

Series on Grey System

Sifeng Liu · Yingjie Yang ·
Jeffrey Yi-Lin Forrest

Grey Systems Analysis

Methods, Models and Applications

 Springer

Series on Grey System

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This proposal aims to publish a series on grey system and various applications in the fields of natural sciences, social sciences and engineering.

This series is devoted to the international advancement of the theory and application of grey system. It seeks to foster professional exchanges between scientists and practitioners who are interested in the models, methods and applications of grey system. Through the pioneering work completed over 40 years, grey data analysis methods have become powerful tools in addressing system with poor information.

Books published with this series will explore the models and applications of grey system, in order to tackle poor information more effectively and efficiently. The series aims to provide state-of-the-art information and case studies on new developments and trends in grey system research and its potential application to solve practical problems.

Coverage includes, but is not limited to:

- Foundations of grey systems theory
- Grey sequence operators
- Grey relational analysis models
- Grey clustering evaluations models
- Techniques for grey system forecasting
- Grey models for decision-making
- Combined grey models
- Grey input-output models
- Techniques for grey control
- Various applications of grey system models in the fields of natural sciences, social sciences and engineering.

Sifeng Liu · Yingjie Yang · Jeffrey Yi-Lin Forrest

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Series Preface

This series will publish the books on grey system theory and various applications in the fields of natural sciences, social sciences and engineering.

It is devoted to the international advancement of the theory and application of grey system theory and seeks to foster professional exchanges between scientists and practitioners who are interested in the models, methods and applications of grey system theory. Through the pioneering work completed over 40 years, grey system analysis methods have become powerful tools in addressing system with poor information.

Books published with this series will explore the models and applications of grey system theory, in order to tackle poor information more effectively and efficiently. The series aims to provide state-of-the-art information and case studies on new developments and trends in grey system research and its potential application to solve practical problems.

In the era of big data, the grey system theory based on poor information data mining has sprung up. It has become an effective tool for people to extract valuable information from massive data. In the past 40 years, grey system method and model have been widely used in many fields, such as social science, natural science and engineering technology, which has led to innovation and progress in various fields. More and more people interested in grey system theory and a lot of new results have been obtained in recent years. In particular, successful applications in many fields have won the attention of the international world of learning.

Scholars from more than 100 countries and regions in the world have published more than 300,000 documents of grey system research and applications.

On the 7th of September 2019, Angela Dorothea Merkel, then German Chancellor, praised grey system theory in her speech at Huazhong University of Science and Technology. She said that the work of Prof. Deng Julong, the founder of grey system theory, and Prof. Sifeng Liu, the editor of this series, “have made a profound impact on the world”.

The coverage of this series includes, but is not limited to:

- Foundations of grey systems theory
- Grey sequence operators
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- Techniques for grey system forecasting
- Grey models for decision-making
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- Grey input–output models
- Techniques for grey control
- Various applications of grey system models in the fields of natural sciences, social sciences and engineering.

If you are interested in the series on grey systems, please contact with Ms. Emily Zhang at emily.zhang@springernature.com or Prof. Sifeng Liu at sfliu@nuaa.edu.cn.

Nanjing, China

Prof. Sifeng Liu, Ph.D.
Editor of the Book Series on Grey
System, Director of Institute for Grey
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of International Association of Grey
System and Uncertain Analysis

Foreword by Dr. James M. Tien

It gives me great pleasure to be introducing this 8th edition of *Grey System Theory and Its Applications* by Prof. Sifeng Liu. The theory of grey systems was first introduced in 1982 by J. L. Deng (1933–2013) at Huazhong University of Science and Technology; it established a relatively new approach for addressing poorly defined problems with a high level of greyness or uncertainty. The theory enables one to model, analyse, monitor and control such partially defined systems by generating, excavating and extracting useful information from what is available. It built on the work of Dr. Lotfi A. Zadeh, who introduced the concept of fuzzy sets in the 1960s that in turn led to breakthroughs in neural networks and soft computing.

Grey System Theory actually combines two critical and overarching areas. The first concerns systems which attempt to synthesize the various components or subsystems into an overall functioning system or system of systems. Systems theory attempts to make transparent the deep connections and interactions among objects and events, all leading to the enrichment and progress of science and technology. Many of the historically difficult, hard-to-solve problems in the different scientific fields have been successfully resolved through the application of systems theory and its allied methodologies, including information theory, cybernetics, combinatorics, genetics, etc. The second concerns the greyness or uncertainty level that is implicit in all natural or man-made systems. Indeed, most modelling techniques assume the existence of uncertainty or stochasticity, as defined by either empirical evidence or assumed distributions, including fuzzy sets.

