Research in Mathematics Education

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Mathematical Creativity

A Developmental Perspective

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Foreword

As you are reading this book, millions of children and young people, who have their whole life laying ahead of them, are being taught in classrooms. Whenever I think of this, I wonder how these young people should be educated, what knowledge and skills do they really need to develop in order to be best equipped for their lives and the future world that we cannot foresee. Unequivocally, in the challenging and changing world that we live in, the role of education should not be restricted to enriching students' knowledge, but it should empower them to adapt to changes and approach problems creatively. Creative thinking constitutes the mechanism to manage change and challenge.

This book offers a widely useful compilation of theoretical frameworks, empirical findings, cases, and approaches to mathematical creativity across various ages. It is, I think, an important resource for those investigating mathematical creativity, for mathematics educators, policy makers, and teachers. First, it provides in a concise way these various aspects of mathematical creativity and an overall view of what is the state of the art on this topic. It highlights the similarities and differences of mathematical creativity across ages and presents some indicative research studies on creativity at different age groups, using different theoretical frameworks, research questions, and methodological tools. It is, therefore, with great pleasure that I accepted the invitation by Dr. Chamberlin, Dr. Liljedahl, and Dr. Savić, three well-known researchers in the field of mathematical creativity, to write the foreword for this book. I was excited to be given the opportunity to read all its chapters in advance.

In 1980, the National Council of Teachers of Mathematics (NCTM) identified gifted students of mathematics as the most neglected segment of research in mathematics education. Since then, a vast amount of quality research was developed for the identification of gifted students, and for creating appropriate materials for helping talented students to enrich their mathematical abilities. Nowadays, much focus is placed on the teaching of mathematics which provides for creativity not only for the gifted and talented students, but provides for all students the opportunity to appreciate the beauty of mathematics and to fully develop their talents and abilities. Creativity is a way of thinking in mathematics through different lenses. Mathematics understanding requires creative applications in the exploration of mathematics problems. Traditional teaching methods involving demonstrations and drill and practice using closed problems with predetermined answers insufficiently prepare students in mathematics. The essence of mathematics is thinking creatively, not simply arriving at the right answer.

In this line of thought, this book offers a detailed and elaborate picture of research on mathematical creativity, setting off from its origins, walking us through some of its major advances and bringing us to its current status, and finally openhandedly offering possible avenues for new research. The chapters blend nicely the theoretical background and literature of some of the most eminent theories in mathematics education and also present findings of some current empirical studies. The book is organized in four sections:

Section 1: History and Background of Mathematical Creativity
Section 2: Synthesis of Literature Finding for Researchers
Section 3: Recently Completed Empirical Studies in Mathematics Education
Section 4: Research Application and Editors' Summative Considerations

When discussing mathematical creativity, it is useful to start from its history. Thus, the first section refers to the history of research and definitions of creativity, providing at the same time the background of mathematical creativity. In the second chapter, one of the editors, Peter Liljedahl, provides an overview of the various strands of creativity research that have influenced mathematics education. He elaborates on the directions that research in mathematics creativity has taken and reveals the links among various theoretical frameworks in general, and specifically in mathematical creativity. This chapter is a useful tool for researchers to look at research on creativity in mathematics education, through different aspects that had been studied as well as through many underlying theoretical assumptions on creativity.

The history of research on creativity is closely related to the topic of the second chapter of the book, which reflects on mathematics creativity and society. The chapter by Chamberlin and Payne reveals various conceptions of creativity and their implicit and explicit value in society. This is mainly examined through the lens of national standard documents and international competitions. The researchers highlight the fact that one of the main reasons for which mathematical creativity is not advancing in the way we might have expected, is that rarely time and money are invested in the development of mathematical creativity in classrooms. Moreover, although the value of creativity is proclaimed in many curricula and policy documents, mathematics creativity is almost never or very rarely assessed in national or international level and thus teachers do not place adequate emphasis during their teaching. The authors aimed at showing the impetus connection of mathematics creativity to the society by underlying the chasm between the emphasis on creativity in curricula on one hand, and the resources invested by educational administrations on the other hand. It is not, of course, possible in a short article to deal with all aspects of the relation of mathematics creativity with the varsity of effects of society. Despite that, the article contributes to a fruitful discussion in answering questions such as how to best nurture mathematical creativity of students, and most importantly, why in most societies the administration undervalues the focus and significance of mathematics education creativity.

