Studies in Systems, Decision and Control 445

Narayanaswamy Balakrishnan ·
María Ángeles Gil · Nirian Martín ·
Domingo Morales ·
María del Carmen Pardo *Editors*

Trends in Mathematical, Information and Data Sciences

A Tribute to Leandro Pardo

Springer

Studies in Systems, Decision and Control

Volume 445

The series "Studies in Systems, Decision and Control" (SSDC) covers both new developments and advances, as well as the state of the art, in the various areas of broadly perceived systems, decision making and control–quickly, up to date and with a high quality. The intent is to cover the theory, applications, and perspectives on the state of the art and future developments relevant to systems, decision making, control, complex processes and related areas, as embedded in the fields of engineering, computer science, physics, economics, social and life sciences, as well as the paradigms and methodologies behind them. The series contains monographs, textbooks, lecture notes and edited volumes in systems, decision making and control spanning the areas of Cyber-Physical Systems, Autonomous Systems, Sensor Networks, Control Systems, Energy Systems, Automotive Systems, Biological Systems, Vehicular Networking and Connected Vehicles, Aerospace Systems, Automation, Manufacturing, Smart Grids, Nonlinear Systems, Power Systems, Robotics, Social Systems, Economic Systems and other. Of particular value to both the contributors and the readership are the short publication timeframe and the world-wide distribution and exposure which enable both a wide and rapid dissemination of research output.

Indexed by SCOPUS, DBLP, WTI Frankfurt eG, zbMATH, SCImago.

All books published in the series are submitted for consideration in Web of Science.

More information about this series at https://link.springer.com/bookseries/13304

Narayanaswamy Balakrishnan ·
María Ángeles Gil · Nirian Martín ·
Domingo Morales · María del Carmen Pardo
Editors

Trends in Mathematical, Information and Data Sciences

A Tribute to Leandro Pardo

Springer

Editors
Narayanaswamy Balakrishnan
Department of Mathematics and Statistics
McMaster University
Hamilton, ON, Canada

María Ángeles Gil
Department of Statistics and OR and TM
University of Oviedo
Oviedo, Asturias, Spain

Nirian Martín
Department of Financial and Actuarial Economics & Statistics
Complutense University of Madrid
Madrid, Spain

Domingo Morales
Department of Statistics, Mathematics and Informatics
Miguel Hernández University of Elche
Elche, Spain

María del Carmen Pardo
Department of Statistics and Operational Research
Complutense University of Madrid
Madrid, Spain

ISSN 2198-4182 ISSN 2198-4190 (electronic)
Studies in Systems, Decision and Control
ISBN 978-3-031-04136-5 ISBN 978-3-031-04137-2 (eBook)
https://doi.org/10.1007/978-3-031-04137-2

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Leandro Pardo in the advertisement of the homage paid to him in his *alma mater*, the Universidad Complutense de Madrid, by December 2019

Leandro Pardo entering the room for his UCM homage (source: https://www.ucm.es/tribunacomplutense/261/art3944.php#.YcoUO2jMJnI)

To Marisa and Julio,
the closest collaborators of Leandro
in research and life

Preface

This scientific edited book has been prepared as a tribute to our beloved friend and colleague, Leandro Pardo. To a certain extent, the book is related to the Symposium on Information Theory with Applications to Statistical Inference that was held at the Universidad Complutense de Madrid on December 2, 2019. The book was planned to be published by either late 2020 but the circumstances associated with the COVID-19 situation have often caused unavoidable delays. Anyway, the book is now ready and we should deeply thank all the contributors for their helpful support and understanding.

The contributions to this book involve ideas/results from the topics of mathematical, information and statistical sciences, to which Leandro has devoted immense research endeavors. Although the book has been structured so that papers have been classified as those related to either Mathematical, Information or Data Science, this should not be considered as a classical partition since classes definitely overlap and many contributions could be properly included in more than one of them.

In the scientific world, there are colleagues who stand out for combining the originality and impact of their research with the generosity and human quality of the character. One of these colleagues is undoubtedly Prof. Leandro Pardo. Since in 1980 he obtained a doctorate in Mathematics at the Department of Statistics and Operations Research of the Complutense University of Madrid, Leandro has been an enthusiastic and tireless researcher. He has made remarkable contributions in fields as diverse as fuzzy sets, Bayesian nonparametric statistics, survival and reliability analysis, information theory, estimation and hypothesis testing procedures based on divergences and entropies, categorical data analysis, statistical models and their diagnosis, or robust statistical inference, among others. This book contains a variety of articles written by authors who want to pay tribute to Leandro for his professional career and his humanity.

It is difficult to summarize in a brief space the professional life of Leandro Pardo and therefore we will limit ourselves to highlighting some important aspects. By the end of 2021, databases show that Leandro has published more than 250 research papers in the field of Statistics with more than 1700 citations and co-authored by around 45 different authors. He is the author of two reference books on Statistical Information Theory, which establish the methodological bases for the statistical inference based on divergence measures. It is remarkable that he has coordinated more than 12 research and development projects funded by the Spanish Government and participated in international research projects funded by NATO.

