Harish Garg *Editor*

q-Rung Orthopair Fuzzy Sets

Theory and Applications

Springer

q-Rung Orthopair Fuzzy Sets

Harish Garg
Editor

q-Rung Orthopair Fuzzy Sets

Theory and Applications

Springer

Editor
Harish Garg
Thapar Institute of Engineering
and Technology
Patiala, Punjab, India

ISBN 978-981-19-1448-5 ISBN 978-981-19-1449-2 (eBook)
https://doi.org/10.1007/978-981-19-1449-2

Mathematics Subject Classification: 03E72, 62A86, 90C27, 90C29, 90B50, 91A35, 91B06, 90C70

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

Preface

Many decision-making problems in real-life scenarios depend on how to deal with uncertainty, which is typically a big challenge for decision-makers. Mathematical models are not common, but where the complexity is not usually probabilistic, various models emerged along with fuzzy logic approach. The classical approaches are typically based on crisp sets and crisp models that overlook vagueness, hesitancy, and uncertainties. Because of the uncertainty of today's decision-making issues, it is not possible for a decision-maker to interpret all of the information relevant to all decision-making strategies. For handling uncertain real-life problems, researchers have introduced various set-theoretic models. In 1965, Lotfi A. Zadeh introduced the fuzzy set theory as an extension of crisp sets to overcome the uncertainty of real-world problems.

Since the appearance of fuzzy set theory, many applications and extensions of it have been developed which are found in both theoretical and practical studies from engineering to arts and humanities, and from life sciences to physical sciences. In this book, a new extension of the fuzzy sets, entitled q-rung orthopair fuzzy set (q-ROFS), which is an extension of the crisp set, intuitionistic fuzzy set, and the Pythagorean fuzzy sets, is introduced by eminent researchers with several applications. In q-ROFS, each element is characterized by two degrees, namely membership degrees μ and non-membership degree υ, such that the restriction $\mu^q + \upsilon^q \leq 1$ for $\mu, \upsilon \in [0, 1]$, where $q > 0$. The objective of this book is to present some new advanced technologies and algorithms for solving real-life decision-making problems by using the features of the q-ROFSs.

This book consists of 19 chapters. The first two chapters present the mathematical aspects and topology of the q-rung orthopair fuzzy sets. The next five chapters relate to the implementation of the q-rung orthopair set to decision-making problems. These chapters include the application to medical diagnosis, inventory model, and multi-attribute decision-making. The next four chapters are related to generalized q-rung orthopair set and its application which include generalized orthopair fuzzy information measures, CODAS method, generalized orthopair fuzzy 2-tuple linguistic information, etc. Application of the q-rung orthopair fuzzy set to the real-life industrial problems involves five chapters related to green campus transportation, social responsibility evaluation pattern, insurance companies in the healthcare sectors, application to the cite selection of electric vehicles, and supplier selection problem in Industry 4.0 transition. Finally, the last three chapters are on the theory of the extension of q-rung orthopair fuzzy sets as well as their applications to the decision-making process. These extensions include pentagonal orthopair numbers, fuzzy soft sets, and interval-valued dual hesitant q-rung pairs.

Chapter 1 "q-Rung Orthopair Fuzzy Supra Topological Applications in Data Mining Process" introduces the q-rung orthopair fuzzy supra topological spaces and defines their certain mapping. To illustrate the stated topological structure, an algorithm related to multiple attribute decision-making problems is presented and applied to solve the problems related to medical diagnosis and data mining. Chapter 2 "q-Rung Orthopair Fuzzy Soft Topology with Multi-attribute Decision-Making" introduces the concept of q-rung orthopair fuzzy soft topology including sub-topology, interior, exterior, boundary, closure, etc. Moreover, grey relational analysis (GRA) approach, generalized choice-value method (GCVM), and aggregation operators-based techniques accompanied by three algorithms are stated to address q-rung orthopair fuzzy soft uncertain information. Numerical examples of these methods are also presented in real-life problems to discuss their impacts.

Chapter 3 "Decision-Making on Patients' Medical Status Based on a q-Rung Orthopair Fuzzy Max-Min-Max Composite Relation" presents a max–min–max composition relation under q-ROFSs with some theoretic characterizations. Based on these stated relations, authors have presented an algorithm for solving the decision-making problems in the field of medical diagnosis. Chapter 4 "Soergel Distance Measures for q-Rung Orthopair Fuzzy Sets and Their Applications" defines the Soergel-type distances and weighted distances to determine the metric between two q-ROFSs. Corresponding similarity measures are also derived from it. The validity and efficiency of the stated measures are demonstrated through a case study related to the multi-attribute decision-making process. Chapter 5 "TOPSIS Techniques on q-Rung Orthopair Fuzzy Sets and Its Extensions" integrates the concept of the TOPSIS method with q-ROF set for solving MCDM problems with hesitant fuzzy information. An example related to military aircraft overhaul effectiveness is taken to demonstrate it. Additionally, the authors have also introduced the TOPSIS technique for solving decision-making problems by using the q-rung orthopair fuzzy soft sets (q-ROFSfS) features. A case of a medical clinic under certain criteria is presented to illustrate the application of the proposed TOPSIS approach. In Chap. 6 "Knowledge Measure-Based q-Rung Orthopair Fuzzy Inventory Model", the authors have analysed the economic order quantity (EOQ) model with faulty products and screening errors under q-ROFS environment. In the study, the authors develop the EOQ model under two cases, such as replacement warranty claiming strategy with mending option and replacement warranty claiming strategy with emergency purchase option. For both cases, the q-ROF inventory model is framed by presuming the proportion of faulty products and the proportion of misclassification errors as q-rung orthopair fuzzy variables. The knowledge measure-based q-rung orthopair fuzzy inventory model is proposed by computing the knowledge measure of the variables. Finally, the sensitivity analysis with respect to the various parameters is provided for both the cases to strengthen the results. In Chap. 7 "Higher Type q-Rung Orthopair Fuzzy Sets: Interval Analysis", some cross-entropy and Hausdorff distance measures for q-rung interval-valued orthopair fuzzy set (q-RIVOFS) are formulated to scale the divergence and similarity measure between the pairs of q-RIVOFSs. In addition, an integrated q-RIVOFS-TODIM approach is constructed to display the valid applications of q-RIVOFSs in MADM areas. In the method, a linear programming model is established to derive the attribute weights, which can consider not only attributes' contribution to decision-making process but also the credibility of attribute evaluation information. A case of medical waste disposal method selection is presented to illustrate the advantages of q-RIVOFSs and the practicability of the proposed q-RIVOFS-TODIM approach.

In the context of the generalized q-ROFSs, Chap. 8 "Evidence-Based Cloud Vendor Assessment with Generalized Orthopair Fuzzy Information and Partial Weight Data" provides a new decision framework for cloud vendor (CV) selection based on the quality of service offered by the vendor. In this framework, the authors include methods for weight calculation and ranking based on decision-maker's (DM's) data. Also, preferences from each DM are given as input to the evidence-Bayesian approximation algorithm, which determines ordering of cloud vendors based on an individual's opinion. A case study related to CV adoption by an academic institution is provided to illustrate the model. In Chap. 9 "Supplier Selection Process Based on CODAS Method Using q-Rung Orthopair Fuzzy Information", the COmbinative Distance-based ASsessment (CODAS) method is extended to its q-rung orthopair CODAS version for handling the impreciseness and vagueness in decision-making process. A case study related to the supplier selection problem is taken to illustrate the approach. Chapter 10 "Group Decision-Making Framework with Generalized Orthopair Fuzzy 2-Tuple Linguistic Information" deals with the study of the Maclaurin symmetric mean (MSM) operator to the generalized orthopair fuzzy 2-tuple linguistic (GOFTL) set. In this chapter, GOFTL-MSM and the GOFTL weighted MSM, GOFTL dual MSM, and GOFTL weighted dual MSM operators are proposed along with desirable properties to aggregate the fuzzy information. Based on these proposed operators, a group decision-making algorithm is presented and illustrates them with a case study of the selection of the most preferable supplier(s) in enterprise framework group (EFG) of companies. In Chap. 11 "3PL Service Provider Selection with q-Rung Orthopair Fuzzy Based CODAS Method", authors have developed the q-ROF-CODAS method by adapting the CODAS method to q-ROFS. A case of third-party logistics (3PL) service provider selection for a retail company is presented to illustrate the application of the proposed q-ROF CODAS method.

