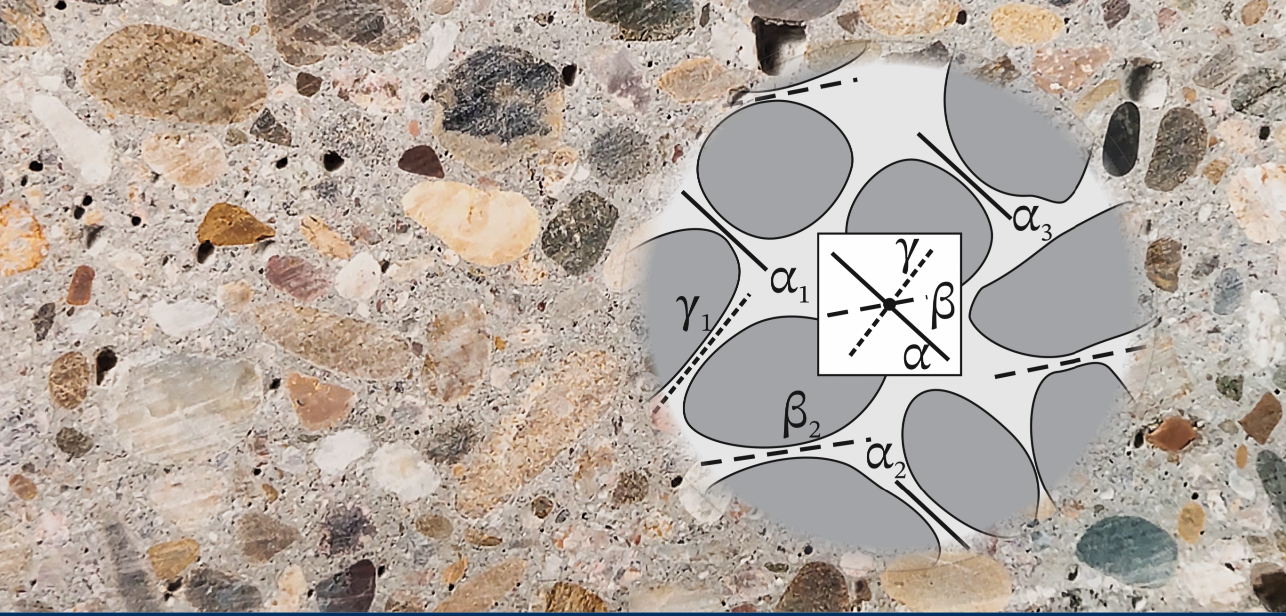

Ulrich Häussler-Combe

Computational Structural Concrete

Theory and Applications

Second Edition

Ernst & Sohn
A Wiley Brand
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Author

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Preface

This book grew out of lectures that the author gave at the Technische Universität Dresden. These lectures were entitled "Computational Methods for Reinforced Concrete Structures" and "Design of Reinforced Concrete Structures." Reinforced concrete is a composite of concrete and reinforcement connected by bond. Bond is a key item for the behaviour of the composite, which utilises the compressive strength of concrete and the tensile strength of reinforcement while allowing for controlled crack formation. This makes reinforced concrete unique compared to other construction materials such as steel, wood, glass, masonry, plastic materials, fibre reinforced plastics, geomaterials, etc. The theory and use of reinforced concrete in structures falls in the area of structural concrete.

Numerical methods like the finite element method, on the other hand, basically allow for a realistic computation of the behaviour of all types of structures. But the implementations are generally presented as black boxes in the view of the users. Input is fed in and the output has to be trusted. The assumptions and methods in-between are not transparent. This book aims to provide transparency with special attention being paid to the unique properties of reinforced concrete structures. Corresponding methods are described with their potentials and limitations while integrating them into the larger framework of computational mechanics connected to reinforced concrete. This is aimed at advanced students of civil and mechanical engineering, academic teachers, designing and supervising engineers involved in complex problems of structural concrete, and researchers and software developers interested in the broader picture. Most of the methods described are complemented with examples computed with a PYTHON software package developed by the author and coworkers. Program package and example data should be available at https://www.concrete-fem.com. The package exclusively uses the methods described in this book. It is open for discussion with the disclosure of the source code and should give stimulation for alternatives and further developments.

This book represents a fundamental revision of the book COMPUTATIONAL METHODS FOR REINFORCED CONCRETE STRUCTURES, which was published in 2014. In particular, the chapter on multi-axial concrete material laws was expanded, and the topics of crack formation and the regularisation of material laws with strain softening were dealt with in a separate chapter. Thanks are given to the publisher

Ernst & Sohn, Berlin, and in particular to Mrs Claudia Ozimek for the engagement in supporting this work. My education in civil engineering and my professional and academic career were guided by my academic teacher Prof. Dr.-Ing. Dr.-Ing. E.h. Dr. techn. h.c. Josef Eibl[1], former Head of the Department of Concrete Structures at the Institute of Concrete Structures and Building Materials at the Technische Hochschule Karlsruhe (nowadays KIT – Karlsruhe Institute of Technology). Further thanks are given to former coworkers Patrik Pröchtel, Jens Hartig, Mirko Kitzig, Tino Kühn, Joachim Finzel, Tilo Senckpiel-Peters, Daniel Karl, Ahmad Chihadeh, Ammar Siddig Ali Babiker, Evmorfia Panteki, and Alaleh Sehni for their specific contributions. I deeply appreciate the inspiring and collaborative environment of the Institute of Concrete Structures at the Technische Unversität Dresden, which is directed by Prof. Dr.-Ing. Dr.-Ing. E.h. Manfred Curbach. It was my pleasure to teach and research at this institution. And I have to express my deep gratitude to my wife Caroline for her love and patience.