In the fourth chapter of this section, the three editors, Chamberlin, Liljedahl, and Savić, present the framework of the book and highlight its contribution. The editors wish to emphasize that mathematical creativity is for all students and not only restricted to a few talented students. To do so, they argue, that one needs to realize that mathematical creativity is not homogenous but its process and products differ in different ages. Therefore, they chose to discuss in this book, mathematical creativity and its development in ages 5–12 elementary school years, 13–18 secondary school years, and 19–23 tertiary education years.

In the second section, a synthesis of literature findings of three age groups are presented. The editors propose that research which focuses on mathematical creativity should take into consideration that persons, products, and processes are different at different ages. It appears that we need a more fine-grained analysis of what creativity may look like at different ages and how it might be developed. For instance, we need to provide a more detailed account of what mathematical creativity may look like at different ages, how it may develop and what the effects of various types of instruction on students' development of mathematical creativity are. The three chapters that follow offer a detailed account of what the empirical studies have shown until now, and what information is available about creativity in the three specific age groups. Understanding the development of creativity, learning about various attempts that were made to develop mathematical creativity and the impact that these attempts had, constitute important first steps for the development of better instruction for the development of students' mathematical creativity. The three chapters also present promising directions for future research which can be useful to people who want to pursue research in this field.

The second chapter of this section by Kozlowski and Chamberlin explores the way in which literature influenced research in mathematics creativity for individuals 5–12 years old. The literature explored in this article is organized in two main categories: academic oriented research and practice oriented research. In the third chapter of this section, Joklitschke offers a systematic overview of current empirical insights on mathematical creativity among secondary school students, while in the fourth chapter, Savić, Satyam, El Turkey, and Tang provide a broad view of research on mathematics creativity among students at the tertiary level. The authors indicate that far fewer research studies explored mathematical creativity among students of tertiary education in comparison to students of elementary and secondary education.

Actually, the second section of the book suggests that individuals are able to be creative in the sense that they are able to come up with novel ideas in the context of their age and abilities. Although there is a general agreement about which processes and abilities are important for the development of creativity, fully understanding the development of each process and its role in creativity is a more complex task. Research is not conclusive as to precisely indicating how creativity develops and what exactly is essential in fostering this development. Thus, the third section of the book, presents empirical studies which are related to a degree with developmental theories and processes thought to be important in the study of creativity in individuals. Specifically, in the third section of the book, five recent empirical research studies in mathematics education are presented. In a broad sense, section three presents research that highlights practices which contribute to the development of students' creativity. The authors refer to episodes in classrooms, to the differences of convergent and divergent thinking, to the progression of creativity, to the concept of group creativity, and finally to approaches which contribute to our understanding of the creative processes at play in educational environments. The chapter by Crespo and Dominguez presents the benefits of using different analytic lenses to understand children's creative mathematical thinking. The researchers invite readers to see through some episodes how children are working and what they are saying from different theoretical lenses. The realization that different theoretical frameworks could reveal or disguise the causes or the results of any learning experience is of fundamental importance. As teachers and researchers, we need to embrace this challenge and invest time and effort in making the right choices.

In the third chapter of this section, de Vink, Lazoner, Willemsen, Schoevers, and Kroesbergen investigate the contribution of convergent and divergent thinking in upper-elementary school children while working on problem posing and multiple solution tasks. This is a worthwhile topic, since we often see creative thinking being associated only with divergent thinking and even equated with divergent thinking. The researchers found that generally divergent and convergent thinking is evolving in a nonlinear process. Students often start from divergent thinking and then move to convergent thinking. The authors found that students with high achievements in mathematics tended to use more convergent thinking or a combination of convergent and divergent thinking. The realization of the important role that convergent thinking plays for the development of creative ideas may be an eye-opening experience and also reveals new directions for instruction which aim towards the development of mathematical creativity. It is possible that most often instruction that tried to facilitate mathematical creativity emphasized mostly divergent thinking without appreciating the combination and cyclic blending of convergent and divergent thinking.