Grey System Theory, then, provides a realistic approach to modelling, analysing, monitoring and controlling systems. Professor Sifeng Liu has greatly extended, if not expanded, his earlier efforts. In the 1980s, he put forward a series of new models and concepts, including sequence operator, absolute degree of grey incidence, grey cluster evaluation model with fixed weight, and positioned coefficient of grey matrix. In the 1990s, he proposed a buffer operator and its axiom, generalized degree of grey incidence, grey number and measurement of its information content, drifting and positioning solution, the grey econometrics model GM(1,1), the grey Cobb–Douglas model, etc. More recently, he proposed the concept of general grey numbers, the grey

algebraic system based on a kernel and degree of greyness, and different variations of the model GM(1,1).

The widespread recognition and application of grey system theory reflect its growing acceptance. A number of universities from around the world have adopted Prof. Sifeng Liu's monographs, both in Chinese and English, as their textbooks. In 2002, he won the World Organization of Systems and Cybernetics (WOSC) Prize. In 2008, as a preeminent Chinese scholar, he was elected an Honorary WOSC Fellow. In 2013, after a strict review by the European Commission, he was selected to be a Marie Curie International Fellow, thus honouring him as the first such Fellow with grey systems expertise.

As a systems scientist and engineer, I am honoured to write this preface for the 8th edition of *Grey System Theory and Its Applications*. I look forward to its widespread dissemination and its promulgation of grey system applications in science and engineering.

James M. Tien, Ph.D., D.Eng. (h.c.),
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Note Professor James M. Tien prepared this note for 8th edition of *Grey System Theory and Its Applications* (in Chinese) by the same authors, published in 2016. With his permission, it is printed here as a foreword for this current book.

Foreword by Dr. Keith William Hipel

Grey Systems: Theory and Applications

Written by Sifeng Liu and Jeffrey Yi-Lin Forrest

Springer-Verlag: Berlin, Heidelberg

2010, 379 pages, ISBN 978-3-642-16158-2 (cloth)

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Professors Sifeng Liu and Yi-Lin have written another pioneering book on the important topic of grey systems. In 2006, the same authors wrote the well-received book entitled *Grey Information: Theory and Practical Applications* which was also published by Springer-Verlag. I am pleased to say that their second book on Grey Systems constitutes a significant expansion and improvement of their previous fine book. Accordingly, if you already possess a copy of the 2006 book, you can make a worthwhile academic investment by obtaining a copy of their recent book in order to be cognizant of the latest ideas and advancements in the crucial field of grey systems.

The question that naturally arises is why grey systems are of such great import at this point in history. The answer is quite straightforward: many challenging problems facing society consist of interconnected complex systems of systems exhibiting high uncertainty and having few measurements. For example, in order to effectively combat climate change, one must understand as much as possible the complex interactions among natural systems such as atmospheric, oceanic, geological, and hydrological systems, with societal systems including energy production, industrial, agricultural and city systems. The deep uncertainty involved with these interconnected systems of systems and their potential emergent behaviour, coupled with a dearth of observations, mean that formal tools for handling this uncertainty are in high demand. Fortunately, an arsenal of mathematically based methodologies and techniques have been developed over the years: a rich variety of probabilistic-based tools, fuzzy sets founded by Lotfi Zadeh, rough sets started by Z. Pawlak, information-gap modelling perfected by Yakov Ben-Haim, uncertainty theory developed by Baoding Liu, and grey systems established by Julong Deng in 1982. The foregoing and other approaches to describing uncertainty are based upon different axioms and are thereby highly complementary for tackling a wide variety of uncertain situations.

Grey systems are purposefully designed for modelling uncertain systems, or systems of systems, problems having small samples and low-quality information. Grey systems are capable of dealing with partially known information through generating, excavating and extracting useful information from what is available. How this is accomplished is explained in depth in the timely grey systems book of Profs. Liu and Lin.

In their contemporary textbook, Liu and Lin systematically present the theory and practice of grey systems. In fact, the excellent ideas and applications contained in their book are based upon the authors' many years of developing theoretical concepts, applying their methods to real-world applications, testing and refining their new techniques with actual data, carrying out stimulating research with their students and colleagues, teaching their students about their exciting work and delivering research papers at international conferences around the globe. Their comprehensive book contains the latest theoretical and applied advances created by the authors and other scholars around the world in order to place the readers at the forefront of international research in grey systems.