Dresden, Spring 2022 *Ulrich Häussler-Combe*

1) He passed away in 2018.

Contents

List of Examples

Notation

The same symbols may have different meanings in some cases. But the different meanings are used in different contexts, and misunderstandings should not arise.

		firstly used
General		
$\bullet^{\mathrm{T}}$	transpose of vector or matrix $\bullet$	Eq. (2.5)
$\bullet^{-1}$	inverse of quadratic matrix $\bullet$	Eq. (2.13)
$\delta\bullet$	virtual variation of $\bullet$, test function	Eq. (2.5)
$\delta\bullet$	solution increment of $\bullet$ within iterations	Eq. (2.75)
$\tilde{\bullet}$	$\bullet$ transformed in (local) coordinate system	Eq. (6.14)
$\dot{\bullet}$	time derivative of $\bullet$	Eq. (2.4)
$\bullet_e$	$\bullet$ related to single finite element	Eq. (2.18)
Normal lowercase italics		
a_s	reinforcement cross-section per unit width	Eq. (9.61)
b	cross-section width	Eq. (4.9)
b_w	crack band width	Eq. (3.6)
d	cross-section effective height	Eq. (9.67)
e	element index	Eq. (2.18)
f	strength condition	Eq. (6.48)
f_c	uniaxial compressive strength of concrete (unsigned)	Eq. (3.2)
f_{ct}	uniaxial tensile strength of concrete	Eq. (3.4)
f_t	uniaxial failure stress of reinforcement	Eq. (3.41)
f_y	uniaxial yield stress of reinforcement	Eq. (2.48)
f_E, f_R	probability density functions of random variables E, R	Eqs. (11.2), (11.3)
g_f	specific crack energy per unit volume	Eq. (3.7)
h	cross-section geometric height	Eq. (4.10)
m_x, m_y, m_{xy}	moments per unit width	Eq. (9.7)
n	total number of degrees of freedom in a discretised system	Eq. (2.70)
n_E	total number of elements	Section 4.3
n_i	order of Gauss integration	Eq. (2.69)
n_N	total number of nodes	Section 4.3
n_x, n_y, n_{xy}	normal forces per unit width	Eq. (9.7)
p	pressure	Eq. (6.8)
p_f	failure probability	Eq. (11.19)
$\overline{p}_x, \overline{p}_z$	loading distributed along beam	Eq. (4.49)

		firstly used
r, s, t	local spatial coordinates	Eq. (2.15)
s	slip	Section 3.4
s_{bf}	slip at residual bond strength	Section 3.4
$s_{b\,\max}$	slip at bond strength	Section 3.4
t	clock time or loading time	Eq. (2.4)
t_x, t_y, t_{xy}	couple force resultants per unit width	Eq. (9.58)
u_i	i-th displacement component	Eq. (6.1)
v_x, v_y	shear forces per unit width	Eq. (9.7)
w	deflection	Eq. (2.56)
w	fictitious crack width	Eq. (3.5)
w_{cr}	critical crack width	Eq. (3.9)
x, y, z	global spatial coordinates	Eq. (2.14)
x, $\overline{x}$	compression zone height	Eqs. (4.29), (9.66)
z, $\overline{z}$	internal lever arm	Eqs. (4.115), (9.58)

Bold lowercase roman

$\mathbf{b}$	body forces	Eq. (2.5)
$\mathbf{f}$	internal nodal forces	Eq. (2.9)
$\mathbf{p}$	external nodal forces	Eq. (2.9)
$\mathbf{n}$	normal vector	Eq. (6.5)
$\mathbf{s}$	slip	Eq. (8.53)
$\mathbf{t}$	surface tractions	Eq. (2.5)
$\mathbf{t}_b$	bond force	Eq. (8.54)
$\mathbf{t}_{cL}$	crack traction in local system	Eq. (7.3)
$\mathbf{t}_c$	crack traction in global system	Eq. (7.133)
$\mathbf{u}$	displacement field	Eq. (2.1)
$\boldsymbol{v}$	nodal displacement vector	Eq. (2.1)
$\boldsymbol{v}_e$	nodal displacement vector related to a single element	Eq. (2.18)
$\mathbf{w}_{cL}$	fictitious crack width in local system	Eq. (7.2)
$\mathbf{w}_c$	fictitious crack width in global system	Eq. (7.133)

