) Mathematics and Arts, (ii) Mathematics in Multidisciplinary Contexts, (iii) Mathematics, Understanding, Motivation, and Empathy, (iv) Mathematics and Science, (v) Teacher Education, and (vi) Mathematics, Problem Solving, and Modeling (Sriraman et al., 2008).

4 MACAS 2009, Moncton

The progress in both the number of participants and submitted articles at the symposium in Denmark showed that the MACAS idea was sustainable and had the potential to set an international agenda and strengthen a relatively unexplored field within mathematics education. In this context, it should be noted that after the ICMI congress in Copenhagen there have been no topic study groups addressing relations between mathematics and other subjects of science or art at the subsequent ICMI congresses. Despite this strange and incomprehensible disposition by the International Commission on Mathematical Instruction, so much momentum had been created by the MACAS initiative that the planning of the next symposium was already initiated during the symposium in Denmark. The initiative group was expanded with Viktor Freiman from the Université de Moncton in Canada, and it was decided to hold the next MACAS symposium at the Université de Moncton.

The third MACAS Symposium at the Université de Moncton, New Brunswick, Canada, took place in May 2009 and included 30 participants from Canada, Denmark, Germany, Mexico, Poland, and the USA interested in connections between mathematics and the arts and sciences in order to exchange their research results and continue to work together on a collaborative research agenda. There were three plenary lectures by invited international researchers:

- David Wagner, Canada: Intercultural positioning in mathematics
- Tinne Hoff Kjeldsen, Denmark: History in mathematics—why bother? Interdisciplinarity, mathematical competence and the learning of mathematics
- Wolff-Michael Roth, Canada: Mathematics in the everyday world and at work. Prolegomena for rethinking the concept of interdisciplinarity

From an ethnomathematical position, Wagner argued in his plenary lecture that intercultural and interdisciplinary experiences are fertile ground for developing alternative forms of positioning, for provoking innovative practice, and for understanding the cultures and disciplines and their relationships. Wagner emphasized that humans who do mathematics in a particular cultural or disciplinary context, trying to respect the contextual values, develop a deeper understanding of the values of both mathematics and the context in which it is applied. Kjeldsen discussed how and in what sense mathematics can engage in interdisciplinary work with history to contribute to general educational goals and to the learning of mathematics. Two cases, the application of mathematics in biology in the 1930s and the influence of physics on the development of differential calculus in mathematics in the 1690s, were used by Kjeldsen to illustrate that within a theory of learning for developing a certain discourse a multiple perspective approach to the history of the practice of mathematics might provide special opportunities for the learning of mathematics. Roth presented empirical findings across very different contexts in which mathematical entities are employed and mathematical practices dealing with modeling fish are enacted and asked the question: If these practices are different, should we still be using the same term, mathematics, to denote them? Roth answered that we have to be cautious about the nature of mathematical practices and how those found in one context articulate with the mathematical practices in another context.

The symposium in Moncton was part of the project Interdisciplinary Networks for Better Education in Mathematics, Science, and Arts funded by the Canadian Social Sciences and Humanities Research Council as part of its program for the International Opportunities Fund in 2008. For the first time, the MACAS Symposium was a part of a larger conference, SMART, in which other groups took part. The proceedings from the Symposium contained 34 peer-reviewed articles and had the title "Interdisciplinarity for the Twenty-First Century." The word diverse was still the best way to characterize the symposium, but the three symposia proceedings were seen as the start of creating a scholarly publication through which the work of the MACAS group, which had gathered more than just math educators and mathematics education researchers over the last four years, could be transmitted (Sriraman & Freiman, 2011).

5 MACAS 2015, Schwäbisch Gmünd

Following the Moncton Symposium, the group of MACAS initiators were preoccupied with tasks other than research, primarily university management, which led to the cancelation of the planned symposium in Montana, USA in 2011. As the ten-year anniversary of the first symposium approached, Astrid Beckman, Viktor Freiman, and Claus Michelsen decided to resume the symposia, and during 2014, it was decided to announce the fourth MACAS symposium in 2015.