The main body of their book contains ten well-explained and interconnected chapters: Introduction to Grey Systems Theory, Basic Building Blocks, Grey Incidence and Evaluation, Grey Systems Modelling, Discrete Grey Prediction Models, Combined Grey Models, Grey Models for Decision Making, Grey Game Models, Grey Control Systems and Introduction to Grey Systems Modelling Software. Moreover, the book includes a computer software package developed for grey systems modelling to permit both researchers and practitioners to use the new methodologies. Their book concludes with three appendices. The first appendix compares grey systems theory and interval analysis while revealing the fact that interval analysis is a part of grey mathematics. The second presents an array of different approaches to studying uncertainties. Finally, the last appendix shows how uncertainties occur using a general systems approach.

The book contains a wealth of mathematical results, techniques and algorithms which are presented by the authors for the first time. These contributions include an axiomatic system of buffer operators and a series of weakening and strengthening operators; axioms for measuring the greyness of grey numbers; general grey incidences (grey absolute incidence, grey relative incidence, grey comprehensive incidence, grey analogy incidence and grey nearness incidence); discrete grey models; fixed weight grey cluster evaluation; and grey evaluation methods based on triangular whitenization weight functions, multi-attribute intelligent grey target decision models, applicable range of the $G(1, 1)$, grey econometrics (G-E), grey Cobb–Douglas (G-C-D), grey input–output (G-I-O) and grey game models (G-G).

In their well-written book, Drs. Liu and Lin do a thorough job in their presentation of many difficult technical concepts. The authors are able to convince the readers of their book regarding the power and usefulness of their new theory by presenting many interesting examples of practical applications to real-life problems. The challenging practical problems addressed in their book include urban economic planning, downtown traffic design, natural disaster prediction, relative strength evaluation of a state, investment projection of a company and employee performance evaluation.

The depth and scope of the advancements in grey systems covered in this book, in conjunction with clarity of explanation, make this seminal book attractive to researchers, students, teachers and practitioners working in many different fields. These areas of endeavour include image processing, video processing, multimedia security, computer vision, machinery, control, agriculture, water resources, medicine, astronomy, earth science, economics and management. I personally found grey systems useful for accurately forecasting wastewater time series for which there is a scarcity of data. I intend to keep a copy of this valuable book easily accessible in my university office and purchase more copies of the book for use by my students.

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Note Professor Keith William Hipel prepared this note for one of the earlier book by the same authors, published in 2010. It is published in *Grey Systems: Theory and Application*, 2011, Vol. 1, No. 3. With his permission, it is printed here as a foreword for this current book.

Foreword by Dr. Hermann Haken

With human knowledge maturing and scientific exploration deepening and largely expanding in the course of time, mankind finally realizes the fundamental fact that due to both internal and external disturbances and limitations of human and technical sensing organs, all information received or collected contains some kind of uncertainty. Accompanying the progress of science and technology and the aforementioned realization, our understanding about various kinds of uncertainties has gradually been deepened. Attesting to this end, in the second half of the twentieth century, the continual appearance of several influential and different types of theories and methods on unascertained systems and information has become a major aspect of the modern world of learning. Each of these new theories was initiated and followed-up by some of the best minds of our modern time.

In their recent book, entitled “*Grey Information: Theory and Practical Applications*”, published in its traditionally excellent way by Springer, Profs. Sifeng Liu and Yi-Lin presented in a systematic fashion the theory of grey system, which was first proposed by J. L. Deng in early 1980s and enthusiastically supported by hundreds of scientists and practitioners in the following years. Based on the hard work of these scholars in the past (nearly) thirty years, scholars from many countries currently are studying and working on the theory and various applications of this fruitful scientific endeavour. With this book published by such a prestigious leading publisher of the world, it can be expected that more scientific workers from different parts of the world will soon join hands and together make grey system and information a powerful theory capable of bringing forward practically beneficial impacts to the advancement of the human society.

This book focuses on the study of such unascertained systems that are known with small samples or “poor information”. Different of all other relevant theories on uncertainties, this work introduces a system of many methods on how to deal with grey information. Starting off with a brief historical introduction, this book carries the reader through all the basics of the theory. And, each important method studied is accompanied with a real-life project the authors were involved in during their professional careers.