Related to the application of q-ROFSs to real-life industrial problems, Chap. 12 "An Integrated Proximity Indexed Value and q-Rung Orthopair Fuzzy Decision-Making Model for Prioritization of Green Campus Transportation" considers the application of q-ROFSs to sustainable campus transportation. Global warming and air pollution are two of the most severe problems, requiring sustainable and environmentally friendly measures. To address this completely, the authors have proposed a hybrid multi-criteria framework based on the q-Rung orthopair proximity indexed value (q-ROF PIV) method and the logarithm methodology of additive weights (q-ROF PIV-LMAW). In this study, a q-ROF PIV method is combined with an algorithm for determining the weight coefficients of the criteria, based on the application of a logarithmic additive function to define the relationship between the criteria. Chapter 12 "An Integrated Proximity Indexed Value and Q-Rung Orthopair Fuzzy Decision-Making Model for Prioritization of Green Campus Transportation" deals with the application of q-ROFSs to the corporate social responsibility (CSR) evaluation mechanism which helps manage the platform-based enterprises. In Chap. 13 "Platform-Based Corporate Social Responsibility Evaluation with Three-Way Group Decisions under q-Rung Orthopair Fuzzy Environment", authors propose a multi-criteria decision-making method with three-way group decisions in q-rung orthopair fuzzy environment to assess and classify the CSR of platform-based enterprises. Chapter 14 "MARCOS Technique by Using q-Rung Orthopair Fuzzy Sets for Evaluating the Performance of Insurance Companies in Terms of Healthcare Services" extends the measurement of alternatives and ranking according to the compromise solution (MARCOS) technique under the consideration of q-rung orthopair fuzzy numbers and discuss their application to the performance of insurance companies in healthcare sectors. In Chap. 15 "Interval Complex q-Rung Orthopair Fuzzy Aggregation Operators and Their Applications in Cite Selection of Electric Vehicle", the authors have presented the study of the interval complex q-ROFS and investigated their properties. A decision-making algorithm is presented under the uncertain complex features and it validates the study through a case study of site selection of the electric vehicles. In Chap. 16 "A Novel Fermatean Fuzzy Analytic Hierarchy Process Proposition and Its Usage for Supplier Selection Problem in Industry 4.0 Transition", the authors have extended the analytic hierarchy process (AHP) to the Fermeatean fuzzy number, which is a special case of the q-ROFSs, and hence solves the decision-making process problems. A suggested process has been demonstrated through a case study of a real supplier selection problem for Industry 4.0 transition.

The book also deals with the theory of the extensions of the q-ROFS sets such as pentagonal q-ROFS, q-rung orthopair fuzzy soft set, dual hesitant, etc., and their applications to solve the decision-making problems in the last three chapters. In this context, Chap. 17 "Pentagonal q-Rung Orthopair Numbers and Their Applications" deals with the pentagonal q-Rung orthopair fuzzy numbers (Pq-ROFN) and normal Pq-ROFN by using the concept of the norm operations. Some operations on the stated concept are defined and based on it, a multi-attribute decision-making algorithm is presented to solve the problems. Chapter 18 "q-Rung Orthopair Fuzzy Soft Set-Based Multi-criteria Decision-Making" is on to introduce the hybrid structure named q-Rung orthopair fuzzy soft sets (q-ROFSSs) by combining the features of q-ROFSs and Molodtsov's soft sets. Various algebraic properties of the set are stated. Later, certain mathematical models for multi-criteria decision-making (MCDM) problems such as TOPSIS, VIKOR, and similarity measures are defined to solve the MCDM problems. A case study related to the selection of appropriate persons for key ministries of a country, selection of agricultural land, and diagnosis during COVID-19 are investigated. In Chap. 19 "Development of Heronian Mean-Based Aggregation Operators Under Interval-Valued Dual Hesitant q-Rung

Orthopair Fuzzy Environments for Multicriteria Decision-Making", some aggregation operators with Heronian mean concept are defined to aggregate the interval-valued dual hesitant q-rung orthopair fuzzy (IVDHq-ROF) environment. Based on the proposed operators, it presents an approach for multi-criteria decision-making problems to solve problems.

We hope that this book will provide a useful resource of ideas, techniques, and methods for the research on the theory and applications of q-rung orthopair fuzzy sets. We are grateful to the referees for their valuable and highly appreciated work contributed to select the high-quality chapters published in this book. We would like to also thank Springer Nature and the team for their supportive role throughout the process of editing this book.

Patiala, India Harish Garg

Contents

About the Editor

Harish Garg is Associate Professor at the School of Mathematics, Thapar Institute of Engineering and Technology, Patiala, India. He completed his Ph.D. in Mathematics from the Indian Institute of Technology Roorkee, India, in 2013. His research interests include soft computing, decision making, aggregation operators, evolutionary algorithm, expert systems, and decision support systems. He has authored around 330 papers, published in international journals of repute, and has supervised 7 Ph.D. dissertations. He is Recipient of the Top-Cited Paper by India-based Author (2015–2019) from Elsevier. He serves as Editor-in-Chief for *Journal of Computational and Cognitive Engineering; Annals of Optimization Theory & Practice* and Associate Editor for several renowned journals. His google citations are over 13000 with h-index 65. For more details, visit http://sites.google.com/site/harishg58iitr/.

Chapter 1
q-Rung Orthopair Fuzzy Supra Topological Applications in Data Mining Process

Mani Parimala, Cenap Ozel, M. A. Al Shumrani, and Aynur Keskin Kaymakci

Abstract The idea of q-rung orthopair fuzzy sets is an extension of intuitionistic and Pythagorean fuzzy sets. The main goal of this manuscript is to present the notion of q-rung orthopair fuzzy supra topological spaces (q-rofsts), a hybrid form of intuitionistic fuzzy supra topological spaces and Pythagorean fuzzy supra topological spaces. In addition, several contradictory examples and their assertions in fuzzy supra topological spaces of Abd El-Monsef and Ramadan (Indian J Pure Appl Math 18(4):322–329, 1987, [9]) are produced using q-rung orthopair fuzzy mappings. Finally, a new multiple attribute decision-making technique based on the q-rung orthopair fuzzy scoring function is suggested as an application to tackle medical diagnosis issues.

Keywords Fuzzy topology · Intuitionistic topology · Pythagorean fuzzy topology · q-rung orthopair fuzzy topology · q-rung orthopair fuzzy supra topology

---

M. Parimala (✉)
Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam, Tamil Nadu, India
e-mail: rishwanthpari@gmail.com

C. Ozel · M. A. A. Shumrani
Department of Mathematics, King Abdulaziz University, 80203, Jeddah 21589, Saudi Arabia
e-mail: maalshmrani1@kau.edu.sa

A. K. Kaymakci
Department of Mathematics, Faculty of Sciences, Selcuk University, 42030 Konya, Turkey
e-mail: akeskin@selcuk.edu.tr

H. Garg (ed.), *q-Rung Orthopair Fuzzy Sets*,
https://doi.org/10.1007/978-981-19-1449-2_1

1.1 Introduction

In 1965, Zadeh [1] developed the notion of fuzzy set, an expansion of the crisp set for analyzing unreliable mathematical information. Fuzzy logic and fuzzy set theory concepts were used by Adlassnig [2] to establish medical connections in a computerized diagnosis system. The idea of [3–5] has been said in the disciplines of control theory, biology, artificial intelligence, economics and probability to name a few. Chang [6] proposed the concept of fuzzy topological spaces, and Lowen [7] investigated its characteristics further. In [8], Mashhour et al. developed supra topological space by weakening one topological postulate and analyzing its characteristics. Abd El-Monsef and Ramadan et al. [9] presented fuzzy supra topological spaces (fsts). Atanassov [10] and Yager [11] developed intuitionistic fuzzy sets and Pythagorean fuzzy sets, respectively, that addressed both membership degree and non-membership degree of an element. Intuitionistic fuzzy topology was presented by Coker [12]. Saadati and Park [13] went on to research the fundamental notion of intuitionistic fuzzy poiq-rofts. De et al. [14] were the first to explore intuitionistic fuzzy set applications in medical diagnosis. Several researchers [15–18] explored intuitionistic fuzzy sets in medical diagnostics in more depth. In 2013, Yager [11] proposed the concept of Pythagorean fuzzy sets. Later in 2019, the concept of Pythagorean fuzzy topological space was proposed by Olgun et al. [19].

The q-rung orthopair fuzzy set (q-ROFS) is a notion that may be used for real-world engineering and scientific applications. Yager [20] was the first to introduce and begin the q-rung orthopair fuzzy set in 2017. Since its appearance, researchers are engaged in the extensions and their applications to the decision-making process. For instance, Garg [21] presented a decision-making algorithm by defining the concept of sine-trigonometric operational laws. Wang et al. [22] discussed the green supplier selection problem using the concept of q-ROFS. Wei et al. [23] discussed the Heronian mean operators for the decision-making problems under the pairs of q-ROFSs. Garg [24] integrated the concept of the q-ROFS with the connection number (CN) of the set pair analysis and hence defined the idea of CN-qROFS. The applicability of this concept has been demonstrated through a decision-making process. Wang et al. [25] presented a group decision-making algorithm with q-ROFS and linguistic features. Related to the topological spaces, Turkarslan et al. [26] in 2021 introduced the concept of q-rung orthopair fuzzy topological spaces.