Normal uppercase italics

A	cross-sectional area of a bar or beam	Eq. (2.54)
A_s	cross-sectional area reinforcement	Section 3.6
A_t	part of surface with prescribed tractions	Eq. (2.5)
A_u	part of surface with prescribed displacements	Eq. (2.53)
C	material stiffness coefficient	Eq. (3.35)
C_T	tangential material stiffness coefficient	Eq. (3.37)
D	scalar damage variable	Eq. (6.105)
E	Young's modulus	Eq. (2.43)
E_0	initial Young's modulus	Eq. (3.16)
E_c	initial Young's modulus of concrete	Eq. (3.1)
E_s	initial Young's modulus of steel	Eq. (3.41)
E_T	tangential hardening material stiffness coefficient	Eq. (3.41)
F	yield function	Eq. (6.64)
F	damage function	Eq. (6.108)
F_E	distribution function of random variable E	Eq. (11.1)
G	shear modulus	Eq. (4.8)

		firstly used
G	flow potential	Eq. (6.63)
G_f	specific crack energy per surface	Eq. (3.8)
I_1	first invariant of stress	Eq. (6.19)
J	determinant of Jacobian matrix	Eq. (2.37)
J_2, J_3	second, third invariant of stress deviator	Eq. (6.19)
K	slab bending stiffness	Eq. (9.12)
L_c	characteristic length of an element	Eq. (7.18)
L_e	length of bar or beam element	Eq. (2.23)
M	bending moment	Eq. (4.9)
N	normal force	Eq. (4.9)
P	probability	Eq. (11.1)
T	natural period	Eq. (4.209)
V	shear force	Eq. (4.9)
V	volume	Eq. (2.5)

Bold uppercase roman

$\mathbf{B}$	matrix of spatial derivatives of trial functions	Eq. (2.2)
$\mathbf{C}$	material stiffness matrix	Eq. (2.47)
$\mathbf{C}_T$	tangential material stiffness matrix	Eq. (2.50)
$\mathbf{C}_{cLT}$	tangential local crack stiffness matrix	Eq. (7.9)
$\mathbf{D}$	material compliance matrix	Eq. (2.51)
$\mathbf{D}_T$	tangential material compliance matrix	Eq. (2.51)
$\mathbf{D}_{cT}$	tangential crack band compliance matrix	Eq. (7.38)
$\mathbf{D}_{cLT}$	tangential local crack compliance matrix	Eq. (7.9)
$\mathbf{E}$	isotropic linear elastic material stiffness matrix	Eq. (6.23)
$\mathbf{G}_1, \mathbf{G}_2, \mathbf{G}_3$	unit vectors of covariant system	Eq. (10.16)
$\mathbf{G}^1, \mathbf{G}^2, \mathbf{G}^3$	unit vectors of contravariant system	Eq. (10.17)
$\mathbf{I}$	unit matrix	Eq. (6.100)
$\mathbf{J}$	Jacobian matrix	Eq. (2.20)
$\mathbf{K}$	stiffness matrix	Eq. (2.11)
$\mathbf{K}_T$	tangential stiffness matrix	Eq. (2.67)
$\mathbf{M}$	mass matrix	Eq. (2.61)
$\mathbf{N}$	matrix of trial functions	Eq. (2.1)
$\mathbf{Q}$	stress / strain rotation matrix	Eq. (6.14)
$\mathbf{T}$	element rotation matrix	Eq. (4.105)
$\mathbf{V}_n$	shell director	Eq. (10.2)
$\mathbf{V}_\alpha, \mathbf{V}_\beta$	unit vectors of local shell system	Eqs. (10.2), (10.3)

Normal lowercase Greek

α	for several purposes in a local context	
α_E, α_R	sensitivity parameters	Eq. (11.14)
α_s	shear retention factor	Eq. (7.7)
β	for several purposes in a local context	
β_t	tension stiffening coefficient	Eq. (3.65)
ϵ	uniaxial strain	Eq. (2.43)
ϵ	strain of a beam reference axis	Eq. (4.4)
$\epsilon_1, \epsilon_2, \epsilon_3$	principal strains	Section 6.2.3
ϵ_{ct}	concrete strain at uniaxial tensile strength	Figure 3.3

		firstly used
ϵ_{cu}	concrete failure strain at uniaxial tension	Figure 3.3
ϵ_{c1}	concrete strain at uniaxial compressive strength (signed)	Eq. (3.1)
ϵ_{cu1}	concrete failure strain at uniaxial compression (signed)	Eq. (3.1)
ϵ_I	imposed uniaxial strain	Eq. (3.35)
ϵ_V	volumetric strain	Eq. (6.101)
ϕ	cross-section rotation	Eq. (4.1)
ϕ	angle of external friction	Eq. (6.90)
φ	for several purposes in a local context	
φ_c	orientation of concrete principal compression	Eq. (8.5)
φ_s	orientation of reinforcement	Eq. (8.6)
γ	shear angle	Eq. (4.1)
γ_E, γ_R	partial safety factors	Eqs. (11.58), (11.59)
κ	curvature	Eq. (4.4)
κ_p	internal state variable for plasticity	Eq. (6.64)
κ_d	internal state variable for damage	Eq. (6.107)
μ_R, μ_E	means of random variables R and E	Eqs ((11.3), (11.6))
ν	Poisson's ratio	Eq. (2.44)
ν_R, ν_E	coefficients of variation	Eq. (11.60)
θ	Lode angle	Eq. (6.45)
ϑ	angle of internal friction	Eq. (6.88)
ρ	deviatoric length	Eq. (6.44)
ρ_s	reinforcement ratio	Eq. (8.8)
ϱ_s	specific mass	Eq. (2.52)
σ	uniaxial stress	Eq. (2.43)
$\sigma_1, \sigma_2, \sigma_3$	principal stresses	Section 6.2.3
σ_R, σ_E	standard deviations of random variables R, E	Eqs. (11.3), (11.6)
τ	bond stress	Eq. (3.47)
τ	for several purposes in a local context	
τ_{bf}	residual bond strength	Figure 3.13
$\tau_{b\,\max}$	bond strength	Figure 3.13
ω	circular natural frequency	Eq. (4.209)
ω	related crack width	Eq. (7.5)
ξ	hydrostatic length	Eq. (6.43)