Thinking of environments that will support the development of creativity and also of group creativity in school classrooms appears to be challenging and necessary. This is the topic that Liljedahl explores in the fourth chapter of this section. He uses the term burstiness, to describe the role of environment on group creativity. He outlines some of the key ingredients that are necessary for an environment to form fruitful ground for group creativity to occur such as the structure, diversity, psychological safety, welcome criticism, freedom to shift attention, focus, and opportunity for non-verbal communication. Liljedahl, presents an episode from secondary education and masterfully illustrates what these ingredients may look like in the mathematics classroom.

Although numerous studies explored what mathematics creativity may look like and how it may progress through various learning environments, we rarely find any studies that show how the perception of individuals' creativity changes during a learning course. A reason for this might be that most of the studies were conducted with young students who may not be mature enough to discuss their perceptions of mathematical creativity and also reflect on them.

In the fifth chapter of this section Karakok, Tang, Cilli-Turner, El Turkey, Satyam, and Savic explore the progression of four undergraduate students' perspectives on mathematical creativity. The original perspective of mathematical creativity of these four undergraduate students is that it involves unique, innovative, and original approaches. After the completion of the course, these students' perspective of creativity changed with the incorporation of different mathematical actions which appear to be more mathematically creative. Undergraduate mathematics students' perception of what mathematical creativity is and how it changes during a course is of outmost importance. Many of these undergraduate mathematics students will become mathematics teachers for the next generation. The way they perceive creativity will dictate the methods they will use to develop it. Thus, the successful development of mathematical creativity and the interruption of any vicious circles that inhibit its development, depend greatly on the perceptions that future mathematics teachers hold. Thus, we need to invest in such studies, and most importantly invest on future mathematics teachers who will take on the responsibility to educate future minds.

Numerous attempts have been made to develop mathematical creativity. Changes of available means and tools also have an impact on the methods used for the development of mathematical creativity. A book written in 2022 would not have been complete if it did not address a main concern and shift in the educational approaches that occurred worldwide as a result of the Covid-19 pandemic. Undoubtably, the Covid-19 pandemic brought to the forefront the need and possibilities of online learning. This raises the question whether online teaching will restrict development of mathematical creativity and if there are any ways in which one could develop mathematical creativity through online learning. In the sixth chapter of this section, Monahan and Munakata investigated through interviews the way in which seven instructors tried to incorporate creative teaching and learning in an online course which was prompted by the Covid-19 pandemic. The course was designed to support students to see the connections between mathematics and creativity. The researchers discuss the affordances and limitations of the online environment. It appears that online learning which so forcefully entered all levels of education in 2020, will not only constitute a teaching environment which was dictated by the restrictions imposed by Covid-19 but, looking at it more optimistically, it may offer new possibilities for the teaching of students of all ages worldwide. Of course, it is likely, that different methods and approaches will be needed for different age groups and mathematical processes and products may also be different among these populations.

The fifth and final section of the book offers an overview of the book and concluding thoughts on application, implications, and future directions. The authors discuss indicators, stages, assessment, processes, and products of creativity in the light of the development/maturation of creativity across the three age groups 5–12, 13–18, and 19–23. The authors discuss what they feel is still needed in research by highlighting application of research to scholars and practitioners.

This book is an important resource. It provides a useful compilation of ideas, theoretical backgrounds, empirical studies which address mathematical creativity of the general population across different ages. The literature review chapters, empirical studies presented, and reflective chapters offer the potential to researchers, mathematics educators, policy makers, teachers, and students to go beyond what they may learn from isolated research articles. The chapters of this book facilitate the reader to explore the field of mathematical creativity, make connections, and feed the development of new studies and theories in mathematical creativity. I hope that this book will become a useful tool for mathematics education researchers, teacher educators, professional developers, teachers, and students to learn and nurture mathematical creativity and creativity in general.