For its 10th anniversary, the MACAS Symposium turned back to the University of Education Schwäbisch Gmünd in Germany. The Symposium was held from May 28 to May 30 with 33 participants, and a total of 17 abstracts were submitted. There were five plenary lectures, held by:

- Jean-Luc Dorier, Switzerland: Vectors and translations in mathematics and physics
- Gesche Pospiech, Germany: Interplay of mathematics and physics in physics education
- George Gadanidis, Canada: Aesthetic attention and young mathematicians

- Irene Neumann, Germany: The role of mathematics in science education
- Uffe Thomas Jankvist, Denmark: On the use of primary historical sources in the teaching and learning of mathematics—short- and long-term effects.

Three of the plenary lectures addressed the relation between mathematics and physics. Dorier used the example of vectors to illustrate how mathematics can be actually connected to physics and to make propositions to make this connection more efficient for the benefit of both mathematics and physics. Pospiech analyzed the interplay of mathematics–physics and looked for possible sources of the often-complained-about deficiencies of students in applying mathematical elements in physics. She concluded her lecture with a recommendation to promote the consciousness of teachers concerning the structural role of mathematics with a focus on the differences between the use of mathematical structures in mathematics and in physics in order to enable students to reconstruct the pathway from physics to mathematics and back. Neumann addressed the role of students' mathematical abilities in their endeavor to develop competences in science. Gadanidis shared his work with elementary schools, collaborating with teachers to develop an aesthetic dimension for engaging young children with big math ideas, a holistic look at the parallels between "story" and "mathematics." He concluded that the aesthetic dimension of mathematics is a natural human quality and that the aesthetic is common to mathematics, the arts, and other disciplines, and it is the aesthetic that makes the experience of the disciplines human. The starting point of the lecture by Jankvist was that the study of primary historical sources in mathematics is often described as a rewarding pursuit worth the effort, despite being extremely demanding for both teachers and students. Jankvist discussed teaching modules on aspects of the History, Application, and Philosophy (HAPh-modules) of mathematics which allowed upper-secondary mathematics students to work with historical primary sources and addressed short-term effects, e.g., the experience of mathematics as "created" by human beings, and long-term effects, e.g., an image of what mathematics is like as a discipline as well.

The proceedings from the symposium present 16 peer-reviewed papers grouped into five sections: (i) Mathematics and Science, (ii) Mathematics and Language, (iii) Mathematics and Arts, (iv) Mathematics and Technology, and (v) New Paths in Pedagogy (Beckmann, Freiman & Michelsen, 2016). The successful revival of the MACAS Symposium made it an obvious decision to hold biannual symposia again. It was decided that the next MACAS Symposium should be held in Denmark, 10 years after the second symposium was held in Odense, Denmark. Uffe Thomas Jankvist from Aarhus University in Denmark offered to arrange the symposium, and he therefore joined the MACAS initiative group.

6 MACAS 2017, Copenhagen

The MACAS 2017 Symposium took place at the Danish School of Education, Aarhus University in Copenhagen, June 27–29, 2017. It included 42 participants from Canada, China, Denmark, the Faroe Islands, Germany, Mexico, Russia, Sweden, Switzerland, Ukraine, and the UK, and a total of 17 abstracts were submitted. The three plenary lectures of the Symposium were held by:

- Annie Savard, Canada: Making decisions in a complex world: Teaching how to navigate using mathematics
- Jens Højgaard Jensen, Denmark: Mathematical modeling—Hiding or guiding?
- Paul Ernest, UK: Mathematics, Beauty and Art.

In her plenary lecture, Savard discussed how critical thinking using mathematics might support the decision-making process from an ethnomathematical perspective. Savard argued that we need critical thinking to develop mathematical knowledge and we need mathematics to develop critical thinking. Navigating our world means being able to quantify, measure, estimate, classify, compare, find patterns, conjecture, justify, prove, and generalize within critical thinking and when using critical thinking. Critical thinking and decision making can also be developed in the mathematical context, so there is no excuse to not explicitly support students in developing them. Jensen shared his reflections about the distinction between theory-derived mathematical models and ad hoc mathematical models as a way to help ordinary people, not to distinguish between trustworthy and non-trustworthy models, but to distinguish between the different qualities of the evaluation processes behind different sorts of models. Ernest addressed the questions of what beauty in mathematics is and what dimensions of mathematical beauty can be distinguished? Ernest offered provisional answers to these questions and illustrated mathematical beauty by means of an example from visual art. Since beauty is shared by both mathematics and art, Ernest also asked the question of what parallels, including similarities and differences, can be drawn between mathematics and art?