Leandro is closely linked to the *Spanish Society of Statistics and Operations Research* (SEIO), in which he has held relevant management positions. For instance, he was President of SEIO and Editor-in-Chief of *TEST*, the statistical journal sponsored by SEIO and in 2020 he has been awarded with SEIO's Medal. He was also Associate Editor of *Communication in Statistics-Theory and Methods*, *Communication in Statistics-Simulation and Computation* and *Journal of Statistical Planning and Inference*. Currently, he is Associate Editor of *Journal of Multivariate Analysis*, *TEST* and *Complutense Mathematical Magazine*. Special mention deserves his time as the "2004 Distinguished Eugene Lukacs Professor" at Booling Green University (Bowling, Green, Ohio), a worldwide recognition established 1990 to 2007 and awarding outstanding scientists on the basis of their distinguished records of research in the application or theory of probability or statistics.

This book, categorized into three parts, Trends in Mathematical Sciences (First Part), Trends in Information Sciences (Second Part) and Trends in Data Sciences (Third Part), brings together 38 contributions, authored by colleagues, students, descendants and friends of Leandro. Throughout these parts the reader will encounter that many of the research works citate his prominent book *"Statistical Inference based on Divergence Measures*" edited by Chapman & Hall/CRC (2006). As mentioned earlier, the three parts are not exhaustive, they have in common several branches of Statistics, namely,

- Big Data/High-dimensional Statistics:
  - José Miguel Angulo and María Dolores Ruiz-Medina (Second Part)
  - Daniel Barreiro-Ures, Ricardo Cao and Mario Francisco-Fernández (Third Part)
  - Wenceslao González-Manteiga, Rosa M. Crujeiras and Eduardo García-Portugués (Third Part)
  - Abhijit Mandal and Samiran Ghosh (Third Part)

- Categorical Data Analysis:
  - Maria Kateri (First Part)
  - Mara Virtudes Alba-Fernández and María Dolores Jiménez-Gamero (Second Part)
  - Juana M. Alonso, Aída Calviño and Susana Muñoz (Second Part)
  - Apostolos Batsidis and Polychronis Economou (Second Part)

  - Pedro Miranda, Ángel Felipe and Nirian Martín (Second Part)
  - María Jaenada (Third Part)

- Fuzzy Data Analysis:
  - María Asunción Lubiano, Manuel Montenegro, Sonia Pérez-Fernández, and María Ángeles Gil (Third Part)
  - Beatriz Sinova (Third Part)

- Mathematical Economics:
  - Emilio Cerdá and Xiral López-Otero (First Part)
  - Ahmed Shatla, Carlos Carleos, Norberto Corral, Antonia Salas and María Teresa López (Second Part)

- Multivariate Data Analysis:
  - Béatrice Byukusenge, Dietrich von Rosen and Martin Singull (Third Part)
  - Miquel Salicrú, Ferran Reverter, Mireia Besalú and Moises Burset (Third Part)

- Spatial Statistics:
  - Alfonso García-Pérez (First Part)
  - Noel Cressie, Alan R. Pearse and David Gunawan (Second Part)
  - Ana F. Militino, María Dolores Ugarte and Unai Pérez-Goya (Third Part)

- Stochastics Processes:
  - Antonio Gómez-Corral, María Jesús López-Herrero and María Teresa Rodríguez-Bernal (First Part)
  - Miguel Lafuente, David Ejea, Raúl Gouet, F. Javier López and Gerardo Sanz (First Part)
  - Javier Villarroel and Juan A. Vega (First Part)

- Survey Data Analysis:
  - María Dolores Esteban, Tomáš Hobza, Domingo Morales and Agustín Pérez (Third Part)
  - Tomáš Hobza and Domingo Morales (Third Part)
  - Juan Luis Moreno-Rebollo, Joaquín Muñoz-García and Rafael Pino-Mejías (Third Part)

  - María del Mar Rueda, Beatriz Cobo and Antonio Arcos (Third Part)

- Survival/Reliability Analysis:
  - Elena Castilla and Pedro J. Chocano (Second Part)
  - Narayanaswamy Balakrishnan, Tian Feng and Hon-Yiu So (Third Part)
  - Alba M. Franco-Pereira, María Carmen Pardo and Teresa Pérez (Third Part)

- Theoretical Statistical Inference with Divergence Measures:
  - Kostas Zografos (First Part)
  - Abhik Ghosh and Ayanendranath Basu (Second Part)
  - Marianthi Markatou and Anran Liu (Second Part)

- Time Series:
  - Lajos Horváth and Gregory Rice (First Part)
  - Carlos Maté (First Part)
  - J. Fernando Vera (Second Part).

Either as main interest or as a complementary tool, "Robust Statistics" has been considered in most of the contributions of the previous branches, as it constitutes the principal research area in which has been published, in the last decade, almost all of Leandro's research work.

Linked specifically to first part, the work of Arturo Hidalgo and Lourdes Tello falls into Partial Differential Equations, while the work of Julián Costa, Ignacio García-Jurado and Juan Carlos Gonçalves-Dosantos falls into Project Management. Design of Experiments is the main area of the contribution of Carlos de la Calle Arroyo, Jesús López-Fidalgo and Licesio J. Rodríguez-Aragón, in third part. The previous challenging and appealing topics have offered either new developments from a theoretical and/or computational point of view, or reviews of recent literature of outstanding developments. They have been applied through examples in Climatology, Chemistry, Economics, Engineering, Geology, Health Sciences, Physics, Pandemics and Socioeconomic indicators.

The authors of this edited multiauthors book should deeply thank to all those contributing it. We know well how much affection for Leandro is involved in all the papers in it. We must also express our special gratitude to Asun Lubiano and Antonia Salas for their meticulous proofreading of the whole book.