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Contents

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Demetra Pitta-Pantazi is Full Professor of Mathematics Education in the Department of Education at the University of Cyprus. Her research interests focus on students' cognitive development of mathematical concepts, problem solving, mathematical creativity, integration of ICT in mathematics teaching, mathematics curriculum, and assessment. She is a member of the scientific committee for the development of the mathematics curriculum and textbooks, which are currently used in all primary schools in Cyprus. She is one of the associate editors of *Educational Studies in Mathematics* and a member of the editorial boards of the *International Journal of Science and Mathematics Education* and the *Journal of Mathematical Behavior*.

Part I
History and Background of Mathematical Creativity

Chapter 1
Creativity and Mathematics: A Beginning Look

Alane Jordan Starko

I am, perhaps, a strange person to be writing the first section of a book about creativity and mathematics. I am not a mathematician. I am a teacher and a teacher educator who is fascinated by creativity, particularly creativity in schools. Over more than 30 years, I've had the opportunity to speak to thousands of students and educators about the nature and support of creativity. In scores of presentations, conferences, and classes, I've begun by asking the group to name individuals or endeavors they believed to be creative. In all those efforts, I've never had anyone name a mathematician or a mathematical idea. Not ever. They have named individuals whose work was grounded in math to be sure; often the first person named is Albert Einstein, a theoretical physicist who spoke the language of mathematics fluently. But when the general person-on-the-street envisions creativity, they are much more likely to think about artists, musicians, and inventors than mathematicians.

There are many reasons for this. Most children develop their concept of mathematics in elementary school. There, for many years, school-math entailed rows of calculations to be completed with maximum speed and accuracy. Math problems always had a correct answer, easily located in the teacher's version of the text, and the students' job was to replicate it. Anything that deviated from that path was not considered creative; it was considered a mistake. The problem, of course, is that memorizing number facts has little to do with actual mathematics.

When I began studying creativity and tried to envision creative mathematics in schools, I came face-to-face with the notion of math-is-not-calculations. Early in the process, I interviewed a mathematician friend who talked about beauty and truth in equations in terms reminiscent of artists, composers, or philosophers. In the years since then, I've learned more about what mathematics is and is not. To readers who are mathematicians, this is painfully obvious. But for the rest of us, it is essential to

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understanding that creativity is fundamental to mathematics. In 1968, Halmos explained the nature of mathematics by first describing some of the things it is not.

> As a first step toward telling you what mathematicians do, let me tell you some of the things they do not do. To begin with, mathematicians have very little to do with numbers. You can no more expect a mathematician to be able to add a column of figures rapidly and correctly than you can expect a painter to draw a straight line or a surgeon to carve a turkey. Popular legend attributes such skills to these professions, but popular legend is wrong. (p. 376)
>
> Mathematics—this may surprise you or shock you some—is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and he becomes convinced of their truth long before he can write down a logical proof. The conviction is not likely to come early—it usually comes after many attempts, many failures, many discouragements, and many false starts (p. 380–81).

Like creativity in any other discipline, creativity in mathematics entails new ideas, new applications, new discoveries of beauty. It supports our understanding of the universe and inspires awe in those who see its implications. Sadly, many of us learned mathematics without either a sense of wonder or belief in the value of guesses, failures, or false starts.

Fortunately, mathematics education has progressed dramatically since the days of math=number facts. Still, the journey from early number concepts to creative mathematics is a complex one. The authors of this book intend to guide readers on that journey, considering the development of mathematical creativity as a process of maturation and growing sophistication over time. It entails understanding both the nature of mathematics and the nature of creativity. Here, we'll start with creativity.