The proceedings from the 2017 symposium collected papers corresponding to the plenary lectures and presentations given at the MACAS 2017 symposium and included 19 peer-reviewed papers. The papers can be grouped into five sections: (i) Geometrical Objects, (ii) Mathematics and Science, (ii) Mathematics and Technology, (iii) Mathematics and Literature, (iv) Mathematics, Creativity, and Aesthetic, and (v) Students' Beliefs about Mathematics (Michelsen, Beckmann, Freiman & Jankvist, 2018). The overall success of the symposium in Copenhagen promoted a continuation of the MACAS symposia. Annie Savard from McGill University in Canada offered to arrange the next symposium and joined the MACAS initiative group.

7 MACAS 2019, Montréal

In June 2019, the MACAS Symposium returned to Canada 10 years after the symposium in Mocnton, this time at McGill University, Montréal, Canada. Forty-seven international participants hailed from Austria, Belgium, Brazil, Canada, China, Denmark, Germany, the Philippines, Russia, Spain, Switzerland, the USA, and

Vietnam and a total of 42 abstracts were submitted. The three plenary lectures of the Symposium were held by:

- Richard Barwell, Canada: In Dialogue with Planet Earth: Thoughts on a Mathematics Education for a Sustainable Future
- Cécile de Hosson, France: When the Student Becomes an Author: Didactic Analysis of Comics and Science Workshops
- Michelle Wilkerson, USA: Putting Artistic and Mathematical Expression into Conversation Through Computing.

With critical mathematics education, post-normal science, and a dialogic epistemology as a basis, Barwell addressed in his plenary lecture how mathematics plays a central role in thinking about sustainability. He argued for the need for a relational approach, in which knowing is seen as contingent, situated, discursive, and multivoiced. As a result, the human activity of mathematics is understood as being in relation to the planetary ecosystem. In her lecture, Hosson presented an analysis of the impact of workshops with the aim of enabling students to become authors of a comic strip involving scientific knowledge. Being involved in the creative process likely allowed the students to understand the reasons for some scientific illustration or narrative choices. Hosson pointed out that the approach could encourage the emergence of a critical spirit toward reading scientific stories created in other contexts. Wilkerson focused on her work on how math educators can introduce learners to the expressive value of mathematics by connecting mathematical representation to other familiar forms of expression, such as sketching and storytelling, through computing. Wilkinson described her work as an attempt to bridge art and mathematics, with a focus on expressivity and pointed out that it demonstrates a sophisticated interplay between students' artistic and epistemic activities, whereby decisions in one domain motivate or constrain possibilities and discoveries within the other.

The proceedings from the symposium contained 19 peer-reviewed papers. While the papers are diverse, they can be grouped into five general areas, including: (i) Mathematics and the Arts, (ii) Mathematics, Games and Technology, (iii) Collaborative Teaching, (iv) Integrative Learning in Mathematics and Science, and (v) STEM (Savard & Pearce, 2020).

8 15 Years of MACAS

With the symposia in Schwäbisch Gmünd, Copenhagen, and Montreal, the MACAS Symposium was now again in a two-year cycle, with the next symposium planned to take place in 2021. But as we all know, the outbreak of COVID-19, among many other things, changed the planning and holding of international scientific conferences; e.g., the ICME-14 was postponed by one year to 2021. In light of the situation, the initiative group for MACAS decided to postpone the next MACAS Symposium for one year, and the next MACAS Symposium will be held in 2023. However, to mark the publication of this book, an online MACAS symposium will be held in 2022 with Moncton University as host.

From its historical and cultural development, the heritage of mathematics reveals itself as a highly connected field of study. Over the past decade or more, there has been a rapid increase in the number of ways mathematics is used and the types of mathematical ideas being applied. Obviously, there are substantial implications for the teaching of mathematics and its connections to the arts, sciences, and other human endeavors. The MACAS proceedings show that there is more than one way of achieving this. The papers of the MACAS proceedings are very diverse in nature and convey real differences in style, culture, and methodology, reflecting the fact that although mathematical entities appear in different contexts, mathematics in one discipline and context is very different from mathematics in another discipline and context. The MACAS proceedings underscore the need for joint empirical investigations that operationalize, model, and study the rich ideas presented at the symposia. The MACAS initiative group is trying to meet this need with the present book, which marks that in 2020 it was 15 years since the first MACAS Symposium was held. The book contains contributions from the initiative group as well as from invited authors, all of whom have participated in one or more of the MACAS symposia. In this way, we intend to gather the threads from the symposia and provide a picture of the state of the art of mathematics and its connections to the sciences and arts from a mathematics education research position.