The rest of the chapter is summarized as follows: In Sect. 1.2, we presented some essential preliminaries of fuzzy, intuitionistic fuzzy, Pythagorean fuzzy and q-rung orthopair fuzzy sets and topological spaces. In Sect. 1.3, the idea of q-rung orthopair fuzzy supra topological spaces is defined. We introduce supra continuity and S^*-q-rung orthopair fuzzy continuity giving some contradicting examples in [9] in Sect. 1.4. In Sect. 1.5, an algorithm for data processing in a supra topological of q-rung orthopair fuzzy environment is provided as a real-world application. We solve a numerical example of the above-suggested technique in Sect. 1.6. The conclusion and future work of this article is stated in Sect. 1.7.

1.2 Preliminary

This part of the paper includes some of the preliminary definitions of fuzzy, intuitionistic, Pythagorean, q-rung orthopair fuzzy sets and respective topological spaces that will be utilized in this work.

Definition 1.2.1 ([1]) $A = \{(x, \mu_A(x)) : x \in X\}$ is said to be a fuzzy set on the non-void set X; for every $x \in X$, the membership function is $\mu_A(x) \in [0, 1]$ of A.

Definition 1.2.2 A set $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, where X is a non-void set, is called

1. an intuitionistic fuzzy set (IFS) [10], where $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The set of all intuitionistic sets of X is denoted by $IFS(X)$.
2. a Pythagorean fuzzy set (PyFS) [11], where $0 \leq \mu_A^2(x) + \nu_A^2(x) \leq 1$. $PyFS(X)$ represents all the sets of Pythagorean fuzzy sets of X.
3. a q-rung orthopair fuzzy set (qROFS) [20], where $0 \leq \mu_A^q(x) + \nu_A^q(x) \leq 1$. The set of all q-rung orthopair fuzzy sets of X is denoted by $qROFS(X)$.

For all $x \in X$, membership function degree is $\mu_A(x)$ and non- membership function $\nu_A(x)$ of A.

We simply argue that qROFS may be divided into classes of orthopair fuzzy numbers with unique q values by definition.

When $q = 1$, for example, it becomes IFS and when $q = 2$, it turns into a PyFS. As a result, IFS and PyFS are subtypes of qROFS. The diagrammatic representation of these sets is given in Fig. 1.1.

Definition 1.2.3 ([20]) The below statements hold for qROFS sets A and B on X:

1. $A \cup B = \vee(\mu_A(x), \mu_B(x)), \wedge(\nu_A(x), \nu_B(x))$.
2. $A \cap B = \wedge(\mu_A(x), \mu_B(x)), \vee(\nu_A(x), \nu_B(x))$.
3. $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
4. $A \oplus B = (\sqrt[q]{\mu_A^q(x) + \mu_B^q(x) - \mu_A^q(x)\mu_B^q(x)}, \nu_A(x)\nu_B(x))$.
5. $A \otimes B = (\mu_A(x)\mu_B(x), \sqrt[q]{\nu_A^q(x) + \nu_B^q(x) - \nu_A^q(x)\nu_B^q(x)})$.
6. $A^c = (\nu_A(x), \mu_A(x))$.
7. $\alpha A = \sqrt[q]{1 - (1 - \mu_A^q(x))^\alpha}, \nu_A^\alpha(x)$, for $\alpha \geq 1$.
8. $A^\alpha = \mu_A^\alpha(x), \sqrt[q]{1 - (1 - \nu_A^q(x))^\alpha}$, for $\alpha \geq 1$.

Notation 1.2.4 1. $1_q = (1, 0)$ is the qROFS whole set.
2. $0_q = (0, 1)$ is the qROFS empty set.

Definition 1.2.5 ([20]) Let $A = (\mu_A(x), \nu_A(x))$ be a q-ROF number, then the

1. score function is $S(A) = \frac{1+\mu_A^q(x)-\nu_A^q(x)}{2}$.
2. accuracy function is $H(A) = \mu_A^q(x) + \nu_A^q(x)$.

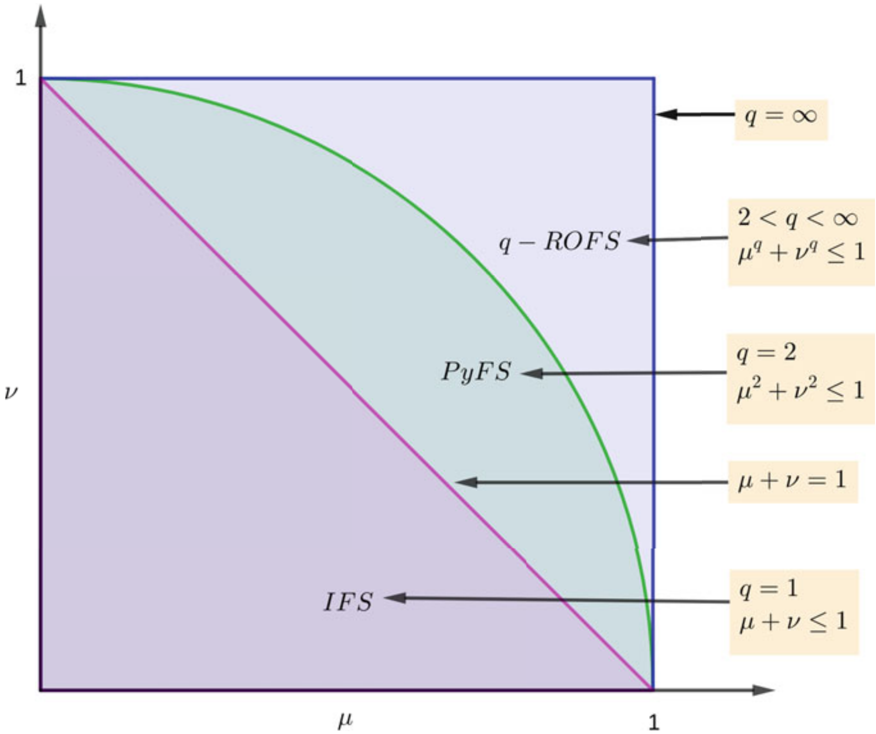

Fig. 1.1 Spaces of IFS, PyFS and qROFS

Definition 1.2.6 ([6]) A sub-collection τ_f of I^X is called a fuzzy topology on X, if

1. $X, \emptyset \in \tau_f$,
2. $\cup A_f \in \tau_f$ and
3. $\cap_{f=1}^{n} A_i \in \tau_f$.

Then (X, τ_f) is called fuzzy topological space (fts).

Definition 1.2.7 ([12]) A subfamily τ_i of $I^X$is called intuitionistic fuzzy topology on X if

1. $X, \emptyset \in \tau_i$,
2. $\cup A_i \in \tau_i$ and
3. $\cap_{i=1}^{n} A_i \in \tau_i$.

Then (X, τ_i) is called intuitionistic fuzzy topological space (ifts).

Definition 1.2.8 ([19]) A subfamily τ_p of I^X is said to be Pythagorean fuzzy topology on X if

1. $X, \emptyset \in \tau_p$,

2. $\cup A_p \in \tau_p$ and
3. $\cap_{p=1}^{n} A_p \in \tau_p$.

Then (X, τ_p) is called Pythagorean fuzzy topological space (Pyfts), elements of τ_p are called open Pythagorean fuzzy sets and their complements are closed Pythagorean fuzzy sets.

Definition 1.2.9 ([26]) A subfamily τ_q of $N(X)$ is said to be q-rung orthopair fuzzy topology (q-roft) on X if

1. $X, \emptyset \in \tau_q$,
2. $\cup A_q \in \tau_q$ and
3. $\cap_{q=1}^{n} A_q \in \tau_q$.

Then (X, τ_q) is known as q-rofts, each member of τ_q is called open q-rung orthopair fuzzy set and the complement q-roft of an open q-rung orthopair fuzzy set is called a closed q-rung orthopair fuzzy set of qROF.

If A is a qROFS, then A has the following closure and interior defined as follows:

1. $\mathrm{int}_q(A) = \cup\{G : G \subseteq A, G \in \tau_q\}$.
2. $\mathrm{cl}_q(A) = \cap\{F : A \subseteq F, F^c \in \tau_q\}$.

Definition 1.2.10 ([26]) Let $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$, $B = \{(y, \mu_B(y), \nu_B(y)) : y \in Y\}$ $f : X \to Y$ be a function between two qROFS.

1. B has a pre-image under the map f, then $f^{-1}(B) = \{(x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)) : x \in X\}$ is a qROFS on X.
2. A has an image of the map f, then $f(A) = \{(y, f(\mu_A)(y), f(\nu_A)(y)) : y \in Y\}$ is a qROFS on Y, where

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \nu_A(x), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

1.3 q-Rung Orthopair Fuzzy Supra Topological Spaces

We introduce q-rung orthopair fuzzy supra topological spaces (q-rofsts) and study their properties in this section.