Many of the methods and techniques the reader will learn in this book were originally introduced by the authors. They show how from our knowledge based on partially and poorly known information can be obtained to accurate descriptions and effective controls of the systems of interest. Because this book shows how the theory of grey system and information was established and how each method could be practically applied, this book can easily be used as a reference by scholars who are interested in either theoretical exploration or practical applications or both. I recommend this book highly to anyone who has either a desire or a need to learn.

Stuttgart, Germany
July 2007

Prof. Dr. Dr. h.c. mult. Hermann Haken
Founder of Synergetics

Note Professor Hermann Haken prepared this note for one of the earlier book by the same authors, published in 2006. It is published in *Grey Systems: Theory and Application*, 2011, Vol. 1, No. 1.

Foreword by Dr. Robert Vallée

I am much interested and impressed by Dr. Sifeng Liu and Dr. Yi-Lin's recently published monograph on grey information, dealing with the theory and practical applications.

This book encompasses many aspects of mathematics under the aegis of uncertain information. I am greatly in favour of this attitude, concerning the uncertainty of information, which has been mine since a long time ago. Also, this book focuses on practice and aims at explorations of new knowledge. It is a comprehensive, all-in-one exposition, detailing not only with the theoretical foundation but also real-life applications. Because of this characteristic of quality and usefulness, Liu and Lin's book possesses the value of the widest possible range of reference by the workers and practitioners from all corners of natural and social sciences and technology.

In this book, Liu and Lin present the theory of grey information and systems starting on such background information as the relevant history, an attempt to establish a unified information theory, the basics of grey elements, and reaching all the most advanced topics of the theory. Complemented by many first-hand and practical project successes, the authors developed an organic theory and methodology of grey information and grey system, dealing with errors. In fact, there is much more to tell about error than about truth. Error (inexactitude) can be met everywhere and truth (exactitude) nowhere. But inexactitude contains a part of the truth. Greyness is the field we live in. Extremes, as whiteness and blackness, are inaccessible, but very useful, ideal concepts.

With the publication of such a book that contains not only a theory, aspects of magnificent real-life implications and explorations of new research, but also the history, the theorization of various difficult concepts, and directions for future works,

there is no doubt that Drs. Liu and Lin have made a remarkable contribution to the development and applications of systems science.

Paris, France
June 2007

Prof. Robert Vallée
President of the World Organisation
of Systems and Cybernetics, Université
Paris-Nord

Note This note is a book review written by Prof. Robert Vallée for one of the earlier book by the same authors, published in 2006. It is published in *Kybernetes: The International Journal of Cybernetics, Systems and Management Science*, 2008, Vol. 37, No. 1.

Preface

In this book, we answer the calls of the readers of our previous publications and systematically present the main advances in grey system theory and applications. By following our readers' feedback and suggestions, this volume introduces the most recent research results and updates on what is presented in our earlier books. In particular, the following content, which represents the authors' recent research, is highlighted in the book: general grey numbers and their operations, negative grey relational analysis models and grey relational analysis models based on similarity and closeness, three-dimensional grey relational analysis models, grey clustering evaluation models based on mixed possibility functions, original difference grey model (ODGM), even difference grey model (EDGM), discrete grey model (DGM), fractional grey models, self-memory grey models, multi-attribute intelligent grey target decision models, weight vector group with kernel and weighted comprehensive clustering coefficient vector. We also attach a software designed for grey system modelling, which was developed by Bo Zeng using Visual C#, the widely employed C/S software tool. This user-friendly software allows users to conveniently input and/or upload data and clearly distinguish module functions. Also, the software has the ability to present users with operational details, as well as periodic and partial results. Additionally, users can adjust the levels of computational accuracy based on their practical needs.

During the writing of this book, we prioritized theoretical simplicity and clarity to make it easy for the reader to follow the main arguments made. With a good number of practical applications, we intended to illustrate the methodology of grey system theory and modelling techniques so that we could emphasize the practical applicability of grey system thinking. We drew on the most recent research developments from various research groups around the world and tried to present the most complete picture of this new area of scientific endeavour in a concise manner.