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The Community of MACAS

MACAS as a Collaboration Hub

Astrid Beckmann, Viktor Freiman, and Claus Michelsen

The many outcomes of MACAS extend far beyond the proceedings of the conferences themselves. From the very beginning, MACAS has served as a collaboration hub promoting national and international projects and networks, research and teaching, interdisciplinary models and further education activities in schools and universities, and a lively exchange between the members of the MACAS community with a steady stream of new participants. MACAS connects researchers and educators who hail from all corners of the globe and work in interdisciplinary contexts across a wide range of fields with a link to mathematics. On top of this, it has always served as a very special inspiration—and a fruitful source of support for many young researchers completing their doctoral theses over the last 15 years. Iversen recounts a typical example in *Model Eliciting Activities Revisited* (2008, p. 165): "The case study reported in this paper found its inspiration at MACAS 1 …. After the symposium a continued collaboration … was initiated and this enabled the planning, carrying through and evaluation of the case study presented below." And during his 2019 talk on *An Interdisciplinary Approach between Modelling Processes in Music and Mathematics*, Nutzinger delivered—because of the new research inspirations—a toast to the program: "Thank you, MACAS!" (Nutzinger, 2020). In this chapter, we will discuss some of the more large-scale activities inspired by MACAS conferences and made possible through the MACAS network.

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C. Michelsen et al. (eds.), *Mathematics and Its Connections to the Arts and Sciences (MACAS)*, Mathematics Education in the Digital Era 19,
https://doi.org/10.1007/978-3-031-10518-0_2

1 International Projects

MACAS conferences attract participants from all continents and feature contributions from the Americas, Asia, Africa, Oceania, and Europe alike. The MACAS program introduces participants to their counterparts from other continents and promotes an atmosphere of mutual respect and opening to a diversity of views and perspectives. This exchange gives attendees the chance to take their research to the next level, especially if their home country maintains less of a focus on mathematics in interdisciplinary contexts. There are also several researchers and educators who, despite coming from neighboring regions, first got to know each other at a MACAS event. MACAS has helped kick-start collaborative research and teaching projects in all manner of ways on a regional, national, and international level.

One example of this is *ScienceMath—Mathematical Literacy and Cross Curricular Competencies through Mathematization and Modeling Science* (www.sciencemath.ph-gmuend.de), a well-received EU project funded by the European Commission. Bringing together mathematics and the sciences, this interdisciplinary cooperation project is coordinated by the University of Education Schwäbisch Gmünd in conjunction with various partner universities and schools[1] in Denmark, Germany, Finland, and Slovenia. Its overarching goal is the development and testing of teaching modules based around subject-specific teaching methodology. Module development is guided by the *ScienceMath approach*, which promotes mathematics learning through scientific content and methods. This enables comprehensive and multidimensional learning of mathematics content and concepts, thereby boosting mathematical literacy. The sciences open the door to teaching in applied and realistic contexts, with non-mathematical references helping learners explore mathematical content and methods to the fullest extent in a suitable, interesting, and meaningful way. Scientific content and methods can likewise contribute to closing the gap between formal mathematics and authentic experience. The variety of scientific phenomena allows for open terms of reference and thus the independent development of mathematics. The theoretical background for this approach is the *Realistic Mathematics Education concept*: Developed at the Freudenthal Institute in the Netherlands, this is founded on the idea that different realistic references inspire different models and can therefore help distinguish conceptual attributes.

Another European project *Cross-Curricular Teaching (CROSSCUT)* had a more general approach to interdisciplinary teaching with a focus on all core subjects of secondary education. The objective of the project was to contribute to the improvement of teacher training in the implementation of innovative pedagogical approaches based on interdisciplinarity. CROSSCUT was funded by the EU

[1] Project partners: University of Education (PH) Schwäbisch Gmünd (coordination), University of Southern Denmark Odense, University of Ljubljana, University of Turku, Staufer-Gymnasium Waiblingen, Mulernes Legatskole Odense, Turun Normaalikoulu/Turku teacher training school, and the Diocesan Classical Gymnasium at St. Stanislav's Institution, Ljubljana.