Definition 1.3.1 A q-rofst on a non-void set X is a collection τ_q^* of qROFS of X having the following properties:

1. the sets $1_q, 0_q \in \tau_q^*$.
2. $\cup_{i=1}^{\infty} A_i \in \tau_q^*$ for $\{A_i\} \in \tau_q^*$.

Then (X, τ_q^*) is said to be q-rofsts on X. τ_q^*'s members are called q-rofs-open sets and q-rofs-closed set is a complement set. A q-rofsts τ_q^* on X is known to be an associated q-rofsts with qroft τ_q if $\tau_q \subseteq \tau_q^*$. Every qroft on X is q-rofst on X.

Definition 1.3.2 ([9]) A subfamily $\tau_f^*$of I^X is called fuzzy supra topology on X if

1. the sets $1_f, 0_f \in \tau_q^*$.
2. $\cup_{i=1}^{\infty} A_i \in \tau_f^*$ for $\{A_i\} \in \tau_f^*$.

Then (X, τ_f^*) is said to be fuzzy supra topological space (fsts) on X. The members of τ_f^* are called fuzzy supra open sets.

Definition 1.3.3 ([27]) A subfamily $\tau_i^*$of I^X is called intuitionistic supra fuzzy topology on X if

1. the sets $1_i, 0_i \in \tau_q^*$.
2. $\cup_{i=1}^{\infty} A_i \in \tau_i^*$ for $\{A_i\} \in \tau_i^*$.

Then (X, τ_i^*) is said to be intuitionistic supra fuzzy topological space (isfts) on X. The members of τ_i^* are called isf-open sets.

Proposition 1.3.4 *The $(\tau_q^*)^c$ of all q-rofs-closed sets in (X, τ_q^*) holds: $\emptyset, X \in (\tau_q^*)^c$ and $(\tau_q^*)^c$ is closed under the arbitrary intersection.*

Proof The result of the proposition is trivial. ∎

Definition 1.3.5 The q-rofst interior $\text{int}_{\tau_q^*}(A)$ and closure $\text{cl}_{\tau_q^*}(A)$ operators of a q-rofs A are, respectively, defined as

1. $\text{cl}_{\tau_q^*}(A) = \cap\{F : A \subseteq F \text{ and } F^c \in \tau_q^*\}$ and
2. $\text{int}_{\tau_q^*}(A) = \cup\{G : G \subseteq A \text{ and } G \in \tau_q^*\}$.

Theorem 1.3.6 *Let (X, τ_q^*) be a q-rofsts. Let A and B be q-rofs in X. Then*

1. $A = \text{cl}_{\tau_q^*}(A)$ *iff A is q-rofs closed.*
2. $A = \text{int}_{\tau_q^*}(A)$ *iff A is q-rofs open.*
3. $\text{cl}_{\tau_q^*}(A) \subseteq \text{cl}_{\tau_q^*}(B)$, *if $A \subseteq B$.*
4. $\text{int}_{\tau_q^*}(A) \subseteq \text{int}_{\tau_q^*}(B)$, *if $A \subseteq B$.*
5. $\text{cl}_{\tau_q^*}(A) \cup \text{cl}_{\tau_q^*}(B) \subseteq \text{cl}_{\tau_q^*}(A \cup B)$.
6. $\text{int}_{\tau_q^*}(A) \cup \text{int}_{\tau_q^*}(B) \subseteq \text{int}_{\tau_q^*}(A \cup B)$.
7. $\text{cl}_{\tau_q^*}(A) \cap \text{cl}_{\tau_q^*}(B) \supseteq (A \cap B)$.
8. $\text{int}_{\tau_q^*}(A) \cap \text{int}_{\tau_q^*}(B) \supseteq (A \cap B)$.
9. $\text{int}_{\tau_q^*}(A^c) = (\text{cl}_{\tau_q^*}(A))^c$.

Proof Only we show (iii), (v) and (ix).

(iii): $\text{cl}_{\tau_q^*}(B) = \cap\{G : G^c \in \tau_q^*, B \subseteq G\} \supseteq \cap\{G : G^c \in \tau_q^*, A \subseteq G\} = \text{cl}_{\tau_q^*}(A)$. Thus $\text{cl}_{\tau_q^*}(A) \subseteq \text{cl}_{\tau_q^*}(B)$.

(v): Since $A \cup B \supseteq A, B$, then $\text{cl}_{\tau_q^*}(A) \cup \text{cl}_{\tau_q^*}(B) \subseteq \text{cl}_{\tau_q^*}(A \cup B)$.

(ix): $\text{cl}_{\tau_q^*}(A) = \cap\{G : G^c \in \tau_q^*, G \supseteq A\}$, $(\text{cl}_{\tau_q^*}(A))^c = \cup G^c : G^c$ is a q-rung orthopair fuzzy supra open in X and $G^c \subseteq A^c\} = \text{int}_{\tau_q^*}(A^c)$. Thus, $(\text{cl}_{\tau_q^*}(A))^c = \text{int}_{\tau_q^*}(A^c)$. ∎

Remark 1.3.7 In q-rofts, we have $\mathrm{cl}_{\tau_q}(A \cup B) = \mathrm{cl}_{\tau_q}(A) \cup \mathrm{cl}_{\tau_q}(B)$ and $\mathrm{int}_{\tau_q}(A \cap B) = \mathrm{int}_{\tau_q}(A) \cap \mathrm{int}_{\tau_q}(B)$. The example below shows that these equalities are not true in q-rofsts.

Example 1.3.8 Let $X = \{a, b\}$ with q-rofs (q=3) $A = ((0.6, 0.85), (0.5, 0.9))$, $B = ((0.6, 0.9), (0.7, 0.85))$ and $C = ((0.7, 0.95), (0.8, 0.7))$, and let 3-rofst be $\tau_q^* = \{\emptyset, X, A, B, C\} = \{\emptyset, X, ((0.6, 0.85), (0.5, 0.9)), ((0.6, 0.9), (0.7, 0.85)), ((0.7, 0.95), (0.8, 0.7))\}$. Then $(\tau_q^*)^c = \{X, \emptyset, ((0.85, 0.6), (0.9, 0.5)), ((0.9, 0.6), (0.85, 0.7)), ((0.95, 0.7), (0.7, 0.8))\}$. Now $cl_{\tau_q^*}(A \cup B) = ((0.85, 0.6), (0.7, 0.85))$ and $cl_{\tau_q^*}(A) \cup cl_{\tau_q^*}(B) = ((0.9, 0.6), (0.9, 0.5))$. Therefore, $\mathrm{cl}_{\tau_q^*}(A \cup B) \neq \mathrm{cl}_{\tau_q^*}(A) \cup \mathrm{cl}_{\tau_q^*}(B)$. And now $int_{\tau_q^*}(A \cap B) = ((0.6, 0.85), (0.5, 0.85))$ and $int_{\tau_q^*}(A) \cap int_{\tau_q^*}(B) = ((0.6, 0.9), (0.5, 0.9))$. Therefore, $int_{\tau_q^*}(A \cap B) \neq int_{\tau_q^*}(A) \cap int_{\tau_q^*}(B)$.

1.4 Mappings of q-Rung Orthopair Fuzzy Spaces

The characteristics of various mappings in q-rung orthopair fuzzy supra topological spaces are defined and established in this section.

Definition 1.4.1 Let τ_q^* and σ_q^* be associated q-rofst to τ_q and σ_q, respectively. A mapping f from a q-rofts (X, τ_q) into q-rofts (Y, σ_q) is called S^*-q-rof-open if every image of q-rof-open set in (X, τ_q) is q-rofs-open in (Y, σ_q^*) and $f : X \to Y$ is known as S^*-q-rof continuous if each inverse image of q-rof-open set in (Y, σ_q^*) is q-rofs-open in (X, τ_q^*).

Definition 1.4.2 Let τ_q^* and σ_q^* be associated q-rofst to q-rofts's τ_q and σ_q, respectively. A mapping f from a q-rofts (X, τ_q) into a q-rofts (Y, σ_q) is called supra q-rof-open if every image of q-rofs-open set in (X, τ_q^*) is a q-rofs=open in (Y, σ_q^*) and $f : X \to Y$ is known as supra q-rof continuous if every inverse image of q-rofs-open set in (Y, σ_q^*) is q-rofs-open in (X, τ_q^*).

A mapping f of q-rofts (X, τ_q) into q-rofts (Y, σ_q) is known as a mapping of q-rof subspace $(A, (\tau_q)_A)$ into q-rof subspace $(B, (\sigma_q)_B)$ if $f(A) \subset B$.