As is evident from the contents of this volume, Leandro has had varied research interests in the field of Statistics and has made many incisive contributions to many different areas of Statistics over the years. Likewise, he has had numerous successful collaborations and relationships with many people in the statistical community at large, both within Spain and outside. So, the diverse editorial team of this volume should come as no surprise! Some of us are among his colleagues, some are among his doctoral students, and some others are his collaborators, but one thing we all share in common is that we are all among his close friends and well-wishers. It is this that brought us together to work closely as a team to bring out this volume to honor our friend, Prof. Leandro Pardo, and it is our sincere hope that it will put a smile on his face and bring him many fond memories and a lot of happiness!

Hamilton, Canada — Narayanaswamy Balakrishnan
Oviedo, Spain — María Ángeles Gil
Madrid, Spain — Nirian Martín
Elche, Spain — Domingo Morales
Madrid, Spain — María del Carmen Pardo
February 2022

Contents

Trends in Mathematical Sciences

Using Taxes to Manage Energy Resources Related to Stock Pollutants: Resource Cartel *versus* Importers

Emilio Cerdá and Xiral López-Otero

Abstract This chapter analyzes, through a two-period model, the interaction between a producing cartel and a country (or coalition of countries) that import an energy-related natural resource whose consumption generates a stock pollution. The chapter has been inspired by and aims to contribute to the design of climate change strategies, as greenhouse gas emissions are accumulated in the atmosphere and are brought about by the use of fossil fuels that in many cases are unevenly distributed across the world. A particular attention is paid to the use of taxes on natural resources in the context of simultaneous decisions by countries and different concern regarding the environmental problem.

1 Introduction

In the last few years there has been an increased attention to the management of stock pollutants as a most significant environmental problem, climate change, is due to the accumulation of greenhouse gas (GHG) emissions in the atmosphere. At the same time, GHG emissions are mainly related to the use of natural resources (fossil fuels) that are unevenly distributed and in some ocassions highly concentrated across the planet. This raises important issues and potential trade-offs as, on the one hand, climate change has the characteristic of a global public good (i.e. affecting both fossil-fuel producers and consumers) but, on the other hand, producers of certain fossil fuels behave strategically to maximize their rents from such natural resources.

E. Cerdá (✉)
Department of Economic Analysis and ICEI, Universidad Complutense de Madrid, Campus de Somosoguas, 28223 Madrid, Spain
e-mail: ecerdate@ucm.es

X. López-Otero
Department of Economic Theory and Mathematical Economics, UNED, Senda del Rey, 11, 28040 Madrid, Spain
e-mail: xiral@outlook.es

N. Balakrishnan et al. (eds.), *Trends in Mathematical, Information and Data Sciences*, Studies in Systems, Decision and Control 445,
https://doi.org/10.1007/978-3-031-04137-2_1

This chapter is interested in exploring this setting, with a particular attention to the use of taxes on the natural resource. Indeed, fossil fuel taxes are quite common in reality and respond to the growing concerns on climate change (carbon taxes) or other environmental problems, and also to strategic reaction by consuming countries to price manipulation by fossil fuel producers. In particular, the chapter analyzes, from a theoretical point of view, the relationship between a producing cartel and a country (or coalition of countries) that imports an energy-related natural resource. It does so by considering different attitudes towards the stock pollutant (global public bad) and by analyzing simultaneous actions by countries. The main objective of the chapter is to study how taxes by importers affect the strategies of producers and eventually the stock of pollution.

Previous literature on these matters includes Bergstrom [1], who studied the capacity of importing countries to obtain rents from the natural resource. Maskin and Newbery [7] and Karp and Newbery [4, 5] analyzed the world oil market in this respect. More related to this paper, Wirl [13, 14] studied the relationship between a producing cartel of a resource that generates pollution with an importing country that uses a resource tax to maximize the welfare of these consumers. Wirl and Dockner [17] explore again this relationship but assuming that the government also values the tax revenues. Rubio [8] extends the interaction between countries when there are many importing countries that behave non-cooperatively. Other papers such as Tahvonen [11], Rubio and Escriche [9] or Strand [10] study the relationship between resource producing and importing countries but incorporating the possibility of sequential choices. Finally, Wirl [15] introduces uncertainty in the model, Daubanes and Grimaud [2] propose a growth model with a rich importing region and a poor exporting region, Dullieux et al. [3] incorporate an upper limit of carbon concentration in the atmosphere, Wirld [16] considers the case in which agents can choose to set prices (taxes) or quantities, while Wei et al. [12] distinguish between domestic marked production and export production, allowing price discrimination between them.

Our contribution to the literature rests on the use of an analytical model in discrete time that, instead of focusing on the evolution path for the different variables in continuous time (as the above.mentioned papers), makes it possible to obtain and compare the optimal solutions to ascertain which situations are better for each side and for the management of the stock of pollution. The chapter also provides some practical and relevant insights. In general it is found that, when countries that import a (energy-related) natural resource are coordinated to set a (environmental) tax on imported energy goods, their welfare is increased and the associated pollution decreased. Yet this does not hold in all situations, so importers should be careful to evaluate each particular setting. Moreover, given that decisions in each period by the involved parties have influence in the future, agents cannot commit to pre-determined actions or commitments.

The chapter is composed of four sections, including this introduction. The following section presents the basic model, which is employed to study particular cases in section three. Finally, the chapter closes with a summary of the main results and conclusions.

2 The Model

There exists a producer cartel of a polluting natural resource (in our case, energy-related, for example oil) and a country or a coalition of countries that imports and consumes such a resource from the cartel.[1] The model considers two periods: in each period the cartel decides the price of the resource, whilst the importer country establishes a tax on the resource.