The overall planning and organization of topics contained in this book were carried out by Sifeng Liu (Nanjing, China), who also authored Chaps. 1, 2, 4, 6, 10 and 12. Yingjie Yang (Leicester, UK) produced Chaps. 3, and 11, Jeffrey Forrest (Slippery Rock, USA) composed Chaps. 7 and 8, Naiming Xie (Nanjing, China) wrote Chap. 9, and the Appendix and the attached computer software were developed by Zeng Bo

(Chongqing, China). Zhigeng Fang, Yaoguo Dang, Lirong Jian and Chunhua Su and colleagues also worked with the authors to refine some of the book's content. Sifeng Liu was responsible for unifying the terms used throughout the book and for finalizing the manuscript.

Finally, we would like to encourage you to communicate with us and send us any comments you might have about this book. It is only by working together, as a team, that we can grow and mature as researchers. Sifeng Liu can be reached at sfliu@nuaa.edu.cn, Yingjie Yang can be reached at yyang@dmu.ac.uk, and Jeffrey Forrest at jeffrey.forrest@sru.edu or jeffrey.forrest@iigss.net.

Nanjing, China/Leicester, UK
June 2022

Sifeng Liu

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Over the years, our research has been highly commended by many first-class scholars, such as Julong Deng, the founder of grey system theory, L. A. Zadeh, the founder of fuzzy mathematics, Hermann Haken, the founder of synergetics, Robert Vallee, former president of the World Organization of Cybernetics and Systems, James Tien, former vice-president of IEEE and member of the American Academy of Engineering, K. W. Hipel, former president of The Academy of Science of the Royal Society of Canada and, Jifa Gu, former president of the International Federation for Systems Research. And praised or cited positively by more than 100 academicians from academies of science and engineering all over the world, such as E. K. Zavadskas, A. Bernard, Guozhi Xu, Qun Lin, Da Chen, Chunsheng Zhao, Haiyan Hu, Suzi Yang, Youlun Xiong, Zhongtuo Wang, Shanlin Yang, Xiaohong Chen and

others. Xuesen Qian, the first winner of the National Highest Science and Technology Award of China, Jinpeng Huai, minister of the Ministry of Education of China, Angela Dorothea Merkel, former German Chancellor, have also praised us for the achievements in this area of research. Indeed, such positive feedback on our work has given us the impetus to develop this book.

Finally, many colleagues and administrators have supported us in the process of writing this book, as the authors have consulted many scholars and a wide range of relevant published literature throughout the development of this project. In particular, they are deeply in debt to Dr. Caroline Moraes, who proofread all chapters of the book, and Ms. Emily Zhang and colleagues of Springer Nature for their great help. The authors would like to express their appreciation to them all.

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Chapter 1

Introduction



1.1 The Scientific Background of the Birth of Grey System Theory

On the basis of dividing the spectrum of scientific and technological endeavors into fine sections, the overall development of modern science has shown the tendency of synthesis at a high level. This higher level synthesis has led to the appearance of various studies of systems science with their specific methodological and epistemological significance. Systems science reveals deep and intrinsic interconnections between objects and events, and has greatly enriched the overall progress of science and technology. Many of the historically difficult problems in different scientific fields have been resolved successfully along with the appearance of systems science and its specific branches. Furthermore, because of the emergence of various new areas in systems science, our understanding of nature and the laws that govern objective evolutions has been gradually deepened. At the end of the 1940s, there appeared systems theory, information theory, and cybernetics. Toward the end of 1960s and the start of 1970s, there appeared the theory of dissipative structures, synergetics, catastrophe, and fractal theory. Then, in the mid to late 1970s, new transfield and interfiled theories of systems science such as the hypercycle theory and dynamical systems theory emerged.

In the process of system research, due to the existence of internal and external disturbances and the limitation of human cognitive ability, the information obtained by people often has some uncertainty. With the development of science and technology and the progress of human society, people have gradually deepened their understanding of various system uncertainties, and the research on uncertain systems is also deepening day by day. Since the 1960s, a variety of uncertain system theories and methods have been proposed one after another. Among them, Fuzzy mathematics founded by Professor L. A. Zadeh in the 1960s (Zadeh, 1965), grey system theory advanced by Professor Deng Julong in the 1980s (Deng Julong, 1982), rough sets theory developed by Professor Z. Pawlak in the 1980s (Pawlak, 1991), etc., are all important achievements in the study of uncertain systems with extensive international

influence. These uncertain theories discussed the theories and methods of describing and processing various kinds of uncertain information from different perspectives and aspects.