ERASMUS+ program and included partners from six European countries: France, Portugal, Poland, Norway, Finland, and Denmark.[2]

As a part of the project, the University of Southern Denmark completed a situational survey to identify obstacles and enablers that enhance effective cross-curricular teaching. The performed investigations in the survey were exploratory and sought to offer insight into the current state of cross-curricular teaching in France, Portugal, Poland, Norway, Finland, and Denmark. The investigations included (i) reviews of national curricula, policy, and resource documents, (ii) interviews with school leaders, (iii) focus group interview with teachers, and (iv) observation of teaching. A meta-analysis of the results was crystallized into five main themes:

1. **Intended cross-curricular teaching**: There is an increasing emphasis on cross-curricular teaching in curriculum in all six countries. Cross-curricular initiatives are being introduced in all investigated countries, and the national curricula often contain learning objectives that would be accommodated well with cross-curricular teaching and learning activities.
2. **Realized cross-curricular teaching**: Cross-curricular teaching activities are generally encouraged by the school leaders. Their understanding of cross-curricular teaching is very broad and range from a teacher making connections to subjects other than their own to multiple teachers from different subjects cooperating on a common theme. However, school leaders across the six countries do not experience many expectations to the school concerning cross-curricular teaching. The teachers have various approaches to cross-curricular teaching and report varying support from their school management. A general approach is projects revolving around a pre-determined or a student-selected theme with multiple teachers and multiple subjects involved. Such projects typically take place during special project weeks. Another widespread approach to cross-curricular teaching is cross-curricular "correlation," where teachers coordinate to teach the same theme in the same time period but do the actual teaching individually and from the perspective of their own subject in their own class time.
3. **Obstacles and enablers of cross-curricular teaching**: The survey led to the identification of several obstacles for cross-curricular teaching: (i) lack of interest and willingness in changes and development, (ii) lack of cross-curricular exams, (iii) changing education reforms makes teachers less willing to get involved in cross-curricular teaching, (iv) lack of support from school management in terms of allocated planning time, and (v) different aspects of insecurity about teaching cross-curricular teaching examples. Similarly, several enablers for cross-curricular teaching could be identified: (i) time for teacher collaboration, (ii) long-term planning and organizational willingness to learn and adapt, and (iv) curricular flexibility and thus greater teacher autonomy.

[2] Project partners: Centre International d'Études Pédagogiques (coordination), Institut français de l'Éducation, École Normale Supérieure de Lyon, University of Southern Denmark, Educational Research Institute Poland, Universidade do Minho, Universidade Aberta, Norwegian Directorate for Education and Training, University of Helsinki.

4. **Required skills for carrying out cross-curricular teaching**: The requirements to teachers as stated by the school leaders encompass: (i) strong pedagogical competencies, (ii) solid subject knowledge, (iv) willingness to collaborate, and (v) openness to changes and development. This should be supplemented with the teachers' requirements: (i) Individual motivation for cross-curricular teaching, (ii) insight in other disciplines, (iii) curiosity and broad intellectual interests, (iv) management skills and project competencies, (v) good relations among colleagues, and (vi) broad life experience.
5. **Teachers' needs for training**: Regarding training and professional development of cross-curricular teachers the situational survey point to this issues: (i) common understanding of "cross-curricular," (ii) concrete examples of cross-curricular teaching activities, and (iii) supportive leaders who take responsibility of organizing time and space for teachers' development of cross-curricular teaching activities is stressed.

The situational survey shows that variety seems to be a keyword when describing the current status of European cross-curricular teaching practice. And though it seems that only especially open-minded, self-confident, and engaged teachers willingly initiate cross-curricular teaching activities, it appears to be a general agreement among both teachers and school leaders that cross-curricular teaching activities have great potential to increase motivation, foster holistic ways of seeing matters and solve problems, provide meaningfulness to students, and develop key transversal competencies (Michelsen, Egemose, & Hjort, 2018). Obviously, this underscores that there are substantially implications for the teaching of mathematics and its connections to the arts, sciences, and other human endeavors.