The consumption of the resource generates a pollution that accumulates in the atmosphere, in such a way that in each period t the stock of pollution (S_t) will be given by

$$S_t = S_{t-1} + q_t \ , \tag{1}$$

q_t being the consumption of the resource in the importing country in period t. We assume that the stock of pollution does not decline. This is a sensible decision if the stock pollutant represents accumulated GHG emissions, as their decline is very slow (about 200 years) and non-linear. In order to consider that all the quantity of the resource consumed enters into the stock of pollution as in (1), a unit of measure of the energy which gives rise to the emission of a unit of pollutant to the atmosphere can be used (see Wirl and Dockner [17]).

That stock of pollution generates a negative externality in each period which is modeled, following the literature (see e.g. Wirl [14] or Liski and Tahvonen [6]), trough a quadratic damage function of the form $\frac{1}{2}cS_t^2$, with $c > 0$.

The cartel tries to maximize its profits, which will be given by the difference between the income and cost of extraction of the resource. We assume that the extraction costs are constant and, since it does not affect the essence of the results, we will consider null costs (see Wirl [13]). For its part, the importer country tries to maximize the welfare of its citizens, which is given by the sum of the consumer's surplus and tax revenues,[2] minus the environmental damage provoked by the consumption of the resource. We assume that the demand function of the resource in each period in the importing country is linear of the form

$$q_t = a - b(p_t + \tau_t) \tag{2}$$

p_t and τ_t being, respectively, the price of the resource, fixed by the cartel, and the tax established by the government of the importing country in period t, $a > 0$, $b > 0$, $a > c$. Although the resource is nonrenewable, following Wirl [13] we assume that the resource constraint is not binding, because the resource will not be consumed totally due to the associated externality. In other words, we limit the externality and do not simply delay its negative environmental effects.

[1] It is assumed that the resource is not consumed in the producer country, whilst the importing country does not produce that resource.

[2] It is assumed that the tax revenues in the importing country are given back to the citizens through lump-sum transfers.

In the following section we present the different cases analyzed using this model, obtaining the analytical expressions of the main variables of the model and studying the relationships between these variables in different cases to see what situations are best for each party and the environment (stock of pollution). As it is a dynamic model with two periods, time-consistent results are presented, i.e. the agents, though initially could commit to certain prices or taxes in the future, may be interested in deviating from that commitment with the passage of time.

3 Analyzed Cases

In this section, we first consider the case in which the government of the importing country does not establish any tax on the consumption of the energy resource. In the second subsection we assume that the government of the importing country decides to introduce a tax in each period on the consumption of the resource. Analytical expressions for prices, quantities, taxes, stock of pollution, profits of the cartel and welfare of the consumers in the importing country in terms of the parameters a, b, c and S_0 are obtained. Finally, a numerical example is given.

3.1 Absence of Taxes (WT)

In this case, there is a static optimization problem in which the producer sets the price that maximizes its profits, and the importing country does not have any capacity to influence that price. So, the problem for the cartel will be

$$\max_{p_1, p_2} \Pi = p_1 q_1 + p_2 q_2 = p_1(a - bp_1) + p_2\,(a - bp_2)\,. \tag{3}$$

From the necessary and sufficient conditions of optimality we obtain that, in absence of taxes, the main variables of the model take the following values, which are time-consistent:

$$\begin{aligned} p_1{}^{WT} &= a/(2b),\, q_1^{WT} = a/2,\, S_1^{WT} = S_0 + (a/2), \\ p_2^{WT} &= a/(2b),\, q_2^{WT} = a/2,\, S_2^{WT} = S_0 + a. \end{aligned} \tag{4}$$

Then, the profits of the cartel will be

$$\Pi^{WT} = p_1^{WT} q_1^{WT} + p_2^{WT} q_2^{WT} = \frac{a^2}{2b}. \tag{5}$$

On the other hand, the welfare of the consumers in the importing country will be given by

$$\begin{aligned} W^{WT} &= u_1 + u_2 - \tfrac{1}{2}cS_1^2 - \tfrac{1}{2}cS_2^2 \\ &= \tfrac{1}{2}\left(\tfrac{a}{b} - p_1^{WT}\right)q_1^{WT} + \tfrac{1}{2}\left(\tfrac{a}{b} - p_2^{WT}\right)q_2^{WT} - \tfrac{1}{2}cS_1^2 - \tfrac{1}{2}cS_2^2 \\ &= a^2/(4b) - c\left(S_0^2 + \tfrac{5}{8}a^2 + \tfrac{3}{2}S_0 a\right), \end{aligned} \tag{6}$$

u_t being the consumer's surplus in the importing country deriving from the consumption of the resource in period t.

3.2 Taxes (T)

Let us now assume that the government of the importing country decides to introduce a tax τ in each period on the consumption of the resource. In this way, the problem of the cartel producer is now

$$\max_{p_1, p_2} \Pi = p_1 q_1 + p_2 q_2 = p_1(a - bp_1 - b\tau_1) + p_2\left(a - bp_2 - b\tau_2\right). \tag{7}$$

The first order conditions are:

$$\frac{\partial \Pi}{\partial p_i} = a - 2bp_i - b\tau_i = 0 \Longrightarrow p_i = \frac{a - b\tau_i}{2b}, i = 1, 2 \tag{8}$$

Sufficient conditions for the maximization problem are also satisfied.