1.1 What Is Creativity?

There are many definitions of creativity (e.g., Kaufman et al., 2017; Kaufman & Sternberg, 2019). Since the mid-twentieth century, most definitions have included two major criteria for judging creativity: novelty and appropriateness. To be considered creative, an idea or product must be new and appropriate to some goal. Random novelty without function, such as might be produced by my cats walking across the keyboard, is not sufficient. As the century continued, it was recognized that novelty and appropriateness must be defined within some environment. Sometimes definitions take aim at the processes involved. An early effort in this direction was made by Guilford (1967, 1988), who defined components of creativity within his Structure of the Intellect model of intelligence. His identification of divergent thinking (fluency, flexibility, originality, and elaboration) as a key element of creativity continues to be important in much creativity research today. More recently, Kounios and Beeman (2015) defined creativity as "the ability to reinterpret something by breaking it down into its elements and recombining these elements in a surprising way to achieve some goal" (p. 9). Here, the elements of surprise and goal directedness echo the two traditional elements of creativity. Simonton (2016) proposed that surprise itself become a third criterion. In that view, being new in a repetitious or mundane way does not define creativity—it requires something that is novel in a surprising way. At its most basic, creativity involves the generation of a new—and possibly surprising—product (idea, artwork, invention, etc.) that is appropriate in some context. It can range from the everyday creativity I exercise when devising a meal from the random contents of my refrigerator or making mosaic switch plate covers, often dubbed "Little c creativity," to the "Big C Creativity" of those whose work changes the direction of their disciplines.

1.1.1 What Creativity Is Not

Though the basics can seem straightforward, popular concepts of creativity are often confusing and prone to mythology (see, for example, Benedek et al., 2021). Some of the difficulties are rooted in the breadth of creative activities in the human experience, some in the varied aspects of creativity addressed in any given research study, some in the awe we feel when faced with the transformative power of "Big C" creativity. While we no longer believe creativity originates in the work of muses, the sense of mystery can remain. The list of creative myths is long, but a few are worth addressing specifically.

1.1.1.1 Creativity Does Not Occur in the Right Brain

Creativity is a complex activity, requiring many kinds of cognitive and affective processes: considering likely areas for activity, producing diverse ideas, selecting from among ideas, viewing ideas from multiple perspectives, linking to prior knowledge and experience, critiquing possibilities, etc. Like any complex activity, it requires the whole brain. It is true that in some creative tasks, highly creative people use the right hemisphere of the brain more than less creative individuals. But everyone who has a whole intact brain uses all of it when attacking creative problems, as documented in activities from musical improvisation to story generation, designing book covers, and traditional creativity measures. In fact, explorations of the neurobiology of creativity, including multiple neural networks and coordination across networks, is some of the most vibrant creativity research today (Abraham, 2018; Vartanian, 2019).

1.1.1.2 Creativity Is Not the Same as Intelligence or Expertise

There are several possible relationships between creativity and intelligence, varying with the measures and definitions used (Sternberg & O'Hara, 2000). Creativity has been hypothesized as part of intelligence; intelligence has been hypothesized as part of creativity. They have been viewed as overlapping in varied ways, or as differing uses of the same cognitive processes. One popular hypothesis postulates a threshold effect, holding that a minimum threshold of IQ of about 120 is necessary for major creative contributions. Above that level, the correlation between creativity and intelligence is seen as limited. This does not suggest that higher intelligence limits creativity (it doesn't), but rather that above 120, other personal and environmental factors may be more important than additional IQ points. The threshold hypothesis, while still popular, continues to be debated, as research is conflicting, particularly when examining real-world creativity rather than standardized assessments (see, for example, Jauk et al., 2013).

Similarly, creativity is not the same as expertise. One can be very knowledgeable about an area without producing original ideas. Unlike intelligence, large amounts of expertise and/or experience can impede creativity, if they lead individuals to routinized problem solving or to become so entrenched in current knowledge that they no longer seek fresh perspectives. Sternberg and Lubart (1995) postulated an upside-down U relationship between creativity and knowledge, in which too little knowledge impedes creativity and too much knowledge can also impede creativity, if it leads an individual to believe they have no need to seek more information or new problem-solving methods. In such cases, it may not be the expertise, per se, that is problematic, but complacency that can set in when individuals believe their knowledge to be sufficient. Expertise plus continued questioning may be a different matter entirely—perhaps as evidenced by Sternberg himself, who continues to develop new theories in an academic career well into its fourth decade.