2 Interdisciplinary Models and Concepts for Pre-Service and In-Service Teacher Training

Researchers all over the world strive to deploy their findings for the benefit of wider society. The academics in the MACAS network demonstrate a particular commitment to this task, as their findings have a direct or indirect impact on the teaching and learning of mathematics. It is our view that current and future social challenges will be easier to overcome if we succeed in providing people with suitable mathematics education in an interdisciplinary context. Since teacher training academics and schoolteachers serve as important multipliers in this regard, the MACAS concept promotes a direct exchange with active teachers at the dedicated conferences and ensures that findings are continuously incorporated into training of pre-service teachers and further education programs for teachers. Some of the participating locations have even hosted the development of special concepts for teacher education and further education of teachers. After all, there is no shortage of evidence that quality teaching materials and interdisciplinary teaching skills are both key prerequisites for successful interdisciplinarity. Yet theoretical models also provide a crucial foundation for the implementation of interdisciplinary teaching. Thanks to MACAS, it has been possible to pool a variety of models for interdisciplinary teaching and develop these for use in the further education of teachers.

We will outline several examples below.

2.1 Models for Interdisciplinary Teaching

Models for interdisciplinary teaching not only represent a useful tool with which to describe and evaluate interdisciplinary teaching, but can also be used to plan interdisciplinary lessons. In her comprehensive model for interdisciplinary/cross-curricular teaching, Beckmann (2003a, b) demonstrated that there are more than 1,000 different ways to engage in interdisciplinary teaching (cf. also *A Conceptual Framework for Cross-Curricular Teaching* in Beckmann, 2009). Since the model she developed arranges these manifold options in a set of clear structures, it serves as an overall theory for interdisciplinary/cross-curricular teaching that both fulfills a heuristic function and also addresses the design of teaching practice, aspects of action and direct planning, and the theoretical rationale for a given approach to teaching (in accordance with Gudjons, 1999). This model can be used to structure interdisciplinary teaching according to the content framework as well as the respective form of cooperation and organization. One approach to interdisciplinary teaching may therefore be based around the commonalities of the subjects in question, such as the circle in mathematics and circular motion in physics ("parallel orientation"). The differences between the subjects can also inspire collaboration, however, with the differing content and methods being used to boost dedicated learning in one's own subject ("subject orientation"). One example is the use of scientific experiments in learning mathematical concepts. A third potential approach starts with a general topic such as the environment, climate, population, nutrition, or epidemic then attempts to solve the problem at hand through the collaboration of multiple subjects ("comprehensive orientation"). Further approaches to interdisciplinary teaching are divided into three categories, namely those that address only one individual topic or example in a cross-curricular manner ("example-oriented approach"), those encompassing an entire study course ("course-oriented approach") and those that concern a project ("project-oriented approach"). In organizational terms, the *Beckmann model* identifies four levels of cooperation. Across the two lowest levels, interdisciplinarity in one's own subject may feature the use of aspects from another subject ("topic- and major-subject-related form") or it may involve fellow teachers addressing the same topic in parallel within their own subjects ("parallel-topic-related form"). On the higher level, participating subject teachers plan the entire unit together and always teach together whenever this is beneficial for the acquisition of skills—which means the teaching participants may vary accordingly ("parallel-planning form"). Teaching on the highest level is always a combined effort incorporating all participating subjects ("joint-planning form").

There are clear criteria for successful interdisciplinary teaching. Interdisciplinarity must have an enriching effect on learning within the respective disciplines, for example by contributing to deeper conceptual understanding. In addition, a corresponding model should be meaningful and useful for teachers while also providing guidance on the implementation of such forms of teaching. Zell has devised four categories for this purpose—holistic learning, constructivist teaching, problem-oriented learning, and recognizing common structures and boundaries of subject-specific views (Zell, 2010, p. 45ff). He presented his specific model "Mathematics and Science under one roof" (Zell, 2018) at the 2017 MACAS conference in Copenhagen, having originally developed this framework in his doctoral thesis by building on more general models (e.g., Lonning & DeFranco, 1997; Berlin & White, 2001, etc.).