Bold lowercase Greek

$\boldsymbol{\epsilon}$	small strain	Eq. (2.2)
$\boldsymbol{\epsilon}$	generalised strain	Eq. (2.33)
$\boldsymbol{\epsilon}_p$	small plastic strain	Eq. (6.61)
$\boldsymbol{\kappa}$	vector of internal state variables	Eq. (6.38)
$\boldsymbol{\sigma}$	Cauchy stress	Eq. (2.3)
$\boldsymbol{\sigma}$	generalised stress	Eq. (2.34)
$\boldsymbol{\sigma}'$	deviatoric part of Cauchy stress	Eq. (6.9)

Uppercase Greek

Φ	standardised normal distribution function	Eq. (11.20)
$\boldsymbol{\Sigma}$	stress extension	Eq. (2.82)

1 Introduction

Why Read This Book?

Concrete is by far the most used building material in the world. Concrete can be given arbitrary forms, its basic constituents are available everywhere, its processing is basically simple, and it is inexpensive. Furthermore, concrete can be customised to fulfil special requirements – e.g. high strength, resistance in rough environments, impermeability, ductility – through adjustment of binder, aggregates, fibres, and additives. Its major characteristic from a mechanical point of view is given by a relatively high compressive strength but a low tensile strength. Thus, it is reinforced with bars, wire mats, fabrics of steel, carbon, glass, and more, which leads to an immense variety of composite building materials.

With this we see architectural landmark buildings like the television tower in Stuttgart, Germany, the first of this type designed and engineered by Fritz Leonhardt and built in 1956, Figure 1.1a, the Palazzetto dello Sport in Rome, Italy, a coliseum for the Olympic games 1960 built in 1956 and engineered by Pier Lucri Nervi, Figure 1.1b, the Ganter bridge within the access road to the Simplon pass in the Swiss Alps built in 1980 and designed and engineered by Christian Menn, Figure 1.2a, and

(a) (b)

Figure 1.1 (a) Stuttgart television tower, from Kleinmanns and Weber (2009), photography: Landesmedienzentrum Baden-Württemberg: Albrecht Brugger. (b) Palazetto dello Sport, from Ehmann and Pfeffer (1999).

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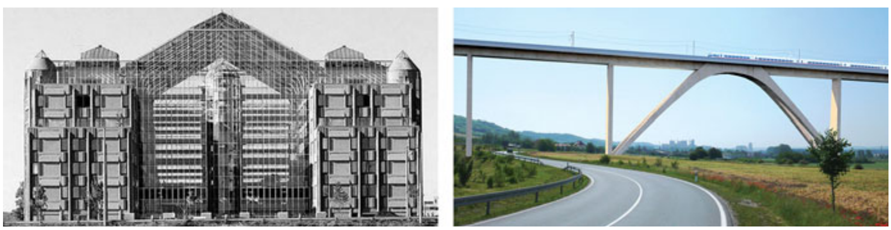
(a) (b)

Figure 1.2 (a) Ganter bridge, from Billington (2014), photography: Nicolas Janberg. (b) National Veterans Memorial and Museum, from Helbig et al. (2020), photography: Knippers Helbig Stuttgart – New York – Berlin.



(a) (b)

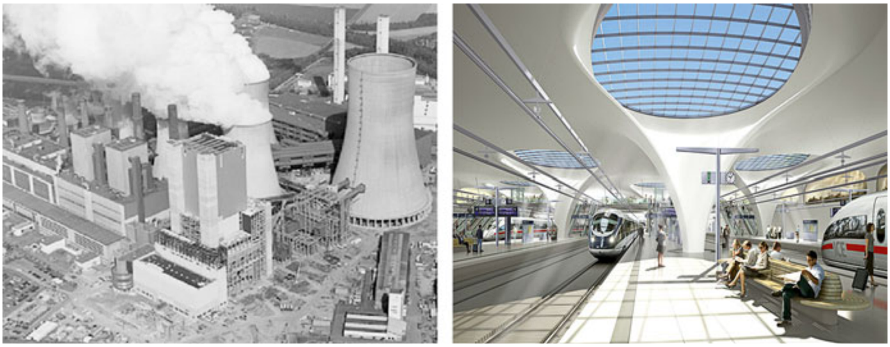
Figure 1.3 (a) Office building: Züblin-Haus, from Bachmann et al. (2021). (b) High-speed railway viaduct over the valley Unstruttal, Germany, photomontage, from Schenkel et al. (2009).

the National Veterans Memorial and Museum, Columbus, Ohio, USA, built in 2018 and engineered by Knippers Helbig, Figure 1.2b, to mention only a few.