Definition 1.4.3 f is a map from q-rof subspace $(A, (\tau_q)_A)$ of q-rofts (X, τ_q) into q-rof subspace $(B, (\sigma_q)_B)$ of q-rofts (Y, σ_q) is known as relatively q-rof continuous if $f^{-1}(O) \cap A \in (\tau_q)_A$ for each $O \in (\sigma_q)_B$. If $f(O') \in (\sigma_q)_B$ for each $O' \in (\tau_q)_A$, then f is called relatively q-rof-open.

Theorem 1.4.4 *If a mapping f is q-rung orthopair fuzzy continuous from q-rofts (X, τ_q) into q-rofts (Y, σ_q) and $f(A) \subset B$, then f is relatively q-rof continuous from q-rof subspace $(A, (\tau_q)_A)$ of q-rofts (X, τ_q) into q-rof subspace $(B, (\sigma_q)_B)$ of q-rofts (Y, σ_q).*

Proof Consider $O \in (\sigma_q)_B$, then $\exists\ G \in \sigma_q \ni O = B \cap G$ and $f^{-1}(G) \in \tau_q$. That is $f^{-1}(O) \cap A = f^{-1}(B) \cap f^{-1}(G) \cap A = f^{-1}(G) \cap A \in (\tau_q^*)_A$. ■

Remark 1.4.5 1. Each q-rof continuous (q-rof open) map is S^*-q-rof continuous (S^*-q-rof open). Generally, the converse of the statement is not true.
2. Each supra q-rof continuous (resp. supra q-rof open) mapping is S^*-q-rung orthopair fuzzy continuous (resp. S^*-q-rung orthopair fuzzy open). Generally, the converse of the statement is not true.
3. All mappings of supra q-rung orthopair fuzzy continuous and q-rof continuous are independent.
4. All mappings of supra q-rung orthopair fuzzy open and q-rof open are independent.

Proof This follows directly from the definitions. ■

Observation 1.4.6 For contradicting the statement q-rofts, we have the following few examples from [9]. In fsts, consider $Y = \{x, y, z\}$, $X = \{a, b, c\}$ with fuzzy topologies $\tau_f = \{\emptyset, X, (0.75, 0.85), (0.95, 0.65)\}$ and $\sigma_f = \{\emptyset, Y, (0.95, 0.65)\}$. Let $\tau_f^* = \{\emptyset, X, (0.75, 0.85), (0.95, 0.65), (0.85, 0.75)\}$ and $\sigma_f^* = \{\emptyset, Y, (0.95, 0.65), (0.3, 0.65), (0.75, 0.65)\}$ be associated fst of τ_f and σ_f, respectively. Let $h : X \to Y$ be a mapping defined by $h(c) = z$, $h(b) = y$, $h(a) = x$. Then $h^{-1}((0.75, 0.85)) = (0.75, 0.85) \notin \tau_f^*$. Hence, h is fuzzy continuous however it is not supra fuzzy continuous. If $g : Y \to X$ is a mapping defined by $g(z) = c$, $g(y) = b$, $g(x) = a$, then g is fuzzy open however it is not supra fuzzy open.

Theorem 1.4.7 *Let (X, τ_q) and (Y, σ_q) be q-rofts and let $f : X \to Y$. Then the following are equivalent.*

1. *$f : X \to Y$ is S^*-q-rung orthopair fuzzy continuous.*
2. *Every inverse image of q-rof closed set in (Y, σ_q) is q-rofs closed in (X, τ_q^*).*
3. *Every q-rof set A in Y, $\mathrm{cl}_{\tau_q^*}(f^{-1}(A)) \subseteq f^{-1}\mathrm{cl}_{\sigma_q^*}(A))$.*
4. *Every q-rof set B in X, $f(\mathrm{cl}_{\tau_q^*}(B)) \subseteq \mathrm{cl}_{\sigma_q^*}(f(B))$.*
5. *Every q-rof set A in Y, $\mathrm{int}_{\tau_q^*}(f \to (A)) \supseteq f^1(\mathrm{int}_{\sigma_q^*}(A))$.*

Proof (i) $\Rightarrow$ (ii): Assume f is a S^*-q-rof continuous and A is a q-rof closed set in (Y, σ_q). $f^{-1}(Y - A) = X - f^{-1}(A)$ is q-rofs open in (X, τ_q^*) and so $f^{-1}(A)$ is q-rofs closed in (x, τ_q^*).
(ii) $\Rightarrow$ (iii): $\mathrm{cl}_{\sigma_q}(A)$ is q-rung orthopair fuzzy closed in (Y, σ_q), for each q-rung orthopair fuzzy set $A \in Y$, then $f^{-1}(\mathrm{cl}_{\sigma_q}(A))$ is q-rung orthopair fuzzy supra closed in (X, τ_q^*). Thus $f^{-1}(\mathrm{cl}_{\sigma_q}(A)) = \mathrm{cl}_{\tau_q^*}(f^{-1}(\mathrm{cl}_{\sigma_q}(A))) \supseteq \mathrm{cl}_{\tau_q^*}(f^{-1}(A))$.
(iii) $\Rightarrow$ (iv): $f^{-1}(\mathrm{cl}_{\sigma_q}(f(B))) \supseteq \mathrm{cl}_{\tau_q^*}(f^{-1}(f(B))) \supseteq \mathrm{cl}_{\tau_q^*}(B)$, for every q-rof set B in X and so $f(\mathrm{cl}_{\tau_q^*}(B)) \subseteq \mathrm{cl}_{\sigma_q}(f(B))$.
(iv) $\Rightarrow$ (ii): Let $B = f^1(A)$, for every q-rof closed set A in Y, then $f(\mathrm{cl}_{\tau_q^*}(B)) \subseteq \mathrm{cl}_{\sigma_q}(f(B)) \subseteq \mathrm{cl}_{\sigma_q}(A) = A$ and $\mathrm{cl}_{\tau_q^*}(B) \subseteq f^{-1}(f(\mathrm{cl}_{\tau_q^* T}(B))) \subseteq f^{-1}(A) = B$. Hence, $B = f^{-1}(A)$ is q-rofs closed in X.
(ii) $\Rightarrow$ (i): Let A be a q-rof open set in Y. Then $X - f^{-1}(A) = f^{-1}(Y - A)$ is q-rofs closed in X, since $Y - A$ is q-rof closed in Y. Therefore, $f^{-1}(A)$ is q-rofs open in X.

(i) $\Rightarrow$ (v): $f^{-1}(\text{int}_{\sigma_q}(A))$ is q-rofs open in X, for every q-rof set A in Y and $\text{int}_{\tau_q^*}(f^{-1}(A)) \supseteq \text{int}_{\tau_q^*}(f^{-1}(\text{int}_{\sigma_q}(A))) = f^{-1}(\text{int}_{\sigma_q}(A))$.
(v) $\Rightarrow$ (i): $f^{-1}(A) = f^{-1}(\text{int}_{\sigma_q}(A)) \subseteq \text{int}_{\tau_q^*}(f^{-1}(A))$, for every q-rof open set A in Y and so $f^{-1}(A)$ is q-rofs open in X. ■

Theorem 1.4.8 *Let (X, τ_q) and (Y, σ_q) be q-rofts and let $f : X \to Y$. Then the following are equivalent.*

1. *A mapping $f : (X, \tau_q^*) \to (Y, \sigma_q^*)$ is q-rofs continuous.*
2. *Every inverse image of q-rofs closed set in (Y, σ_q^*) is q-rofs closed in (X, τ_q^*).*
3. *Every q-rof set A in Y,* $\text{cl}_{\tau_q^*}(f^{-1}(A)) \subseteq f^{-1}(\text{cl}_{\sigma_q^*}(A)) \subseteq f^{-1}(\text{cl}_{\tau_q}(A))$.
4. *Every q-rof set B in X,* $f(\text{cl}_{\tau_q^*}(B)) \subseteq \text{cl}_{\sigma_q^*}(f(B)) \subseteq \text{cl}_{\gamma_q^*}(f(B))$.
5. *Every q-rof set A in Y,* $\text{int}_{\tau_q^*}(f^{-1}(A)) \supseteq f^{-1}(\text{int}_{\sigma_q^*}(A)) \supseteq f^{-1}(\text{int}_{\sigma_q}(A))$.