On the other hand, the government of the importing country sets the taxes in such a way that the welfare of its citizens in maximized, that is

$$\begin{aligned} &\max_{\tau_1,\tau_2} W = u_1 + \tau_1 q_1 - \tfrac{1}{2}cS_1^2 + u_2 + \tau_2 q_2 - \tfrac{1}{2}cS_2^2 \\ &= \tfrac{1}{2}\left(\tfrac{a}{b} - p_1 - \tau_1\right)(a - bp_1 - b\tau_1) + \tau_1(a - bp_1 - b\tau_1) \\ &-\tfrac{1}{2}c(S_0 + a - bp_1 - b\tau_1)^2 + \tfrac{1}{2}\left(\tfrac{a}{b} - p_2 - \tau_2\right)(a - bp_2 - b\tau_2) \\ &+\tau_2(a - bp_2 - b\tau_2) - \tfrac{1}{2}c(S_0 + a - bp_1 - b\tau_1 + a - bp_2 - b\tau_2)^2 \end{aligned} \tag{9}$$

Solving this maximization problem, we obtain the best-response functions of the importer. The open-loop Nash equilibrium is calculated with the best response functions of both agents. However, these results are not time consistent as, given that pollution generated in the first period accumulates in the atmosphere and thus influences the agents when making their decisions in the second period, the result in that period will depend on what happens in the first period and therefore the importer is forced to reduce the tax on the resource. Although the price increase brings about a reduction in the consumed quantity, and hence a reduction in the stock of pollution, it also reduces their utility and tax revenues causing a welfare loss that is greater than the profit obtained by reducing pollution. Therefore, the importer partly offsets the fall in consumption due to the price increase. On the other hand, since decisions in the first period influence the second period through the accumulated stock of pollution, as a result of deviation in the first period, and since the price in the second period depends negatively on the stock of accumulated stock of pollution in the first period, the producer's price will also increase in the second. In this case, however, the reduction in the importer's tax is of such magnitude that it provokes a reduction in the price paid by consumers. This, in turn, increases the quantity consumed during this period, in an attempt to reduce welfare loss. As a result exporters increase their profits, whereas welfare in the importing country is reduced.

Thus, the time consistent results (subgame perfect Nash equilibrium), obtained by backward induction are given below:

$$p_1^T = \frac{4a + 5abc + 2ab^2c^2 - 8bcS_0 - 9b^2c^2S_0 - b^3c^3S_0}{8b + 16b^2c + 7b^3c^2 + b^4c^3} \tag{10}$$

$$p_2^T = \frac{4a + 4abc + ab^2c^2 - 4bcS_0 - 2b^2c^2S_0}{8b + 16b^2c + 7b^3c^2 + b^4c^3} \tag{11}$$

$$\tau_1^T = \frac{10ac + 5abc^2 + ab^2c^3 + 16cS_0 + 14bc^2S_0 + 2b^2c^3S_0}{8 + 16bc + 7b^2c^2 + b^3c^3} \tag{12}$$

$$\tau_2^T = \frac{8ac + 5abc^2 + ab^2c^3 + 8cS_0 + 4bc^2S_0}{8 + 16bc + 7b^2c^2 + b^3c^3} \tag{13}$$

From these expressions, we can see, as in Wirl [13] that the taxes set by the importing country will be purely environmental. Thus, if the importing country had not taken into account the environmental damage that the consumption of the resource provokes ($c = 0$), such taxes would be zero and the government could not use them to influence the price set by the producers.

Substituting expressions (10) and (12) in the demand function (2) for $t = 1$ and expressions (11) and (13) in that demand function for $t = 2$, the following results for the quantities are obtained:

$$q_1^T = \frac{4a + abc - 8bcS_0 - 5b^2c^2S_0 - b^3c^3S_0}{8 + 16bc + 7b^2c^2 + b^3c^3} \tag{14}$$

$$q_2^T = \frac{4a + 4abc + ab^2c^2 - 4bcS_0 - 2b^2c^2S_0}{8 + 16bc + 7b^2c^2 + b^3c^3} \tag{15}$$

In an analogous way, substituting the expressions (10) to (15) in the objective function (7), the mathematical expression of the maximum profit of the cartel producer can be obtained, and after substitution in the objective function (9), the expression of the maximum welfare of the consumers in the importing countries can also be obtained.

The corresponding values of the stock of pollution in periods 1 and 2 are the following:

$$S_1^T = \frac{4a + abc + 8S_0 + 8bcS_0 + 2b^2c^2S_0}{8 + 16bc + 7b^2c^2 + b^3c^3} \tag{16}$$

$$S_2^T = \frac{8a + 5abc + ab^2c^2 + 8S_0 + 4bcS_0}{8 + 16bc + 7b^2c^2 + b^3c^3} \tag{17}$$

The main results are presented in the following proposition.

Proposition 3.1 *The introduction of taxes on the consumption of the resource by the importing country reduces the prices of the producer* ($p_i^T \leq p_i^{WT}$, *with* $p_i^T < p_i^{WT}$, *if* $c \neq 0, i = 1, 2$), *the quantities* ($q_i^T \leq q_i^{WT}$, *with* $q_i^T < q_i^{WT}$, *if* $c \neq 0, i = 1, 2$) *and the stock of the pollution* ($S_i^T \leq S_i^{WT}$, *with* $S_i^T < S_i^{WT}$, *if* $c \neq 0, i = 1, 2$) *in both periods, although the final price paid by the consumers is higher* ($p_i^T + \tau_i^T \geq p_i^{WT}$, *with* $p_i^T + \tau_i^T > p_i^{WT}$, *if* $c \neq 0, i = 1, 2$). *Moreover, the welfare in the importing country is higher* ($W^T \geq W^{WT}$, *with* $W^T > W^{WT}$, *if* $c \neq 0$) *and the profits of the cartel are lower* ($\Pi^T \leq \Pi^{WT}$, *with* $\Pi^T < \Pi^{WT}$, *if* $c \neq 0$).