A countless number of concrete buildings contribute to everyday life; for example, office buildings, Figure 1.3a (Züblin headquarters, Stuttgart, Germany; precast concrete with steel–glass atrium), railway bridges, Figure 1.3b (Unstruttal viaduct, Thuringia, Germany), power plants, Figure 1.4a (RWE, Niederaußem, Germany), station concourses, Figure 1.4b (Stuttgart 21, Germany; final state visualisation, still under construction). This demonstrates some visible contributions of the application of concrete. Indispensable infrastructures providing freshwater, drainage, and wastewater processing, waste disposal processing in general, generation and provision of electricity, support of transport via vehicles, trains, ships, and airplanes are generally hidden from immediate visibility. To sum it up, today's civilisation would be unthinkable without concrete as a building material.

It can be stated that reinforced concrete is the building material of the twentieth century. But will it also be the building material of the twenty-first century?

Presumably yes, due to its advantages listed above. But sustainability has to become a predominant topic also for reinforced concrete besides bearing capacity, usabili-



(a) **(b)**

Figure 1.4 (a) Power plant, RWE, Niederaußem, Germany, from Krätzig et al. (2007), photography: RWE. (b) Underground station concourse, Stuttgart 21, from Bechmann et al. (2019), visualisation: Ingenhoven Architekten, Düsseldorf.

ty, and durability. Production of cement – the predominant binder for concrete – causes a high output of CO_2 due to its energy consumption on the one hand and chemical conversion processes on the other hand. The same also applies to reinforcing steel whereupon its contribution to reinforced concrete is relatively small measured by weight ratio. Construction waste makes up the largest proportion of the total amount of waste. What is the conclusion?

We have to use less concrete and fewer reinforcement materials and at the same time achieve a higher quality of building components.

Structural design plays a key role to reach this goal. We should gain a better understanding of load carrying mechanisms of building components in order to fully utilise load bearing potentials and to optimise structural forms and materials. There is still a lot of room for improvement in this regard.

Computational methods are an extremely important tool for this. Numerical simulation in combination with experimental investigations allows for a comprehensive understanding of the deformation behaviour, force flow, and failure mechanisms of building components. This permits weak points to be identified and eliminated in a targeted manner. New concepts may be initiated, and a simulation-based rapid prototyping may be performed for initial assessments of new innovative structural forms and materials. On the basis of the knowledge gained from this, the design and elaboration of components in building practice can be carried out more efficiently and with higher quality using computational methods.

Topics of the Book

Such methods are generally demanding with respect to methodology, implementation, and application. This is especially true for nonlinear problems as are typical for structural concrete. Computational methods for nonlinear structural analysis offer a wide range of capabilities. But they are made available to users as black boxes.

This hides the fact that numerical methods usually have application limits. If these are not observed, the results become questionable. Often, this is not obvious to users providing input for black boxes and accepting output without hesitation. This motivates the goals and contents of this textbook about computational methods – in particular, the finite element method (FEM) – for reinforced concrete (RC):

- Survey of the key aspects of the FEM.
- Understanding of basic mechanisms of RC regarding interaction of concrete and reinforcement through bond.
- Specifics of FEM regarding structural elements like RC-beams, plates, slabs, and shells.
- Essential characteristics of the multi-axial mechanical behaviour of concrete.
- Pitfalls related to FEM treating structural concrete and in particular the failure behaviour.

Knowing these issues, the black boxes should become more transparent, and their results should be better comprehensible. The finite element method is the preferred method also for the computation of reinforced concrete structures due to its versatility and adaptability.