Proof The proof follows directly from Theorem 1.4.7 ■

Theorem 1.4.9 *If $f : X \to Y$ is S^*-q-rof continuous and $g : Y \to Z$ is q-rof continuous, then $g \circ f : X \to Z$ is S^*-q-rof continuous.*

Proof From the definitions, the proof follows. ■

Theorem 1.4.10 *If $f : X \to Y$ is supra q-rof continuous and $g : Y \to Z$ is S^*-q-rof continuous (or q-rof continuous), then $g \circ f : X \to Z$ is S^*-q-rof continuous.*

Proof From the definitions, the proof follows. ■

Theorem 1.4.11 *If the mappings $f : X \to Y$ and $g : Y \to Z$ are supra q-rof continuous (supra q-rof open), then $g \circ f : X \to Z$ is supra q-rof continuous (supra q-rof open).*

Proof From the definitions, the proof is achieved. ■

1.5 Algorithm for Data Mining Problem Via q-Rung Orthopair Fuzzy Supra Topology

We offer a systematic technique for a multi-attribute decision-making (MADM) problem using q-rung orthopair fuzzy information in this part. The methodical technique to select the appropriate qualities and alternatives in a decision-making scenario is proposed in the following phases.
Step 1: Problem selection:
Consider a MADM problem with m attributes $\alpha_1, \alpha_2, \ldots, \alpha_m$ and n alternatives $\beta_1, \beta_2, \ldots, \beta_n$ and p attributes $\gamma_1, \gamma_2, \ldots, \gamma_p$, $(n \leq p)$.

	β_1	β_2	.	.	.	β_n
α_1	(σ_{11})	(σ_{12})	.	.	.	(σ_{1n})
α_2	(σ_{21})	(σ_{22})	.	.	.	(σ_{2n})
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
α_m	(σ_{m1})	(σ_{m2})	.	.	.	(σ_{mn})

	α_1	α_2	.	.	.	α_m
γ_1	(ζ_{11})	(ζ_{12})	.	.	.	(ζ_{1m})
γ_2	(ζ_{21})	(ζ_{22})	.	.	.	(ζ_{2m})
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
γ_p	(ζ_{p1})	(ζ_{p2})	.	.	.	(ζ_{pm})

Here, all the attributes and ζ_{ki} $(i = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, p)$ are q-rung orthopair fuzzy numbers.

Step 2: Form q-rung orthopair fuzzy supra topologies for (β_j) and (γ_k):

1. $\tau_j^* = A \cup B$, where $A = \{1_q, 0_q, \sigma_{1j}, \sigma_{2j}, \ldots, \sigma_{mj}\}$ and $B = \{\sigma_{1j} \cup \sigma_{2j}, \sigma_{1j} \cup \sigma_{3j}, \ldots, \sigma_{m-1j} \cup \sigma_{mj}\}$.
2. $\nu_k^* = C \cup D$, where $C = \{1_q, 0_q, \zeta_{kl}, \zeta_{k2}, \ldots, \zeta_{km}\}$ and $D = \{\zeta_{k1} \cup \zeta_{k2}, \zeta_{kl} \cup \zeta_{k3}, \ldots, \zeta_{km-i} \cup \zeta_{km}\}$.

Step 3: Find q-ROFSF: q-ROFSF of A, B, C, D, β_j and γ_k is stated in the following way.

1. q-ROFSF$(A) = \frac{1}{2(m+2)}\left[\sum_{i=1}^{m+2}[\frac{1+\mu_i^q-\nu_i^q}{2}]\right]$, and
   q-ROFSF$(B) = \frac{1}{2r}\left[\sum_{i=1}^{r}[\frac{1+\mu_i^q-\nu_i^q}{2}]\right]$. For $j = 1, 2, \ldots, n$,
   $$\text{q-ROFSF}(C_j) = \begin{cases} \text{q-ROFSF}(A), & \text{if q-ROFSF}(B) = 0 \\ \frac{1}{2}[\text{q-ROFSF}(A) + \text{q-ROFSF}(B)], & \text{otherwise} \end{cases}$$
2. q-ROFSF$(C) = \frac{1}{2(m+2)}\left[\sum_{i=1}^{m+2}[\frac{1+\mu_i^q-\nu_i^q}{2}]\right]$, and
   q-ROFSF$(D) = \frac{1}{2s}\left[\sum_{i=1}^{s}[\frac{1+\mu_i^q-\nu_i^q}{2}]\right]$. For $k = 1, 2, \ldots, p$,
   $$\text{q-ROFSF}(D_k) = \begin{cases} \text{q-ROFSF}(C), & \text{if q-ROFSF}(D) = 0 \\ \frac{1}{2}[\text{q-ROFSF}(C) + \text{q-ROFSF}(D)], & \text{otherwise} \end{cases}$$

Step 4: Final Decision

Arrange q-rung orthopair fuzzy score values for the alternatives $C_1 \le C_2 \le \cdots \le C_q$ and the attributes $D_1 \le D_2 \le \cdots \le D_p$. Choose the attributes D_p and D_{p-1} for the alternatives C_1 and C_2 and vice versa. If $n < p$ is true, D_k is ignored.

The proposed Algorithm for MADM via q-rung orthopair fuzzy supra topological spaces is represented as a flowchart in Fig. 1.2.

1.6 Numerical Example

New technologies in medical field have expanded the amount of information available to medical doctors, which includes uncertainty. The practice of categorizing diverse sets of indications under a specific pattern of a disease is a tough challenge in medical

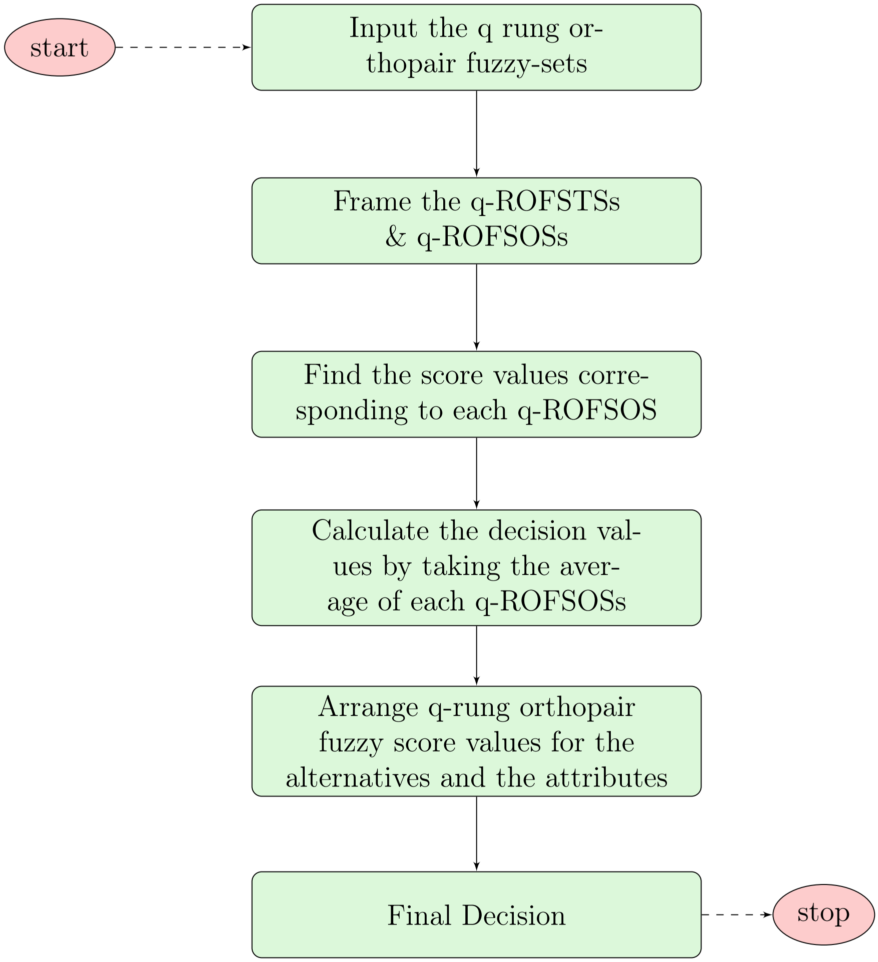

Fig. 1.2 Flowchart of the proposed Algorithm

diagnosis. In this part, we use a clinical diagnosis problem to demonstrate the efficacy and applicability of the above-mentioned technique.

Step 1: Problem field selection: Observe the tables below, which provide information on four individuals P_1, P_2, P_3, P_4 whose symptoms include blood plates, cough, insulin, joint pain and temperature. We need to locate the patient as well as the illness that the patient is suffering from, such as Chikungunya, Dengue, Diabetes, Swine Flu or Tuberculosis. Table 1.1 shows data by q-rung orthopair fuzzy numbers; here, for instance, we have q=3 with the functions of membership (μ) and non-membership (ν). Table 1.2 shows that cough is low for Chikungunya ($\mu = 0.19$, $\nu = 0.92$) but high for Tuberculosis ($\mu = 0.91$, $\nu = 0.23$).

