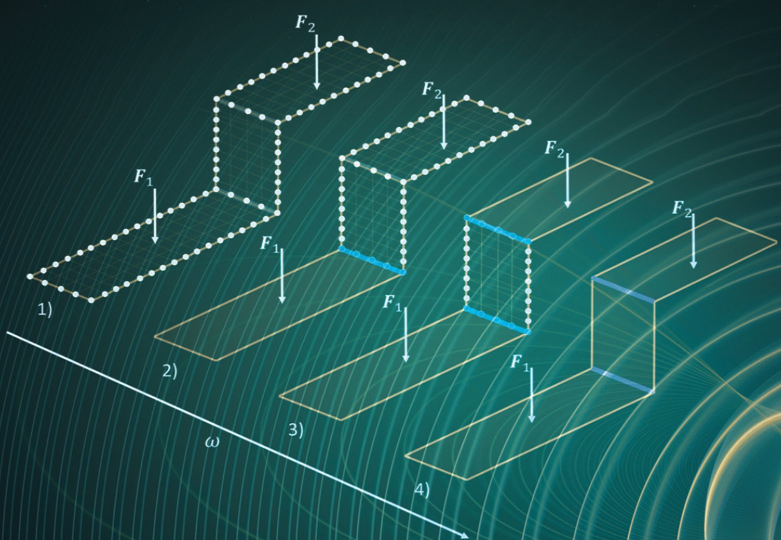
VIBROACOUSTIC SIMULATION

AN INTRODUCTION TO STATISTICAL ENERGY ANALYSIS AND HYBRID METHODS

ALEXANDER PEIFFER



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To my parents and Ivonne my love who always supported me

Contents

Preface

The reason and stimulation for this book was an application for a lecturer position several years ago. I slightly panicked due to the fact that in case of success, I would be totally unprepared in terms of lecturing material. So, I decided to write a script in order to have at least something prepared. But, the more I got into the text and the more I tried to collect content for a script on vibroacoustics, I came to the conclusion that there is a need for a modern text book on the subject of vibroacoustic simulation focussing on statistical energy analyses and hybrid methods.

There are many excellent books on acoustics and vibration, but what is missing to my opinion is an overall treatment of vibroacoustic simulation methods. Especially when we are talking about statistical energy analysis (SEA) and the combination of finite element methods (FEM) and SEA, the hybrid FEM/SEA method. In addition, the hybrid FEM/SEA method allows a much clearer and more systematic approach to SEA compared to the original literature and might help to impart the knowledge to students and professionals. It is my persuasion, that every acoustic simulation engineer shall master these simulation techniques to be prepared for vibroacoustic prediction of the full audible frequency range.

What is so special about vibroacoustics that so many methods are required? One answer is that the dynamic properties of structure and fluid systems are so different. This leads to distinct dynamic behavior. There may fit a lot of wavelengths of acoustic waves into a chamber of a machine or a passenger compartment filled with air, whereas the surrounding structure is often stiff and robust, and only a few wavelengths of the structural bending waves fit into the area of the surrounding walls. This strongly influences how energy is transmitted via the walls into the cavity and how small uncertainties affect the system response.

Additionally, there is often a great variety of materials, like foams, fibers, rubbers, etc. in the structure or applied as noise and vibration control, all having different orders of magnitude in wavelengths or even completely different modes of wave propagation.

As a consequence, vibroacoustics is a complex engineering discipline or science because the engineer has to master all those modes of wave propagation in the different systems and media as far as the coupling between those waves for connected subsystems. A thorough treatment of all wave types, couplings, and properties is not possible in a typical lecture or textbook, but it is possible to explain the main idea of how to deal with vibroacoustic phenomena and which means are required to solve the engineering problem. This book tries to extract the basic concepts, so that candidates are in a position to determine, investigate and categorize vibroacoustic systems and make the right decision on how to simulate them.

The frequency range of interest is covering four orders of magnitude from 20 to 20 000 Hz. That is one further reason why various methods for the description of these phenomena are required. At low frequencies it makes sense to investigate the modal behavior of a structure like the first modes of a string. In contrast to this, calculating all standing waves at high frequencies for a large room is not reasonable, as small changes at the boundaries or even temperature will lead to totally different wave forms in the room. Both regimes are addressed by different approaches categorized as (i) deterministic or (ii) statistic methods. The first occurs normally at lower frequencies, whereas the latter is valid at high frequencies. Because of the different wavelengths, it often appears that both cases occur in one vibroacoustic system, and both approaches are necessary. The combination of the two methods is called hybrid FEM/SEA method.

As there are many books on the subjects of deterministic acoustics and vibration available, this book focuses on SEA and hybrid methods. However, as FEM systems of equations are involved in the hybrid method, a minimum understanding of deterministic systems is required.