Chapter 2 gives an overview of modelling in general and summarises items of FEM as far as is required for its application to reinforced concrete structures. *Chapter* 3 describes basic mechanisms of structural concrete, which relies on the interaction of concrete and reinforcement by continuous transfer of forces through bond. This is restricted to uniaxial behaviour in a first approach to point out essential properties and describes the mechanisms of the reinforced uniaxial tension bar as prototype of structural concrete. In *Chapter* 4, this is extended to reinforced concrete beams and frames, which are characterised by bending that may be superimposed with normal forces whereby still basing on uniaxial behaviour of materials. This also includes first aspects of creep, temperature, and shrinkage. Furthermore, prestressing of beams is treated, which is an important technology to extend the application range of reinforced concrete. The chapter closes with the analysis of large displacements and dynamics, exemplarily in each case with their application to beams. A first extension of bending of beams to high beams and plates is given in *Chapter* 5 with strut-and-tie models, which utilise the uniaxial behaviour of concrete and reinforcement for a design of plane structures with in-plane loading. Furthermore, limit theorems of plasticity – which are an important basis for design in structural concrete – are exemplarily developed within this context. *Chapter* 6 treats multi-axial concrete behaviour as extension of the uniaxial approach applied in the foregoing chapters. Multi-axial material concrete models are the basis for the structural models for plates, slabs, and shells treated in the following chapters. Basic topics of continuum mechanics are described in as far as they are necessary to understand multi-axial nonlinear stress–strain and failure behaviour of concrete. Material models like elasto-plasticity, damage, and microplane are applied with respect to concrete modelling. A major item regarding material modelling occurs with strain softening – increasing strains with decreasing stresses – which requires a regularisation to reach reliable numerical solution. A further major item concerns the cracking of concrete, which separates parts of a continuum into a discontinuum. This couples discretisation issues with material modelling and is described in *Chapter* 7. *Chapter* 8 treats design and simulation of reinforced concrete plates with high beams as a special but common case. In this respect, the design is considered separately, as it may be based on linear solutions for plate stresses utilizing a limit theorem of plasticity. On the other hand, simulation considers nonlinear stress–strain relations additionally leading to solutions for the deformation behaviour. Reinforced concrete slabs, which are treated in *Chapter* 9, extend uniaxial 1D-bending of beams into biaxial 2D-bending. As before with plates, aspects of design and simulation may be separated in an analogous manner. The most general approach for structural analysis is given with shells, which combine in-plane actions of plates and transverse actions of slabs whereby extending flat geometries to folded or curved geometries. Shells require complex mechanical models, which is exemplarily treated in *Chapter* 10 together with the application to reinforced concrete. *Chapter* 11 treats first aspects of randomness, which is a major topic regarding structural concrete behaviour. Deterministic models – however sophisticated they may be – always give a more or less restricted view of the real world. First notions of an extended view are given in this chapter. Finally, a number of topics are treated in the appendices insofar they are reasonable for better understanding of the main text but might disturb the line of concise arguing therein.

How to Read This Book

The treatment of the above combines methods of mechanics, structural analysis, and applied mathematics. This recourse should be self-explanatory and conclusive to a large degree, so that a study of accompanying literature is generally not required. In doing so, essential lines of development are worked out on the one hand, but on the other hand, the available concepts and methods cannot be described with all details. Furthermore, not every problem addressed is provided with a comprehensive solution. The book is intended to encourage the reader to deepen and explore such topics independently.

Nevertheless, the book involves a large volume. Proposals for shorter tracks are given in the following thereby also enlightening the structure of the book content and the relations between sections. Major groups are characterised as

- FEM and reinforced concrete bases, see Figure 1.5a.
- Uniaxial structures, see Figure 1.5b.
- Multi-axial concrete and its implications for numerical methods, see Figure 1.6a.
- Multi-axial structures such as plates, slabs, and shells, see Figure 1.6b.

This includes a short track (left column) and branches (right column) for each of these. Chapter 11.1 *Randomness and Reliability* falls out of this scheme. Nevertheless, basic knowledge of stochastics related to reinforced concrete is considered necessary.

Many topics are illustrated with examples. Most of them are computational and are processed with the Python 3.6 program package ConFem. A few are performed

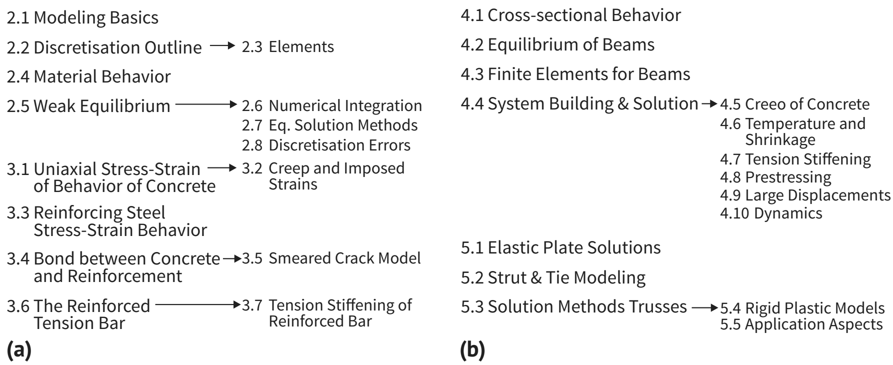


Figure 1.5 (a) FEM and reinforced concrete bases. (b) Uniaxial structures.

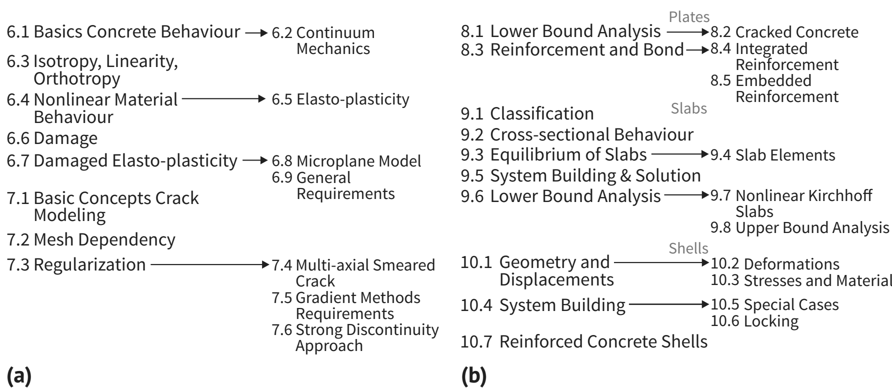


Figure 1.6 (a) Multi-axial concrete and its implications. (b) Multi-axial structures.

with stand-alone PYTHON scripts or are short, illustrating theoretical derivations. Environments to perform PYTHON are freely available on the internet for all common platforms.