Proof We have that

$$p_1^T = p_1^{WT} + \frac{-6ac - 3abc^2 - ab^2c^3 - 16cS_0 - 16bc^2S_0 - 2b^3c^3S_0}{16 + 32bc + 14b^2c^2 + 2b^3c^3} \tag{18}$$

$$p_2^T = p_2^{WT} + \frac{-8ac - 5abc^2 - ab^2c^3 - 8cS_0 - 4bc^2S_0}{16 + 32bc + 14b^2c^2 + 2b^3c^3} \tag{19}$$

$$q_1^T = q_1^{WT} + \frac{-8abc - 7ab^2c^2 - ab^3c^3 - 16bcS_0 - 10b^2c^2S_0 - 2b^3c^3S_0}{16 + 32bc + 14b^2c^2 + 2b^3c^3} \tag{20}$$

$$q_2^T = bp_2^T, q_2^{WT} = bp_2^{WT}, p_2^T < p_2^{WT} \Longrightarrow q_2^T < q_2^{WT} \tag{21}$$

$$p_1^T + \tau_1^T = p_1^{WT} + \frac{14ac + 7abc^2 + ab^2c^3 + 16cS_0 + 12bc^2S_0 + 2b^2c^3S_0}{16 + 32bc + 14b^2c^2 + 2b^3c^3} \tag{22}$$

$$p_2^T + \tau_2^T = p_2^{WT} + \frac{8ac + 5abc^2 + ab^2c^3 + 8cS_0 + 4bc^2S_0}{16 + 32bc + 14b^2c^2 + 2b^3c^3} \tag{23}$$

Moreover

$$W^T = W^{WT} + \frac{A + B}{8(8 + 16bc + 7b^2c^2 + b^3c^3)^2} + \frac{C + D + E}{8(4 + 6bc + b^2c^2)^2}, \tag{24}$$

where

$$\begin{aligned} A &= 224a^2c + 948a^2bc^2 + 1552a^2b^2c^3 + 1082a^2b^3c^4 \\ B &= 381a^2b^4c^5 + 68a^2b^5c^6 + 5a^2b^6c^7 + 384acS_0 \\ C &= 1824abc^2S_0 + 325ab^2c^3S_0 + 2400ab^3c^4S_0 + 876ab^4c^5S_0 \\ D &= 160ab^5c^6S_0 + 12ab^6c^7S_0 + 320bc^2S_0^2 + 1088b^2c^3S_0^2 \\ E &= 1092b^3c^4S_0^2 + 480b^4c^5S_0^2 + 100b^5c^6S_0^2 + 8b^6c^7S_0^2 \end{aligned} \tag{25}$$

As a, b, c and S_0 are positive ($c = 0$ if the importing country does not take into account the environmental damage that the consumption of the resource provokes), with the introduction of the tax, producer prices and quantities are reduced. At the same time, the price paid by consumers and the welfare in the importing country increase. Moreover, as q_1 and q_2 are smaller than in the case without taxes, S_1 and S_2 are also smaller. Finally, prices and quantities are smaller in both periods with respect to the case without taxes, and therefore producer's profits are also smaller. □

By introducing the consumption tax on the resource, prices increase for consumers, who thus reduce the amount of resource consumed. This requires the exporter to reduce the prices to ensure that the fall in consumption is not as sharp and thus minimizes the reduction in profits. In any case, the exporter cannot avoid the decrease of its profits. The importing country manages to increase its welfare because the stock of pollution is reduced and tax revenues are collected, despite the loss of utility caused by the reduction of the amount of resource consumed.

3.3 Numerical Example

For the following values of the parameters, $a = 5, b = 1, c = 0.02, S_0 = 10$, the following results are obtained:

	Without taxes	With taxes
p_1	2.5	2.27
p_2	2.5	2.35
τ_1	0	0.51
τ_2	0	0.29
$p_1 + \tau_1$	2.5	2.78
$p_2 + \tau_2$	2.5	2.65
Π	12.5	10.58

(26)

	Without taxes	With taxes
q_1	2.5	2.22
q_2	2.5	2.35
S_1	12.5	12.22
S_2	15	14.85
W	2.44	3.61

(27)

4 Conclusions

This chapter has analyzed, from a theoretical point of view, the relationship between a producing cartel and a country (or coalition of countries) that imports an energy-related natural resource whose consumption generates a stock of pollution. Although the chapter can be applicable to any environmental problem with these characteristics, we have been inspired by a major contemporary environmental challenge: climate change phenomena. Climate change is caused by the accumulation of GHG emissions in the atmosphere and has a public good nature, so there is room for strategic interaction among countries. Moreover, GHG emissions are related to the consumption of fossil fuels that, in many cases are unevenly distributed and concentrated across the planet, giving also rise to strategic interaction among countries to capture resource rents. An adequate treatment of such a complex and multiple relationships between resource producers and consumers is thus necessary for a proper understanding and management of the stock pollutant.