The topic of interdisciplinary models has been addressed on multiple occasions at the various MACAS conferences, with participants presenting numerous specific approaches to ensure successful interdisciplinary teaching. McFeetors and Kim (2020) have demonstrated how the authentic cognitive process of reasoning can establish a connection between subjects, while Norman, Kern, and Moore (2011) identified the integration of engineering as a fruitful approach to connecting mathematics and science. A host of presentations at MACAS have pointed to the significance of physics experiments in ensuring a deeper understanding of mathematical concepts and contributing to mathematical literacy (e.g., Beckmann & Litz, 2005; Höfer, 2008; Zell, 2011). For example, Golez (2008) explored this fruitful connection in a contribution focusing on the "Horizontal launch" experiment.

Additional authors have repeatedly stressed how important modeling is to establishing a successful interdisciplinary link between different subjects (e.g., Michelsen, 2005, 2016; Doorman & Gravemeijer, 2008; Pospiech, 2008). Indeed, the Danish program "MathBio in the study package"—which deals with further education for teachers—is also guided by the didactic finding that mathematical modeling establishes such a beneficial connection between mathematics and sciences that each discipline profits from the other within the scope of teaching (Michelsen, 2018).

2.2 The "MathBio in the Study Package" Program

Since 2005, Danish upper secondary education has been organized into specialized "study packages" containing compulsory subjects, core subjects, and elective subjects. An important feature of each package is that the core subjects form a coherent program, which is ensured through closer interaction between the subjects. While some of the packages include mathematics and biology as core subjects, interdisciplinary teaching across mathematics and biology is required to fulfill the objectives.

In order to provide upper secondary-level mathematics and biology teachers with didactic tools to prepare them for the practical challenges of interdisciplinary teaching, the *Laboratory of Coherent Teaching and Learning* (head: Claus

Michelsen) at the University of Southern Denmark teamed up with Danish Science Gymnasiums (DASG) in 2016/17. Thanks to the financial support received from DASG, it was possible to organize the professional development program “MathBio in the study package” (Michelsen, 2018). This was aimed at enabling teachers to implement interdisciplinary teaching sequences for mathematics and biology within a day-to-day classroom environment.

Arranged as an intervention project, the program was structured around a combination of four seminars at the university and practical phases at the participating teachers’ schools. The fundamental goal of these practical phases was for the teachers to design and implement an interdisciplinary teaching sequence comprising mathematics and biology. As a result, the teachers were introduced to the didactic model for linking mathematics and biology, different ways of organizing interdisciplinary teaching, examples of teaching, and research results relating to interdisciplinarity. They then presented their interdisciplinary teaching sequences for mathematics and biology at a poster session during the final seminar. Surveys conducted during the program also helped with identifying the necessary conditions for ensuring successful collaboration between teachers of mathematics and biology. Key aspects reported by respondents included the availability of quality teaching materials, the use of shared content across both disciplines as an approach, and for participants to achieve a common understanding and speak the same language (further details on the program available in Michelsen, 2018).

2.3 The ScienceMath Professional Development Concept

The benefits of interdisciplinary teaching can only be felt if interdisciplinarity is successfully realized and implemented in the classroom. Well-prepared and suitable teaching materials are crucial in this regard, as is an implementation concept, and the aforementioned EU project *ScienceMath* delivers on both counts.

ScienceMath involved the development of numerous teaching proposals for interdisciplinary mathematics teaching, which were subsequently fleshed out via teaching materials ready for direct deployment. The project Web site www.sciencemath.ph-gmuend.de provides access to the materials in English, Danish, German, Finnish, French, Turkish, Slovenian, and Spanish.

To promote the skilled use of materials within interdisciplinary cooperation between teachers, one-week further education seminars for European teachers were held over the course of several consecutive summers starting in 2009. Based at the Diocesan Classical Gymnasium at St. Stanislav’s Institution in Ljubljana, Slovenia, this represented a lasting achievement of the EU project. A concept addressing further education for teachers was developed; together with the teaching materials, it provides a suitable and comprehensible framework for teacher-oriented further education programs around the world and has received several positive reviews.

The *ScienceMath Professional Development Concept* is structured around five elements of further education, namely a basis module (further education event) and four additional modules (such as intervening periods at school). These elements can be combined in various ways to suit the respective participants. The basis element is a further education program that lasts for approximately one week and consists of presentations and workshops at the university or coordinating institution. A key aim is for participants to explore the interdisciplinary teaching concept, with teaching modules developed through the EU project ScienceMath serving as teaching materials. The length, content, and intensity of the further education concept can be tailored to the participants—for example, the basis module can also be offered as a single event (sample available online at www.sciencemath.ph-gmuend.de > ScienceMath Teacher Training Concept > Download Example).