How is the book organized? It starts with a simple but excellent example for a vibrating system: the harmonic oscillator. In chapter 1 phenomena such as resonances, off resonance dynamics, and numerous damping mechanisms are explained based on this test case. A first step towards complex and FEM systems is made by introducing multiple coupled oscillators as an example for multiple degree of freedom systems. Real excitations often are of random nature. Hence, this chapter ends with tools and methods to describe random signals and processes as far as the response of linear systems to such signals.

Chapters 2 and 3 deal with wave motion in fluids and structures, respectively. Both chapters bring into focus the physics of sources, because the source mechanisms reveal how energy is introduced into the wavefields and how the feedback to the excitation can be characterized. Furthermore, the source dynamics are required when systems are coupled. The dynamics of acoustic and structure systems are shown in chapters 4 and 5. This includes the natural resonances of such systems that will become important for the classification of random systems. Based on analytical models, the low and high frequency behavior of such systems is presented. One aim of the various examples is to illustrate that when sources are exciting those systems, the high frequency dynamics become similar to the free field results from chapters 2 and 3. Chapter 6 deals with the random description of systems. The concept of ensemble average and diffuse fields is applied to typical example systems by using Monte Carlo simulations. Based on such randomized systems and averaged values, it is shown that we get similar results to those you would get from deterministic methods when the uncertainty of dynamically complex systems is considered. This opens the door to the statistical energy analysis (SEA). Some typical one-, two-, and three-dimensional systems are presented in the very detail, so that the reader gets a feeling when and under which conditions the SEA assumptions are valid. The idea is to provide comprehensive examples for the rules of thumb usually used to determine if random methods are valid or not.

In chapter 7 methods for coupling deterministic (FEM) and random (SEA) systems are presented, and the hybrid FEM/SEA method is introduced by describing the coupling between FEM and SEA systems. Based on this, the effect of random on deterministic systems as far as the impact of deterministic on random subsystems is presented. The chapter closes with the global procedure of hybrid FEM/SEA modelling that calculates the joint response of both types of systems.

Chapter 8 applies these coupling formulas to several options of connections. Especially the coupling sections are often missing in text books on SEA for a certain reason: the calculation of coupling loss factors is not easy. However, as it is important to understand the assumptions and limits, the coupling loss factors of point, area, and line junctions are systematically derived. Since junctions are nothing else than noise paths, this chapter is also useful for practical applications, for example the acoustic transmission loss of plates that is an important quantity for airborne acoustic isolation.

The following chapters apply the theory to pure deterministic (chapter 9), pure random (chapter 10), and hybrid FEM/SEA examples (chapter 11). All examples are worked out in detail and show real engineering systems such as mufflers. In chapter 9 the transfer matrix method is introduced as an example of deterministic methods. This allows the simulation of complex lay-ups of noise control treatments applied in chapters 10 and 11.

The presented theory and the examples are calculated using Python scripts. The scripts and the related toolbox are made available as open source code. The author hopes that this toolbox helps to understand and to apply the presented topics. Further contributions to the code of the toolbox are very welcome. The documentation of the toolbox and the GIT repository can be found on the authors website www.docpeiffer.com.

As an acoustic engineer, I am in the somehow unique situation that I had the chance to work on several means of transportation: trains, aircraft, helicopters, launchers, satellites, and finally cars (mainly electric). Because of this experience I am convinced that a deep knowledge of vibroacoustic simulation methods is mandatory to create excellent *and* low-noise products. This know-how puts the acoustic engineer in the position to apply the right method in the right situation and frequency range. To underline this fact, chapter 12 presents models, basic ideas, pitfalls, and results of some industrial examples from aerospace, automotive, and train industries. Special thanks goes to my former colleague Ulf Orrenius who wrote the train and motivation section of chapter 12. His great experience and knowledge strongly enriched the content of this chapter.

This book is about simulation, but simulation is nothing without validation based on tests. In my view both – simulation and tests – are required to perform a good acoustic design and noise control engineering. Thus, chapter 13 briefly summarizes test and correlation methods together with an outlook to further topics of simulation and ongoing research in the field of acoustic simulation.

In most cases the life of an acoustic engineer means solving the target conflict between the acoustic performance and costs, weight, and space requirements. This is the reason why design engineers are not always the best friends of acousticians during the design phase. The more important it is that you are able to calculate the effect of your decisions for efficient application of the sometimes rare acoustic resources. I hope that this book provides some support for this demanding task.

Coming back to my initial motivation: If I would have to hold a lecture on vibroacoustic simulation now, I would sleep much better.

Alexander Peiffer

Planegg, Germany
15 October 2021

Acknowledgments

Special thanks goes to my former colleague Ulf Orrenius who wrote the train- and motivation section of chapter 12. His great experience and knowledge strongly enriched the content of this chapter.

I would also like to thank the Audi AG who provided the models to present the simulation strategy of automotive systems.