We have paid a particular attention to the use of taxes on the natural resource whose use provokes the stock of pollution. Indeed, fossil fuel taxes are quite common in reality and respond to the growing concerns on climate change (carbon taxes) or other environmental problems, and also to strategic reaction by consuming countries to price manipulation by fossil fuel producers. Tax revenues may also play an important role in welfare-enhancing strategies by countries, either through public expenditure programs or through shifts in the tax system.

The chapter has actually studied the influence of resource taxes, and of environmental concerns by countries, on the price and the quantity consumed of such a resource, thus providing information on the stock of pollution and on the level of welfare of exporters and importers. Unlike most of the literature in this area, which has focused on the evaluation path for the different variables in continuous time, our contribution rests on the use of an analytical model in discrete time that allows us to obtain and compare the optimal solutions to ascertain which situations are better for each side and for the management of the stock of pollution.

Our research has shown that, in the case of simultaneous decisions, the introduction of a tax on resources to correct the environmental externality brings about a decrease in the producer price and in the quantity of the consumed resource, thus reducing the level of pollution, and an increase in the price paid by the consumers. Moreover, there is an increase in the welfare of the importing country, and the profits of the cartel are reduced.

An interesting extension for this piece of research consist of the consideration of sequential choices, instead of simultaneous decisions. The two possible cases are the following: (a) The exporting country is first in deciding its prices (the leader is the exporter). (b) The importing country decides first on the taxes levied on the consumption of the resource (the leader is the importer country).

References

1. Bergstrom, T.: On capturing oil rents with national excise tax. Amer. Econ. Rev. **72**, 194–201 (1982)
2. Daubanes, J., Grimaud, A.: Taxation of a polluting non-renewable resource in the heterogeneous world. Environ. Resour. Econ. **47**, 567–588 (2010)
3. Dullieux, R., Ragot, L., Shcubert, K.: Carbon tax and OPEC's rents under a ceiling constraint. Escand. J. Econ. **113**, 798–824 (2011)
4. Karp, L., Newbery, D.: OPEC and the U.S. oil import tariff. Econ. J. **101**, 303–313 (1991)
5. Karp, L., Newbery, D.: Dynamically consistent oil import tariffs. Can. J. Econ. **XXV**, 1–21 (1992)
6. Liski, M., Tahvonen, O.: Can carbon tax eat OPEC's rents? J. Environ. Econ. Manag. **47**, 1–12 (2004)
7. Maskin, E., Newbery, D.: Disadvantageous oil tariffs and dynamic consistency. Amer. Econ. Rev. **80**, 143–156 (1990)
8. Rubio, S.: On capturing rent from a non-renewable resource international monopoly: prices versus quantities. Dyn. Games Appl. **1**, 558–580 (2011)
9. Rubio, S., Escriche, L.: Strategic pigouvian taxation, stock externalities and polluting non-renewable resources. J. Pub. Econ. **79**, 297–313 (2001)
10. Strand, J.: Importer and producer petroleum taxation: a geo-political model. International Monetary Fund Working Paper WP/08/35 (2008)
11. Tahvonen, O.: Trade with polluting nonrenewable resources. J. Environ. Econ. Manag. **30**, 1–17 (1996)
12. Wei, J., Hennlock, M., Johansson, D., Sterner, T.: The fossil endgame: strategic oil price discrimination and carbon taxation. J. Environ. Econ. Policy **1**, 48–69 (2012)
13. Wirl, F.: Pigouvian taxation of energy for flow and stock externalities and strategic, non competitive energy pricing. J. Environ. Econ. Manag. **26**, 1–18 (1994)
14. Wirl, F.: The exploitation of fossil fuels under the threat of global warming and carbon taxes: a dynamic game approach. Environ. Resour. Econ. **5**, 333–352 (1995)
15. Wirl, F.: Energy prices and carbon taxes under uncertainty about global warming. Environ. Resour. Econ. **36**, 313–340 (2007)
16. Wirl, F.: Global warming: prices versus quantities from a strategic point of view. J. Environ. Econ. Manag. **64**, 217–229 (2012)
17. Wirl, F., Dockner, E.: Leviathan governments and carbon taxes: costs and potential benefits. Eur. Econ. Rev. **39**, 1215–1236 (1995)

A New Shapley Value-Based Rule for Distributing Delay Costs in Stochastic Projects

Julián Costa, Ignacio García-Jurado, and Juan Carlos Gonçalves-Dosantos

Abstract In this paper we propose a new allocation rule for stochastic projects with delays based on the Shapley value and compare it with another Shapley value-based rule introduced in a recent paper. First we justify the interest of considering a new rule of this kind, then we compare it with the old one in some examples and finally we study some theoretical properties that distinguish them.

1 Introduction

Project management is an important and widely used tool in engineering for the successful implementation of complex projects. One of the first contributions to this body of knowledge was the PERT/CPM methodology, developed in the late 1950s (see [8]). Since then, numerous techniques, algorithms and protocols have given rise to modern project management.

An important issue in this field is the planning and time control of a project and, within this issue, a relevant question is how the costs generated by project delays should be distributed.

The first articles that deal with the allocation of delay costs in a project from the perspective of cooperative game theory are [1, 3]. Since then, this topic has been treated by various authors, always in a deterministic setting, until [4] addresses the problem in stochastic projects and proposes a sharing rule based on the Shapley value, inspired by the rule for deterministic projects in [2].