Table 1.1 Input data of the patients in q-ROFS form

Symptoms	Patients			
	P_1	P_2	P_3	P_4
Blood Plates	(0.85, 0.53)	(0.74, 0.69)	(0.22, 0.99)	(0.32, 0.96)
Cough	(0.97, 0.27)	(0.78, 0.63)	(0.45, 0.65)	(0.81, 0.72)
Insulin	(0.59, 0.91)	(0.84, 0.59)	(0.19, 0.98)	(0.36, 0.89)
Joint Pain	(0.68, 0.47)	(0.47, 0.55)	(0.92, 0.36)	(0.29, 0.89)
Temperature	(0.43,0.79)	(0.92, 0.28)	(0.26, 0.97)	(0.43, 0.22)

Table 1.2 Representation of the Symptoms in terms of q-ROFS

Diagnosis	Symptoms				
	Blood Plates	Cough	Insulin	Joint Pain	Temperature
Chikungunya	(0.96, 0.54)	(0.19, 0.92)	(0.44, 0.86)	(0.96, 0.21)	(0.89, 0.25)
Dengue	(0.95, 0.54)	(0.26, 0.78)	(0.22, 0.89)	(0.37, 0.77)	(0.82, 0.37)
Diabetes	(0.19,0.98)	(0.25, 0.89)	(0.36,0.91)	(0.43,0.72)	(0.49,0.92)
Swine Flu	(0.28,0.75)	(0.54,0.84)	(0.27,0.94)	(0.64,0.83)	(0.94,0.23)
Tuberculosis	(0.62,0.73)	(0.91,0.23)	(0.23,0.87)	(0.16,0.99)	(0.22,0.91)

Step 2: Form q-rung orthopair fuzzy supra topologies for (Cj) and (D_k):

1. $\tau_1^* = A \cup B$, where $A = \{(1, 0), (0, 1), (0.85, 0.53), (0.97, 0.27), (0.59, 0.91), (0.68, 0.47), (0.43, 0.79)\}$, and $B = \{(0.97, 0.27), (0.85, 0.47), (0.59, 0.79)\}$.
2. $\tau_2^* = A \cup B$, where $A = \{(1, 0), (0, , 1), (0.74, 0.69), (0.78, 0.63), (0.84, 0.59), (0.47, 0.55), (0.92, 0.28)\}$, and $B = \{(0.74, 0.55), (0.78, 0.55), (0.84, 0.55)\}$.
3. $\tau_3^* = A \cup B$, where $A = \{(1, 0), (0, 1), (0.22, 0.99), (0.45, 0.65), (0.19, 0.98), (0.92, 0.36), (0.26, 0.97)\}$, and $B = \{(0.22, 0.98)\}$.
4. $\tau_4^* = A \cup B$, where $A = \{(1, 0), (0, 1), (0.32, 0.96), (0.32, 0.87), (0.92, 0.49), (0.23, 0.94), (0.22, 0.91)\}$, and $B = \{(0.32, 0.89), (0.81, 0.22)\}$.
5. $\nu_1^* = C \cup D$, here $C = \{(1, 0), (0, 1), (0.96, 0.54), (0.19, 0.92), (0.44, 0.86), (0.96, 0.21), (0.89, 0.25)\}$, and $D = \{(0.96, 0.25)\}$.
6. $\nu_2^* = C \cup D$, where $C = \{(1, 0), (0, 1), (0.95, 0.54), (0.26, 0.78), (0.22, 0.89), (0.37, 0.77), (0.82, 0.37)\}$, and $D = \{(0.95, 0.37)\}$.
7. $\nu_3^* = C \cup D$, where $C = \{(1, 0), (0, 1), (0.19, 0.98), (0.25, 0.89), (0.36, 0.91), (0.43, 0.72), (0.49, 0.92)\}$, and $D = \{(0.36, 0.89), (0.49, 0.89), (0.49, 0.91), (0.49, 0.72)\}$.
8. $\nu_4^* = C \cup D$, where $C = \{(1, 0), (0, 1), (0.28, 0.75), (0.54, 0.84), (0.27, 0.94), (0.64, 0.83), (0.94, 0.23)\}$, and $D = \{(0.64, 0.75), (0.54, 0.84)\}$.
9. $\nu_k^* = C \cup D$, here $C = \{(1, 0), (0, 1), (0.62, 0.73), (0.91, 0.23), (0.23, 0.87), (0.16, 0.99), (0.22, 0.91)\}$, and $D = \{\emptyset\}$.

Step 3: Find q-ROFSF:

1. q-ROFSF(A) = 0.2696 and q-ROFSF(B) = 0.265, where $r = 3$. q-ROFSF(C_1) = 0.2673.
2. q-ROFSF(A) = 0.2861 and q-ROFSF(B) = 0.3091, where $r = 3$. q-ROFSF(C_2) = 0.2976.
3. q-ROFSF(A) = 0.1818 and q-ROFSF(B) = 0.06, where $r = 1$. q-ROFSF(C_3) = 0.1209.
4. q-ROFSF(A) = 0.1975 and q-ROFSF(B) = 0.2525, where $r = 2$. q-ROFSF(C_4) = 0.225.
5. q-ROFSF(C) = 0.2736 and q-ROFSF(D) = 0.4275, where $s = 1$. q-ROFSF(D_1) = 0.3505.
6. q-ROFSF(C) = 0.2239 and q-ROFSF(D) = 0.395, where $s = 1$. q-ROFSF(D_2) = 0.3095.
7. q-ROFSF(C) = 0.1536 and q-ROFSF(D) = 0.1513, where $s = 4$. q-ROFSF(D_3) = 0.1524.
8. q-ROFSF(C) = 0.2171 and q-ROFSF(D) = 0.1988, where $s = 2$. q-ROFSF(D_4) = 0.2079.
9. q-ROFSF(C) = 0.1932 and q-ROFSF(D) = 0, where $s = 0$. q-ROFSF(D_5) = 0.1932.

Step 4: Final Decision:

Arrange q-rung orthopair fuzzy score values for the alternatives C_1, C_2, C_3, C_4 and the attributes D_1, D_2, D_3, D_4, D_5 in ascending order. We get the following sequences $C_3 \leq C_4 \leq C_1 \leq C_2$ and $D_3 \leq D_5 \leq D_4 \leq D_2 \leq D_1$. Patients P_1, P_2, P_3 and P_4 have Diabetes, Swine Flu, Chikungunya and Dengue, respectively.

1.7 Conclusion and Future Work

The q-rofts with the idea of vagueness is one of the present studies in generic fuzzy topological spaces. We introduced the q-rorfsts as well as their real-world applications. Furthermore, certain mappings in q-rung orthopair fuzzy supra topological spaces were studied and some contradictory examples were presented. A multiple attribute decision-making algorithm via q-rung orthopair fuzzy supra topological spaces is presented and a numerical example in medical diagnosis data mining for the proposed algorithm is presented finally. In future, this work may be applied to various fields of topology research including soft, rough, digital topology and interval-valued sets [28, 29].