1.1.1.3 Creativity Is Not Just for a Lucky Few

As noted earlier, creativity takes many forms. While few individuals make the "Big C" contributions that change their disciplines in dramatic ways, there are many opportunities for creativity in smaller professional contributions and in the innovations that make daily life easier. The fact that activities may be "Little c" level in terms of the discipline doesn't limit their opportunity for creativity. In Maslow's words, "a first-rate soup is more creative than a second-rate painting" (1968, p. 136). The universality of creativity may be most easily envisioned in its early stages. Budding creativity evidenced in childhood play crosses time and cultures, while its more mature manifestations are impacted by personal values, characteristics, and experiences.

1.1.1.4 Creativity Is Not Just a Phenomenon in the Arts

For many people, creativity is most immediately associated with the arts. We recognize great painters, poets, composers, and choreographers as creative in their fields. Certainly, creativity is essential in the arts. But additionally, every field needs creativity to move forward. Without creativity, there would be no progress in science, no new literature, no inventions or technology, no problem-solving for our myriad cultural dilemmas. And, fortunately for this book, creativity is important for original theorizing and problem solving in mathematics. The need for creativity across all areas of human endeavor means creativity is not a luxury (or worse yet, "fluff") to be seen as an unnecessary intrusion on education. It is essential to all human progress, and thus, essential in schools.

1.1.2 Mathematical Creativity

One of the great debates of creativity research is whether creativity in particular disciplines, for example, mathematical or musical creativity, represents unique constructs or is simply "general" creativity applied to different content (Kaufman et al., 2017). One model that attempts to bridge the gap is the Amusement Park Theoretical model (APT, Baer & Kaufman, 2017). The APT model conceptualizes creativity as having initial requirements common to all creativity, such as intelligence and motivation (like the entrance tickets to amusement parks), and then increasingly specific general thematic areas, domains, and microdomains, in which the characteristics and requirements for creativity may vary (like varied height requirements for different rides). For example, when considering Katherine Johnson's creativity, one might consider her overall intelligence and motivation, but also how creativity might operate in the general area of mathematics, the domain of early computer science, and the micro domain enabling space exploration in the mid-twentieth century.

However the two are related, the creativity-basics of general creativity undergird concepts of mathematical creativity. Philosophies of mathematics differ as to whether mathematics is discovered, like the nature of sound waves, or invented, like the telephone. Regardless, creativity in mathematics may be seen as having two faces: discovering mathematical facts and creating proofs to support the discovered facts. Just as science requires questioning and data, so mathematics requires exploration, problems, proofs, and generalizations. It searches for new ideas, new processes, and original solutions, and is a far cry from the textbook-driven rows of problems some students have experienced.

In many ways, mathematical creativity resembles models and descriptions of other types of creativity. Hadamard's (1945) description of processes used in mathematical creativity mirrored Wallas' (1926) more general four-stage creativity model that included *preparation* for addressing a creative problem; a period of *incubation* representing time away from conscious consideration of the problem; *illumination* or the "aha!" experience of a new idea; and *verification*, in which the new idea is tested. Mathematical problem solving can involve essential elements of divergent thinking: fluency (many solutions), flexibility (many approaches to solution), and originality. Karwowski et al. (2017) described mathematical creative problem-solving as supported by creative abilities, openness, and independence, characteristics associated with general creativity since MacKinnon's research at the Institute of Personality Assessment and Research (IPAR) beginning in the 1950s (MacKinnon, 1978). Most contemporary models of creativity can be considered systems models, that is, they view creativity as the result of complex interactions of cognitive and affective variables considered in context. For example, Amabile's Componential model (Amabile, 1988; Amabile & Pratt, 2016) was a primary influence in recognizing the role of motivation in creativity. She described the necessity of individual domain skills and knowledge, creativity-relevant processes, and intrinsic motivation within situations conducive to creativity. Grégoire (2016) applied this thinking to mathematics, suggesting that mathematical creativity can be supported by addressing three dimensions of creativity: expertise, original thinking, and intrinsic motivation.

1.2 How Does Creativity Develop?

This book's authors are particular in their definition of "development" as regards creativity.

> In this book, development is not considered to be the development of creativity in the classroom, as influenced by overt pedagogical decisions or carefully selected curricula. Instead, it can be equated with a maturation process, which should not be left completely to chance (Chap. 3).