J. Costa (✉)
Grupo MODES, Universidade da Coruña, Facultad de Informática, 15071 A Coruña, Spain
e-mail: julian.costa@udc.es

I. García-Jurado · J. C. Gonçalves-Dosantos
Grupo MODES and CITMAga, Universidade da Coruña, Facultad de Informática, 15071 A Coruña, Spain
e-mail: ignacio.garcia.jurado@udc.es

J. C. Gonçalves-Dosantos
e-mail: juan.carlos.goncalves@udc.es

N. Balakrishnan et al. (eds.), *Trends in Mathematical, Information and Data Sciences*, Studies in Systems, Decision and Control 445,
https://doi.org/10.1007/978-3-031-04137-2_2

In this paper we propose a new sharing rule for stochastic projects with delays based on the Shapley value. In Sect. 3 we justify the interest of considering a new rule, in Sect. 4 we compare both rules in some examples and, finally, in Sect. 5 we study some theoretical properties that distinguish them.

2 The Problem

In this section we describe the problem we are dealing with, that was first introduced and discussed in [4].

We start from a stochastic project, given by a list of activities, a description of the precedence relations between them, and a specification of the probability distribution of the duration of each of the activities. We also have a cost function that depends on the durations of the activities. Our objective is to propose and analyse a rule for distributing the cost associated with each implementation of the project. Next we give the formal definition of stochastic projects with delays; notice that it generalizes the notion of deterministic projects with delays, dealt with in [1, 3].

Definition 2.1 A stochastic project with delays SP is a tuple $(N, \prec, X^0, x, C)$ where:

- N is the finite non-empty set of activities.
- $\prec$ is a binary relation over N satisfying asymmetry and transitivity; it describes the precedence relations between the activities.
- $X^0 = (X_i^0)_{i\in N}$ is a vector of independent non-negative random variables. Each X_i^0 describes the duration of activity i.
- $x \in \mathbb{R}^N$ is the vector of observed non-negative durations.
- $C : \mathbb{R}^N \to \mathbb{R}$ is the delay cost function. We assume that $C(0) = 0$ and that C is non-decreasing.

We denote by $\mathcal{SP}^N$ the set of stochastic projects with delays with activities set N, and by $\mathcal{SP}$ the set of all stochastic projects with delays.

The problem is to identify a sound rule for distributing the costs associated with the duration of the project and its activities. The following is a formal definition of distribution rule in this context.

Definition 2.2 A rule for stochastic projects with delays is a map ψ on $\mathcal{SP}$ that assigns to each $SP = (N, \prec, X^0, x, C) \in \mathcal{SP}^N$ a vector $\psi(SP) \in \mathbb{R}^N$ satisfying $\sum_{i\in N} \psi_i(SP) = C(x)$.

3 A New Shapley Value-Based Rule

In this section we provide a theoretical rationale for the introduction of a new rule for stochastic projects with delays.

To begin with, we remember the Shapley rule for stochastic projects with delays introduced in [4]. Informally, such a rule is based on associating a cooperative game to each stochastic project with delays and calculating its Shapley value. For an introduction to cooperative games and the Shapley value, the reader can consult [6].

Definition 3.1 The Shapley rule for stochastic projects with delays Sh is defined by $Sh(SP) = \Phi(v^{SP})$ where for all $SP \in \mathcal{SP}^N$:

- v^{SP} is the cooperative game with set of players N given by

$$v^{SP}(S) = \begin{cases} E(C(x_S, X^0_{N\setminus S})) & \text{for all non-empty } S \subseteq N, \\ 0 & \text{for } S = \emptyset, \end{cases}$$

  where x_S and $X^0_{N\setminus S}$ denote the restrictions of x and X^0 to S and $N \setminus S$, respectively.
- $\Phi(v^{SP})$ denotes the proposal of the Shapley value for v^{SP}, i.e.

$$\Phi_i(v^{SP}) = \sum_{S\subseteq N\setminus\{i\}} \frac{|S|!\,(|N|-|S|-1)!}{|N|!}(v^{SP}(S\cup\{i\}) - v^{SP}(S))$$

  for all $i \in N$.

The Shapley rule Sh satisfies a number of reasonable properties (see [4]) and, moreover, it is a natural extension of the Shapley rule for deterministic projects with delays introduced in [2]. However, Sh has the disadvantage that it is based on a game that shows a certain discontinuity, in the sense that its definition for a coalition S is different depending on whether or not S is empty. In fact, it is easy to check that there exist $SP \in \mathcal{SP}^N$ with

$$E(C(X^0_N)) \neq 0 = v^{SP}(\emptyset).$$

To correct this disadvantage, in this paper we study a variant of the Shapley rule that is based on a game whose definition has a single expression for all coalitions. This rule was introduced in [5], but in that paper only its definition is given without any additional comments, except that it has been incorporated to the R package ProjectManagement. The main purpose of this paper is to motivate its interest, to study some of its properties and to compare it with Sh.

Definition 3.2 The modified Shapley rule for stochastic projects with delays $\widehat{Sh}$ is defined by $\widehat{Sh}(SP) = \Phi(\widehat{v}^{SP})$ where for all $SP \in \mathcal{SP}^N$:

- $\widehat{v}^{SP}$ is the cooperative game with set of players N given by

$$\widehat{v}^{SP}(S) = E(C(x_S, X^0_{N\setminus S})) - E(C(X^0_N)) + E(C(X^0_S, 0_{N\setminus S}))$$

  for all $S \subseteq N$.
- $\Phi(\widehat{v}^{SP})$ denotes the proposal of the Shapley value for $\widehat{v}^{SP}$.