All PYTHON sources for CONFEM, a basic documentation, example input data, and reference result data are available at https://www.concrete-fem.com under open-source conditions.

Thus, all book examples should be reproducible by the reader. But the CONFEM project is not finished and may be subject to continuous development. The user should see it as an inspiring challenge to master this tool. The interplay of theory, implementation, and application – possibly with overcoming resistance – ultimately leads to a deeper understanding of numerical methods, structural concrete, and their dependencies.

2
Finite Elements Overview

Numerical methods like the finite element method are outstanding as engineering tools but actually have to be embedded in a larger frame of modelling. Discretisation is a key therein, whereby an infinite number of unknowns is reduced to a finite number. On the one hand, this is based on cornerstones of structural analysis like equilibrium, kinematic compatibility, and material behaviour, and on the other hand, methods of numerical mathematics have to be used to reach solutions. An overview is given in the following in as far it is necessary for the application to structural concrete.

2.1 Modelling Basics

> *There are no exact answers. Just bad ones, good ones and better ones.*
> *Engineering is the art of approximation.*

Approximation is performed with models. We consider a reality of interest, e.g. a concrete beam. In a first view, it has *properties* such as dimensions, colour, and surface texture. From the view of structural analysis, the latter are irrelevant. A more detailed inspection reveals a lot more properties: weight, displacements, stiffness, strength, temperatures, conductivities, capacities, and so on. Only a part of these is essential from the structural point of view. We combine those essential properties to form a *conceptual model*. Whether a property is essential is obvious for some, but the valuation of others might be doubtful. We have to choose. By choosing properties our model becomes an approximation compared to reality. Approximations are more or less accurate.

On the one hand, we should reduce the number of properties of a model. Any reduction of properties will make a model less accurate. Nevertheless, it might remain a good model. On the other hand, an over-reduction of properties will make a model inaccurate and therefore useless. Furthermore, maybe properties that have no counterparts in the reality of interest are introduced. Conceptual modelling is the art of choosing properties. As all other arts, it cannot be performed guided by strict rules.

The chosen properties have to be related to each other in a quantitative manner. This leads to a *mathematical model*. In many cases, we have systems of differen-

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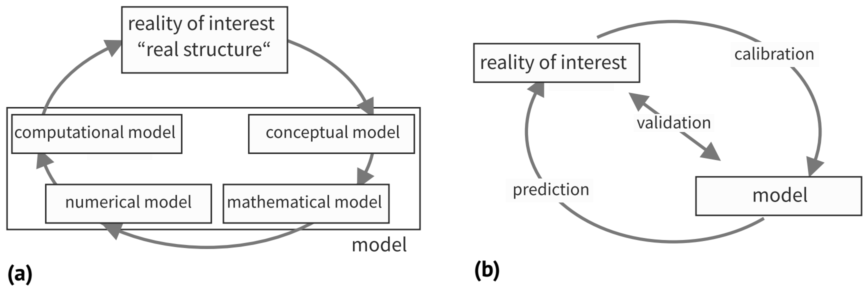


Figure 2.1 Modelling. (a) Type of models following Schwer (2007). (b) Relations between model and reality.

tial equations relating variable properties or *variables*. After prescribing appropriate boundary and initial conditions an exact, unique solution should exist for variables depending on spatial coordinates and time. Thus, a particular variable forms a field. Such fields of variables are infinite, as space coordinates and instants of time are infinite although being bounded.

As analytical solutions are not available in many cases, a discretisation is performed to obtain approximate numerical solutions. *Discretisation* reduces underlying infinite space and time into a finite number of supporting points in space and time and maps differential equations into algebraic equations connecting a finite number of variables. This leads to a *numerical model*. A numerical model needs a completion by means of programming to form a *computational model*. Finally, programs yield solutions through processing by computers. The whole cycle is shown in Figure 2.1. The sequence of partial models forms the *model* as a whole.

A final solution provided after computer processing is approximate compared to the exact solution of the underlying mathematical model. This is caused by discretisation and round-off errors. Let us assume that we can minimise this *mathematical approximation error* in some sense and consider the final solution as a *model solution*. Nevertheless, the relation between the model solution and the underlying reality of interest remains an issue. Both – model and reality of interest – share the same properties by definition or conceptual modelling, respectively. Let us also assume that the real data of properties can be objectively determined, e.g. by measurements.

Thus, real data of variables should be approximated by their computed model counterparts at least. The difference between model solution data and real data yields a *modelling error*. In order to distinguish between bad (inaccurate), good (accurate), and better model solutions, we have to choose a reference to measure the modelling error. This choice has to be made within a larger context, allows for a margin of discretion, and again is not guided by strict rules. Furthermore, the reference may shift while getting improved model solutions due to methodical progress or a better data survey.

A bad model solution may be caused by a bad model – bad choice of properties, poor relations of properties, insufficient discretisation, programming errors – or by incorrect model parameters. *Parameters* are those properties that are assumed to be known in advance for a particular problem and are not subject to a computation.