References

1. L.A. Zadeh, Probability measures of fuzzy events. J. Math. Anal. Appl. **23**(2), 421–427 (1968)
2. K.P. Adlassnig, Fuzzy set theory in medical diagnosis. IEEE Trans. Syst. Man Cybern. **16**(2), 260–265 (1986)
3. M. Sugeno, An Introductory survey of fuzzy control. Inf. Sci. **36**, 59–83 (1985)
4. P.R. Innocent, R.I. John, Computer aided fuzzy medical diagnosis. Inf. Sci. **162**, 81–104 (2004)
5. T.J. Roos, *Fuzzy Logic with Engineering Applications* (McGraw Hill P. C, New York, 1994)
6. C.L. Chang, Fuzzy topological spaces. J. Math. Anal. Appl. **24**, 182–190 (1968)
7. R. Lowen, Fuzzy topological spaces and fuzzy compactness. J. Math. Anal. Appl. **56**, 621–633 (1976)
8. A.S. Mashhour, A.A. Allam, F.S. Mohmoud, F.H. Khedr, On supra topological spaces. Indian J. Pure and Appl. Math. **14**(4), 502–510 (1983)
9. M.E. Abd El-Monsef, A.E. Ramadan, On fuzzy supra topological spaces. Indian J. Pure Appl. Math. 18(4), 322–329 (1987)
10. K. Atanassov, Intuitionistic fuzzy sets. Fuzzy Sets Syst. **20**, 87–96 (1986)
11. R.R. Yager, Pythagorean fuzzy subsets, in *Proceedings of the 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), Edmonton, Canada, June 24–28, 2013* (IEEE, 2013), pp. 57–61
12. D. Coker, An introduction to intuitionistic Fuzzy topological spaces. Fuzzy Sets Syst. **88**(1), 81–89 (1997)
13. R. Saadati, J.H. Park, On the intuitionistic fuzzy topological space. Chaos, Solitons Fractals **27**(2), 331–344 (2006)
14. S.K. De, A. Biswas, R. Roy, An application of intuitionistic fuzzy sets in medical diagnosis. Fuzzy Sets Syst. **117**(2), 209–213 (2001)
15. E. Szmidt, J. Kacprzyk, Intuitionistic fuzzy sets in some medical applications, in *International Conference on Computational Intelligence* (Springer, Berlin, Heidelberg, 2001), pp. 148–151
16. P. Biswas, S. Pramanik, B.C. Giri, A study on information technology professionals' health problem based on intuitionistic fuzzy cosine similarity measure. Swiss J. Stat. Appl. Math. **2**(1), 44–50 (2014)
17. V. Khatibi, G.A. Montazer, Intuitionistic fuzzy set vs. fuzzy set application in medical pattern recognition. Artif. Intell. Med. **47**(1), 43–52 (2009)
18. K.C. Hung, H.W. Tuan, Medical diagnosis based on intuitionistic fuzzy sets revisited. J. Interdiscip. Math. **16**(6), 385–395 (2013)
19. M. Olgun, M. Unver, S. Yardimci, Pythagorean fuzzy topological spaces. Complex Intell. Syst. **5**, 177–183 (2019)
20. R.R. Yager, Generalized orthopair fuzzy sets. IEEE Trans. Fuzzy Syst. **25**, 1222–1230 (2017)
21. H. Garg, A novel trigonometric operation-based q-rung orthopair fuzzy aggregation operator and its fundamental properties. Neural Comput. Appl. **32**(18), 15077–15099 (2020)
22. R. Wang, Y. Li, A novel approach for green supplier selection under a q-rung orthopair fuzzy environment. Symmetry **10**(12), 687–712 (2018)
23. G. Wei, H. Gao, Y. Wei, Some q-rung orthopair fuzzy Heronian mean operators in multiple attribute decision making. Int. J. Intell. Syst. **33**, 1426–1458 (2018)
24. H. Garg, CN-q-ROFS: Connection number-based q-rung orthopair fuzzy set and their application to decision-making process. Int. J. Intell. Syst. **36**(7), 3106–3143 (2021)
25. H. Wang, Y. Ju, P. Liu, Multi-attribute group decision making methods based on q-rung orthopair fuzzy linguistic sets. Int. J. Intell. Syst. **34**(6), 1129–1157 (2019)
26. E. Turkarslan, M. Unver, M. Olgun, q-Rung Orthopair Fuzzy Topological Spaces. Lobachevskii J. Math. **42**, 470–478 (2021)
27. S.E. Abbas, Intuitionistic supra fuzzy topological spaces. Chaos, Solitons Fractals **21**, 1205–1214 (2004)
28. H. Garg, A new possibility degree measure for interval-valued q-rung orthopair fuzzy sets in decision-making. Int. J. Intell. Syst. **36**(1), 526–557 (2021)

29. H. Garg, New exponential operation laws and operators for interval-valued q-rung orthopair fuzzy sets in group decision making process. Neural Comput. Appl. **33**(20), 13927–13963 (2021)

Mani Parimala is an Assistant Professor in the Department of Mathematics at Bannari Amman Institute of Technology. Dr. Parimala received her Ph.D. in Mathematics in 2012 from Bharathiar University. She is one of the Editors in Chief of Asia Mathematika—An International Journal and Journal of Engineering Mathematics & Statistics. She is a reviewer in many reputable journals. She published more than 100 research articles in reputed international journals. Her major researches include general topology, algebraic structures and optimization techniques.

Cenap Ozel is a Professor at the King Abdulaziz University. He received his Ph.D. from the Glasgow University, Scotland, in 1998. His research interests include algebra, topology, differential geometry and mathematical physics. He has published several papers and books.

Mohammed A. Al Shumrani is a Professor of Mathematics at King Abdulaziz University, Jeddah, Saudi Arabia. He got his M.Sc. in Mathematics in 2002 from University of Missouri-Kansas City, USA. He got his Ph.D. in Mathematics in 2006 from University of Glasgow, UK. His research interests include General Topology, Algebraic Topology, Category Theory and Neutrosophic Theory. He has published several papers and one book.

Aynur Keskin Kaymakci is a Professor of Mathematics at Selcuk University, Konya, Turkey. Dr. Keskin Kaymakci got her Ph.D. from Selcuk University in 2003. She is one of the editors of Pakistan Academy of Sciences, Journal of Mathematics and Computational Intelligence, IJMTT, Adıyaman University Journal of Science. She is a reviewer in many reputable journals. Her research subjects are general topology, ideal topological spaces, soft topology and rough sets theory. She has published several papers.

Chapter 2
q-Rung Orthopair Fuzzy Soft Topology with Multi-attribute Decision-Making

Muhammad Tahir Hamid, Muhammad Riaz, and Khalid Naeem

Abstract In this chapter, the idea of q-rung orthopair fuzzy soft sets is extended to introduce the notion of q-rung orthopair fuzzy soft topology together with some interesting results. Certain properties of q-rung orthopair fuzzy soft topology are investigated for their practical applications in multi-attribute decision-making. For these objectives, grey relational analysis, generalized choice value method, and aggregation operators-based technique are proposed to address q-rung orthopair fuzzy soft uncertain information. Numerical examples of these methods are also presented from real-life situations. The validity and efficiency of proposed methods are analysed by their performing comparative analysis.

Keywords q-rung orthopair fuzzy soft topology · Multi-attribute decision-making · GRA · Generalized choice value method · Aggregation operators

2.1 Introduction

Information handling is an important aspect of our life. The classical approaches utilized for manipulating incomplete and inconsistent information are typically based on crisp sets and crisp models that overlook vagueness, hesitancy, and uncertainties. For handling uncertain real-life problems, researchers have introduced various set theoretic models. To handle inconsistent information, Fuzzy sets [1] by Zadeh, rough

M. T. Hamid
Department of Mathematics & Statistics, The University of Lahore, Lahore, Pakistan

M. Riaz (✉)
Department of Mathematics, University of the Punjab Lahore, Lahore, Pakistan
e-mail: mriaz.math@pu.edu.pk

K. Naeem
Department of Mathematics, FG Degree College, Lahore Cantt., Lahore, Pakistan

H. Garg (ed.), *q-Rung Orthopair Fuzzy Sets*,
https://doi.org/10.1007/978-981-19-1449-2_2

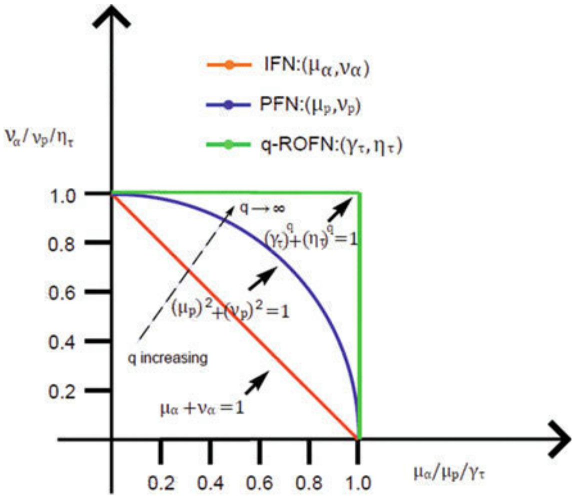

Fig. 2.1 Space for IFN, PFN, and q-ROFN. (The line in green colour may be observed as dotted)

sets [2] by Pawlak, and soft sets [3] by Molodstov are taken as fundamental tools in computational intelligence, artificial intelligence, social sciences, medical diagnosis, and engineering.

With the invention of new theories and models, researchers around the globe started working on different directions to explore new dimensions to cope with real-life problems. Atanassov [4–6], by expanding the notion of Zadeh's fuzzy set, proposed intuitionistic fuzzy sets (IFSs) and intuitionistic fuzzy numbers (IFNs) by supplementing non-membership function in addition to membership function. Atanassov imposed the condition on membership and non-membership functions that the sum of these two must lie in the unit closed interval. Xu [7] proposed IF-aggregation operators. Xu and Yager [8] rendered geometric aggregation operators for IFNs. Zhao et al. [9] presented generalized aggregation operators for IFNs. Yager [10, 11] and Yager and Abbasov [12] broadened the notion of IFSs to Pythagorean fuzzy sets (PFSs), by altering the condition on membership and non-membership function to that sum of their squares should fall in the unit closed interval. Later, in 2017, Yager [13] extended the idea to q-rung orthopair fuzzy sets (q-ROFSs) by amending the restriction on the two functions that sum of their qth powers should lie from 0 to 1. Thus, he broadened the space for the selection of values of the two functions and empowered the decision-makers to choose any values for the two functions from 0 to 1. Ali [14] developed certain features of q-ROFSs and their orbits. The spaces for intuitionistic fuzzy, Pythagorean fuzzy, and q-rung orthopair fuzzy numbers are depicted in Fig. 2.1.

Many researchers around the globe have studied the concepts of soft sets and soft topology. Aygunoglu et al. [15] studied fuzzy topological space; Cagman et al. [16] explored soft topology. Shabir and Naz [17] rendered more results on soft topological spaces. Bashir and Sabir [18] unveiled some structures on soft topological spaces. Roy and Samanta [19] established some important results on soft topological spaces.