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Bo Zhang
Jin Peng

Uncertain Graph and Network Optimization

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Uncertain Graph and Network Optimization

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*We live in a world full of networks and
uncertainties.*

Preface

There exist many network optimization problems in real life. Without historical data to estimate a probability distribution, decisions have to be made based on experts' belief degree. To better deal with such indeterminacy, an uncertainty theory was founded by Liu [1] in 2007 and then became a branch of mathematics for modeling uncertainty by belief degree. Within the framework of uncertainty theory, uncertain programming is a powerful tool to handle decision problems with uncertain variables.

Graph and Network

Graph is one of the most frequently used models in natural and artificial structures. For example, a city traffic graph can be used to describe the route conditions between urban roads. Relationship graphs can be used to describe the relationship between humans. What is network optimization? Network optimization is the study of how to design, manage, control, and optimize the network efficiently. Network optimization is a subset of mathematical optimization that is related to operations research, graph theory, and computational complexity theory. Many network optimization problems, such as the Chinese postman problem, optimal assignment problem, shortest path problem, maximum flow problem, can be studied on a graph or network. Chapter 1 introduces some basic concepts about graph and network.

Uncertainty Theory

We usually make a decision in the state of uncertainty. In order to deal with the uncertainty generated by human behavior, uncertainty theory was founded based on normality, duality, subadditivity, and product axioms. Up to now, uncertainty theory has been developed steadily and has become a branch of mathematics. Chapter 2 is devoted to uncertainty theory.

Uncertain Programming

As an important application of uncertainty theory, uncertain programming was founded in 2009. Since then, uncertain programming has been and continues to be an efficient tool for dealing with uncertain network optimization problems. Chapter 3 will present three types of uncertain programming to deal with optimization problems with uncertain variables: (1) the expected value model; (2) chance-constrained programming; and (3) measure-chance programming.

Uncertain Graph

An uncertain graph is essentially a graph with uncertain edges. Thus, an uncertain graph is usually used to model uncertain phenomena that vary with the topology structure. Chapter 4 is devoted to basic concepts of uncertain graphs as well as the connectivity index, Euler index, matching index, and matching number.

Uncertain Network Optimization

Uncertain network optimization is the study of network optimization with uncertain variables which we often meet when making decisions are under uncertain conditions. Chapter 5 will introduce some classical network optimization problems within the framework of uncertainty theory, such as Chinese postman problem, optimal assignment problem, and shortest path problem.

Applications of Uncertain Network Optimization

Chapter 6 will introduce the applications of uncertain network optimization in the field of transportation problems, dispatching medical supplies, and the covering location problem.

Uncertainty Theory Online

If you want to read more papers related to uncertain graph and uncertain network, please visit the website at <http://www.starfuture.net.cn:2007/online/>.

Purpose

The purpose of this book is to provide a tool for the readers to deal with network optimization problems with uncertainty by belief degree. The book is suitable for researchers, engineers, and students in the field of mathematics, information science, operations research, industrial engineering, computer science, artificial intelligence, economics, and management science.

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Reference

1. Liu B (2007) Uncertainty Theory, 2nd edn. Springer-Verlag, Berlin

About This Book

Bo Zhang and Jin Peng
Uncertain Graph and Network Optimization

In real life world, there usually exist indeterminacy relationships between our research objects. Uncertain graph provides a theoretical tool to describe this type of uncertainty background.

Uncertain network optimization is the study of network optimization with uncertainty which addresses new optimization methodology to a various of network optimization problems in uncertain environment.

This is an introductory textbook on uncertain graphs, uncertain network, and uncertain network optimization. This textbook also shows applications of uncertain network optimization to a lot of real problems such as transportation problem, dispatching medical supplies problem, and location problem.

The book is suitable for researchers, engineers, and students in the field of mathematical science, information science, computer science, decision science, management science and engineering, control science and engineering, artificial intelligence, economics, industrial engineering, and operations research.

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Intelligent Manufacturing, Computers & Industrial Engineering, International Journal of Computer Mathematics, International Journal of Systems Science, International Journal of General Systems, International Journal of Machine Learning and Cybernetics, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, etc. His research has been supported by varieties of research projects including the National Natural Science Foundation, the Humanity and Social Science Foundation of Ministry of Education, and Hubei Provincial Natural Science Foundation, China. Prof. Peng is currently serving as the associate editor, area editor, editorial board member, or peer reviewer of multiple international journals. He has also acted as the chairs, board members of program committees or organizing committees in various conferences or academic organizations at home and abroad.

Frequently Used Symbols

\mathcal{M}	Uncertain measure
$(\Gamma, \mathcal{L}, \mathcal{M})$	Uncertainty space
ξ, η, τ	Uncertain variables
Φ, Ψ, Υ	Uncertainty distributions
$\Phi^{-1}, \Psi^{-1}, \Upsilon^{-1}$	Inverse uncertainty distributions
$\mathcal{L}(a, b)$	Linear uncertain variable
$\mathcal{Z}(a, b, c)$	Zigzag uncertain variable
$\mathcal{N}(e, \sigma)$	Normal uncertain variable
E	Expected value
V	Variance
\mathfrak{R}	The set of real numbers
\vee	Maximum operator
\wedge	Minimum operator