Under the assumption of a good model, the model parameters can be corrected by *calibration*. This is based upon appropriate problems from the reality of interest with known real data. On the one hand, calibration minimises the modelling error by adjustment of parameters. On the other hand, *validation* examines similar problems with modified parameters and known real data and assesses the modelling error. A proper calibration generally does not guarantee a successful validation.

Regarding reinforced concrete structures, calibrations usually involve the adaption of material parameters like strength and stiffness as part of *material models*. These parameters are chosen such that the behaviour of material specimen observed in experiments is reproduced. A validation is usually performed with structural elements such as bars, beams, plates, and slabs. Computational results of *structural models* are compared with the corresponding experimental data. This may lead to basic peculiarities. Reproducible experiments performed with structural elements are of a small, simplified format compared with complex unique buildings. Furthermore, repeated experimental tests with the same nominal parameters exhibit scattering results. Standardised *benchmark tests* carving out different aspects of reinforced concrete behaviour are required. Actually, agreements about such benchmark tests exist only to a limited extent. Regarding a particular structural problem a corresponding model has to be validated on a case-by-case strategy using adequate experimental investigations. Again, there are no strict rules like for the preceding arts.

Complex proceedings have been sketched hitherto outlining a model of modelling; see also Babuska and Oden (2004) for a more comprehensive discussion. Some benefit is finally expected. Thus, a model that passes validations is usable for *predictions*. Structures built along such predictions, hopefully, prove their high quality in the reality of interest.

This textbook covers the range of conceptual models, mathematical models, and numerical models with special attention being paid to reinforced concrete structures. The computational model with PYTHON-sources is available under open-source conditions at https://www.concrete-fem.com. A major aspect of the following is the modelling of *ultimate limit states*: states with maximum bearable loading or acceptable deformations and displacements in relation to failure. Another aspect is given with *serviceability*: deformations, and in some cases oscillations, of structures have to be limited to allow their proper usage and fulfilment of intended services. *Durability* is a third important aspect: deterioration of materials through, e.g. corrosion, has to be controlled. This is connected to cracking and crack width in the case of reinforced concrete structures. These topics are also treated in the following.

2.2 Discretisation Outline

The finite element method (FEM) is the predominant method to derive numerical models from mathematical models. Its basic theory is described in the following sections of this chapter insofar as it is needed for its application to different types of structures with reinforced concrete in the following chapters.

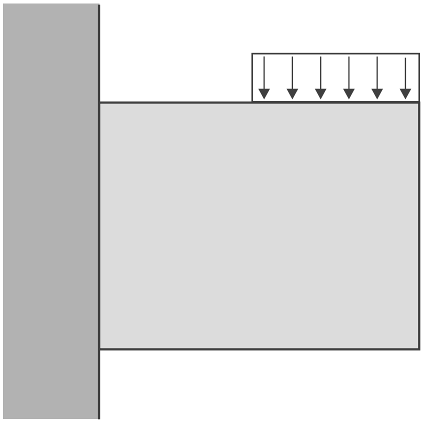
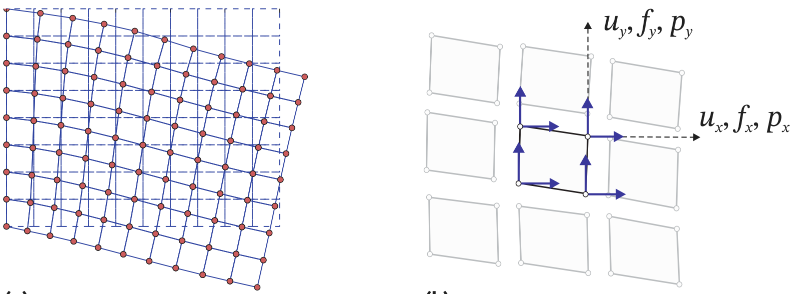


Figure 2.2 Model of a plate.



(a) **(b)**

Figure 2.3 (a) Elements and nodes (deformed). (b) Nodal quantities.

The underlying mathematical model is defined in one-, two-, or three-dimensional fields of space related to a body and one-dimensional space of time. A body undergoes deformations during time due to loading. We consider a simple example with a plate defined in 2D space, see Figure 2.2.

Loading is generally defined depending on time, whereby time may be replaced by a loading factor or an equivalent loading time in the case of quasi-static problems. Field variables depending on spatial coordinates and time are, e.g. given by the displacements.

- Such fields are discretised by dividing space into *elements* that are connected by *nodes*, see Figure 2.3a. Elements adjoin but do not overlap and fill out the space of the body under consideration.
- Discretisation basically means *interpolation*, i.e. displacements within an element are interpolated using the values at nodes belonging to the particular element.

In the following, this is written as

$$\mathbf{u} = \mathbf{N} \cdot \boldsymbol{\upsilon} \tag{2.1}$$

with the displacements $\mathbf{u}$ depending on spatial coordinates and time, a matrix $\mathbf{N}$ of *trial functions* depending on spatial coordinates, and a vector $\boldsymbol{\upsilon}$ depending on time and collecting all displacements at nodes.