Grey system theory takes the “small data and poor information” uncertain system with “some information known and some information unknown” as the research object. It mainly extracts valuable information through the mining of “some” known information, and realizes the correct description of the system operation behavior and evolution law, so that people can use mathematical models to analyze and assess the “small data and poor information” uncertain system, then realize high-precision prediction, scientific decision-making and optimal control of the “small data and poor information” uncertain system. The uncertainty system of “small data and poor information” in the real world provides rich research resources and broad development space for grey system theory.

1.2 The Founder of Grey System Theory

The birth of grey system theory is an outcome of Professor Julong Deng who has been working with perseverance for decades.

Prof. Deng was born in Lianyuan County, Hunan Province of China in 1933. He got his degree in electrical machinery from Huazhong Institute of Technology and then joined the same institute in 1955 as a teaching assistant. Prof. Deng used to keep an eye on new ideas related to his field which led to his later investigation into multi-variable system control problems. In the 1960s, he put forward a new method—“control by removing redundant”. His paper entitled “multivariable linear system shunt calibration device of a comprehensive approach” was published in 1965 (Deng, 1965). By the early 1970s, the method of “control by remove redundant” has been widely recognized as a representative methodology in cybernetics.

In 1965, Prof. L. A. Zadeh proposed Fuzzy Sets (Zadeh, 1965). Prof. Deng was involved in research of fuzzy mathematics. He published some papers in fuzzy mathematics. And served as a member of editorial board for several journals on fuzzy mathematics. In the late 1970s, Prof. Deng devoted himself to the study of “prediction and control problems of economic system”. In dealing with systems where “some information is known, and some information is unknown”, the main challenge is to develop an effective method to represent such systems. Despite the difficulties, Professor Deng and his colleagues have made significant progress in their explorations. In 1982, his pioneering paper titled “The Control Problems of Grey Systems” published by Systems and Control Letters (Deng, 1982). The publication of this seminal article indicated that grey system theory, a new branch of research, came into being.

Since the birth of Grey System Theory, it has received significant attention from academic communities and industries both in China and overseas, especially in real world applications.

So far, Prof. Deng's works has been cited over 50 thousand times. Prof. Deng won the award of founder of Grey System Theory at the 2007 IEEE International Conference on Grey Systems and Intelligent Services which held in Nanjing. In 2011, he was elected as the honor fellow of the World Organisation of Systems and Cybernetics at the joint conference of the 15th WOSC International Congress on Cybernetics and Systems and 2011 IEEE International Conference on Grey Systems and Intelligent Services.

1.3 Development of Grey Systems Theory

1.3.1 Building a Basic Team

In the early 1990s, Professor Julong Deng began to recruit and train doctoral students in the field of grey system theory in the discipline of system engineering of Huazhong University of Science and Technology. He has recruited and trained 10 doctoral students, most of them are young scholars who have been engaged in grey system theory research for many years before entered Prof. Deng's group. These scholars naturally become the first generation of grey system theory. They actively participate in the research of grey system theory, consciously assume the responsibility of developing and disseminating grey system theory, and unswervingly take the research and inheritance of grey system theory as their lifelong career.

In 2000, as the first distinguished professor introduced by Nanjing University of Aeronautics and Astronautics (NUAA), one of Prof. Deng's Ph.D. students, Professor Sifeng Liu joined this university with aerospace characteristics. In the same year, with Professor Sifeng Liu as the chief discipline leader, NUAA submitted an application to the Academic Degrees Committee of the State Council of China for the establishment of a doctoral degree authorization point in management science and engineering, which was successfully approved. Therefore, grey system theory has naturally become the characteristic and leading direction of the doctoral program of management science and engineering of NUAA. At the same time, as the founding director, Professor Liu established the Institute for Grey System Studies at NUAA. IGSS-NUAA has also become the center of grey system scholars. A group of outstanding young scholars gathered in IGSS-NUAA through talent introduction, entering the station to carry out post-doctoral research and pursuing doctoral degree, forming a highland of grey system research. IGSS-NUAA has 12 doctoral tutors (including 6 full-time doctoral tutors). Over the past 20 years, it has recruited and trained more than 200 doctoral students, post-doctors and visiting scholars at home and abroad in the field of grey system theory.

Many other universities are recruiting and funding doctoral and postdoctoral researchers in grey system theory and its application. Examples include Southeast University, Wuhan University of Technology, Fuzhou University, Shantou University, De Montfort University, Bucharest Economics University, Poznań University of