That is, the book is focused on the ways mathematical creativity matures across time. While the authors are careful to distinguish this idea from the notion of developing creativity in the classroom, the definition I'm more likely to utilize, it is clear they do not intend that creativity be ignored or left to develop on its own. This is wise. Virtually all current creativity research recognizes creativity existing within a social and emotional context. Those contexts influence how—and if—creativity will be possible or be recognized. Csiksentmilhalyi's (1988) fundamental question of "Where is creativity?" recognized "Big C's" naissance in the interactions of a person (or persons), a domain (discipline) and a field, the social structure of the domain. In the case of mathematical creativity, such interactions might include a mathematician's personal characteristics, motivation, and creativity; knowledge of the domain; and her interactions with the gatekeepers of the profession, such as journal editors. Hennessey (2015) has described these factors at an earlier educational level, examining the interactions among student characteristics, teachers' characteristics, the culture of the classroom, and the larger surrounding culture. Even as any living thing needs supportive conditions to mature, so does creativity. In considering the development and maturation of creativity, it is essential to consider the circumstances and influences that support it.

1.2.1 Creativity Across Time

Relatively few researchers theorize about the development of creativity over time. Vygotsky situated his view of creativity in his sociocultural analysis of human thought, emphasizing the role of social and cultural interactions in the development of thought. He believed creativity developed in three major stages: the creativity and symbolic play of childhood, the increasingly abstract thinking of adolescence, and consciously purposeful creativity in adulthood. In all stages, it is influenced by surrounding social interactions. In childhood, these could be adults helping a child engage in symbolic play, while in adulthood, surrounding social and cultural needs can give direction to creative thought (Smolucha, 1992; Vygotsky, 1967).

In many ways, Vygotsky foreshadowed Bloom's (1985) studies of talent development, in which Bloom and colleagues studied the processes and influences through which individuals developed high levels of accomplishment in various fields. Those studied were all highly successful: concert pianists, sculptors, research neurologists, tennis champions, Olympic swimmers, and prize-winning research mathematicians. While the role and type of creativity varied across such diverse domains, the trajectories of the careers examined all entailed the development of creativity, and there was surprising consistency in stages of development, particularly considered the wide range of talents studied.

First, the authors recognized the long periods of training and support necessary for exceptional accomplishment, regardless of initial individual abilities. Mature creativity does not grow without care and attention. They also identified patterns of beginning, middle, and later stages of talent (creativity) development, requiring different types of instruction and support. Initial stages of talent development entailed finding and falling in love with a discipline. It was a time of joyful discovery. The timing varied by field. Whereas young people often became engaged in music or sports at a very young age, prospective scientists or mathematicians might not discover their specific area of study until high school or college. Teachers during the early years of talent development, whenever they occurred, helped students experience delight in discovery and envision what the field might be. Learning was often playful and supportive of exploration. The middle years of development entailed more rigorous study mastering the basics of a discipline, often requiring a more expert teacher. Emphasis was generally on precision and accuracy. Later years of talent development, particularly for those aspiring toward "Big C" creativity, often required yet a different type of teaching, supporting young people in finding their own voice, questions, or challenges rather than replicating those of the past. These stages of talent development have been used as an organizing framework for gifted education and for supporting creativity developmentally (Olszewski-Kubilius et al., 2018; Starko, 2018).