In the long-term version of the further education concept, the basis module is followed by one or two intervening periods alternating with one-day events at the university. The first intervening period sees the participating teachers work at home to prepare their specific materials for school use. This involves purchasing or creating the necessary materials, copying, or downloading worksheets, and considering how the materials are to be implemented (based on the room, class, cooperation partner, etc.). During the second intervening period, these materials are then implemented in a live teaching environment. The teachers also document their relevant experiences, successes, and problems, with this input being discussed at the final university event and made available to all other participants.

2.4 *"Modeling and Interdisciplinary Teaching"—A Course for Pre-service Teachers*

The demand for increased interdisciplinary work poses a challenge to teachers who traditionally have had their academic training within one or two mono-disciplinary programs. Further, there are many conceptual pitfalls pertaining to the very idea of interdisciplinary teaching and it seems that there is a genuine need for a scholarly discussion about exactly how teachers could be equipped to implement fruitful interdisciplinary activities. To offer future science and mathematics teachers the possibility to prepare themselves for the practical challenges of interdisciplinary teaching, the University of Southern Denmark offers the graduate course "Modeling and interdisciplinary teaching" for upper secondary education teacher students. The course title explicitly highlights that the main idea of the course is mathematical modeling as a tool for interdisciplinary teaching.

The course is designed upon the conception of mathematical modeling as an interdisciplinary competency. In Denmark, the notion of subject competencies functions as a flexible framework for a description of what it means to master a subject independent of specific topics and specific levels. Competency is someone's insightful readiness to act appropriately in situations in a way that is guided by one's knowledge from a subject. Eight mathematical and four science competencies are identified, and the competency of modeling is identified both as a mathematical and as a science competence. In mathematics, the modeling competence includes structuring an intra- or extra-mathematical situation to be modeled, mathematizing the situation, analyzing, and tackling the model, interpreting the results, validation of the model, communicating about the model, monitoring the modeling activity. The reference to the modeling of extra-mathematical situations underscores that the competency should not be considered as a subject-specific competency, but as an interdisciplinary competency. Modeling is a specific problem-solving strategy with scientific and pragmatic purposes, and as a rule, scientific and everyday life problems call on modeling and do not accept traditional and historical determined boundaries between subjects. Thus, the interdisciplinary competency of modeling is the putty that holds together the interacting subjects.

The course is based on a syllabus of research literature concerning interdisciplinary teaching, and activities involving the student–teachers in developing sequences of interdisciplinary teaching. For the implementation of the course, the students are required to do a final exam project in groups of two to four students, where they design an imagined interdisciplinary teaching module and clearly (i) describe the form and extent of the interdisciplinarity, and the place and role of modeling and (ii) argue for their design choices on the basis of the course literature. Examples of students' exams project are: "Energy and sustainability" incorporating mathematics, physics, and social sciences, "What constitutes a dangerous disease?" incorporating mathematics, chemistry and biology, "Which, style of swimming is the 'best' (crawl, breaststroke, or butterfly)?" incorporating mathematics, physics and sports, and "The development of calculus during the Age of Enlightenment" incorporating mathematics and history.

In their exam reports, the students emphasize the relevance and motivational aspects of an interdisciplinary approach and modeling competence as the link between the interdisciplinary context and the subjects. In addition, the students are aware of the challenges of interdisciplinary teaching. This underscores that the course, which has been offered since 2008, familiarize future teachers with a solid basis of literature, including research literature, on interdisciplinary teaching and related topics, which not only enable them to design their own interdisciplinary teaching activities and to do so on a theoretical foothold, but also to equip them to discuss the benefits of taking such a theoretical approach with future (possibly more experienced and established) colleagues (Jankvist et al., 2013); Michelsen, 2015).

Among the early initiatives at MACAS was the proposal from the University of Moncton, Canada, to establish a methodology for building new didactic collaborative environments using wiki technology (Freiman & Lirette-Pitre, 2005). With two didactics courses in science and mathematics education being taught in parallel, the team created a virtual collaborative space for pre-service teachers to exchange their ideas and acquire new knowledge on ten didactic questions reflecting common issues in two high school subjects.