1.2.2 Talent Development in Mathematics

The mathematicians studied by Bloom and colleagues were winners of the Sloan Research Fellowship, awarded to early career professionals in recognition of their "distinguished performance and a unique potential to make substantial contributions to their field," suggesting significant creative potential (Alfred P. Sloan Foundation, 2022). They came from homes that valued intellectual activity and encouraged curiosity. However, the “early years” teachers for these students were found in middle or high school, when students were first exposed to the patterns and processes of mathematics, and experienced math as problem solving with the opportunity for varied procedures. It is interesting to consider whether this might have been different had students experienced more actual mathematical discourse in their early years. For this group, middle years’ teachers were generally found in college, particularly when undergraduate students had the opportunity to take more advanced graduate classes. In those years, the style of teaching seemed less essential than the knowledge base of the teacher, the commitment of the students to spend hours mastering essential content, and the teacher’s commitment to help them succeed. Finally, the later years of high-level talent development required what Bloom (1985) described as a “master teacher” (p. 524). Only a handful of these individuals were seen to exist in any given field, so being accepted as a student in such a program required both skill and support. Mentorship with a master teacher, typically in a doctoral or post-doctoral environment, allowed young mathematicians to work alongside those who were doing the research that expands the field. In this type of environment, high-level mathematical creativity developed most successfully. Of course, this succession of progressively more expert teachers is not the only path to mathematical creativity. Srinivasa Ramanujan, for example, is known for his extraordinary contributions to mathematics, developed largely in isolation. Still, even he required correspondence with other mathematicians to integrate his ideas with standard procedures and bring his work to the field. As we consider the development of mathematical creativity, from early explorations in number sense to the abstractions of mature creativity, it seems best to consider the concept of “development” in both senses: the maturation and growing abstraction that are the focus of the book, and the supportive environments and actions that can allow it to flourish.

1.3 About This Section

The first section of the book contains three chapters. Chapter 1 (Liljedahl) presents an introduction to mathematical creativity. It overviews theoretical perspectives on creativity grounded in mathematical problem solving and reviews the ways mathematical creativity has been measured. It also describes some of the ways creativity, or creativity studies, can be divided, including the “Big C” “Little c” categories, and studies that emphasize creative persons, processes, products, or press.

Chapter 2 (Chamberlin & Payne) first reviews the development of general creativity research over time. The following section focuses specifically on mathematical creativity. Of particular interest to those focused on mathematics education is information on early interest in creativity by mathematicians. This is a stark contrast to the stereotypes of creativity existing only in the arts, or of mathematics as comprised only of increasing complex calculations. The next section emphasizes the value of mathematical creativity and its limited representation in the curriculum standards that shape today’s education. Chamberlin and Payne emphasize the development of mathematical creativity as focused on maturation, dynamic and changing over time. They also examine factors that influence mathematical creativity, such as intelligence/content knowledge and affective variables. This section includes an introduction to the "Five Legs" theory (Chamberlin & Mann, 2021), which describes five affective factors influencing mathematical creativity. The final section of the chapter examines the application of Rhodes' (1961) creativity categories of person, place, and process to mathematical creativity.

Chapter 3 (Chamberlin, Liljedahl, & Savić) begins with an operational definition of development of mathematical creativity and its relationship to mathematical curriculum rigor. Development is associated with the Kaufman and Beghetto's (2009) 4 c's (levels) of creativity, illustrating how mathematical creativity may mature across time. The authors review the relationship of Rhodes' (1961) 4 P's (person, process, product, press) to creativity and its development. Finally, they discuss barriers to developing mathematical creativity, particularly in schools. These include the focus and/or breadth of standards, limitations in teachers' content or pedagogical content knowledge, developmentally inappropriate materials, and pressure to teach to standardized tests. A particularly striking (and very familiar) description is "When teachers are forced to hastily cover a rather extensive list of mathematical concepts, ample time for mathematical creativity to emerge… [is] compromised" (p. 48–49). The description of basic structural barriers is both realistic and daunting. Finally, the chapter addresses some of the affective variables that impact the development of creativity, including additional information on the "Five Legs Theory" that focuses specifically on mathematical creativity (Chamberlin & Mann, 2021). The theory includes affective dimensions that parallel those often described in general creativity research. For example, Iconoclasm can be seen as a particular aspect of the more general characteristics of risk-taking and courage. Impartiality entails flexible thinking and willingness to examine problems from multiple perspectives. Inquisitiveness mirrors curiosity and openness to experience. Like so much of what we know about mathematical creativity, these factors mirror general creativity characteristics, with a particular mathematical spin.

Considering the inclusion and support of creativity in mathematics education has the potential to transform the mathematics experiences of school children. With that transformation comes the opportunity to build a cohort of individuals with the vision and desire to develop mature mathematical creativity. If we are wise, that cohort will be both larger and more diverse than those who have gone before. The path may take us several steps closer to the "schools of curious delight" that have been my professional aspiration (Starko, 2022). With that hope, read on!

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