Acronyms

2DOF	Two degrees of freedom
ms	Mean square
rms	Root mean square
CFD	Computational fluid dynamics
CFRP	Carbon fiber reinforced plastic
DFT	Discrete Fourier transform
DVA	Dynamic vibration absorber
DOF	Degree of freedom
EMA	Experimental modal analysis
FEM	Finite element method
FFT	Fast Fourier transform
FT	Fourier transform
GFRP	Glass fiber reinforced plastic
HVAC	Heating, ventilation, and air conditioning
MAC	Modal assurance criterion
MDOF	Multiple degrees of freedom
MIMO	Multiple input multiple output
NVH	Noise, vibration, and harshnes
PAX	Passenger
SDOF	Single degree of freedom
SEA	Statistical Energy Analysis
SISO	Single input single output
TBL	Turbulent boundary layer
TPA	Transfer path analysis
TVA	Tuned vibration absorber

1

Linear Systems, Random Process and Signals

Simple systems with properties constructed by lumped elements as masses, springs and dampers are a good playground to understand and investigate the physics of dynamic systems. Many phenomena of vibration as resonance, forced vibration and even first means of vibration control can be explained and visualized by these lumped systems.

In addition, a basic knowledge of signal and system analysis is required to put the principle of cause and effect in the right context. Every vibroacoustic system response depends on excitation by random, harmonic or specific signals in the time domain and we need a mathematical tool set to describe this.

An excellent test case to demonstrate and define the principle effects of vibration is the harmonic oscillator. It consists of a point mass, a spring and a damper. The combination of many point masses connected via simple springs and dampers provides some further insight into dynamic systems.

As those systems are described by components that have no dynamics in themselves they are called lumped systems. In principle all vibroacoustic systems can by modelled and approximated by this simplified approach.

1.1 The Damped Harmonic Oscillator

A realization of the harmonic oscillator is given by a concentrated point mass m fixed at massless spring with stiffness k_s as in Figure 1.1. The static equilibrium is assumed at $u = 0$ being the displacement in x-direction. A damper connecting mass and fixation creates dissipation.

1.1.1 Homogeneous Solutions

Without external excitation as shown in Figure 1.1 a) the motion depends on the initial conditions at time $t = 0$ with the displacement $u(0) = u_0$ and velocity $v_x(0) = v_{x0}$. The damping is supposed to be viscous, thus proportional to the velocity $F_{xv} = -c_v\dot{u}$. The equation of motion

$$m\ddot{u} + c_v\dot{u} + k_s u = 0 \tag{1.1}$$

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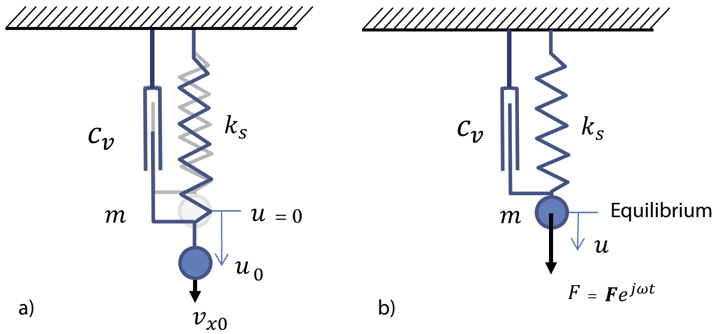

Figure 1.1 Damped harmonic oscillator with initial conditions a) and external force excitation b). *Source:* Alexander Peiffer.

is a homogeneous second order equation with a solution of the form $u = Ae^{st}$. Entering this into Equation (1.1) leads to the characteristic equation

$$ms^2 + c_v s + k_s = 0 \tag{1.2}$$

with the two solutions

$$s_{1/2} = -\frac{1}{2m}\left[-c_v \pm \sqrt{c_v^2 - 4mk_s^{1/2}}\right] \tag{1.3}$$

Hence,

$$u(t) = B_1 e^{s_1 t} + B_2 e^{s_2 t} \tag{1.4}$$

with B_1 and B_2 depending on the initial conditions. The root in Equation (1.3) is zero when c_v equals $\sqrt{4mk_s}$. This specific value is called the critical viscous damping

$$c_{vc} = \sqrt{4mk_s} \tag{1.5}$$

We use the following definitions:

$$\omega_0^2 = \frac{k_s}{m} \qquad \frac{c_v}{m} = 2\zeta\omega_0 \qquad \zeta = \frac{c_v}{\sqrt{4mk_s}} = \frac{c_v}{c_{vc}} \tag{1.6}$$

ω_0 is the natural angular frequency, ζ is ratio of the viscous-damping to the critical viscous-damping. There are additional expressions for the period and frequency

$$f_0 = \frac{\omega_0}{2\pi} \qquad T_0 = \frac{1}{f_0} \tag{1.7}$$

where f_0 is the natural frequency and T_0 the oscillation period. Equations (1.1)–(1.3) can now be written as

$$\ddot{u} + 2\zeta\omega_0\dot{u} + \omega_0^2 u = 0 \tag{1.8}$$

$$s^2 + 2\zeta\omega_0 s + \omega_0^2 = 0 \tag{1.9}$$

$$s_{1/2} = -\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2 - 1} \tag{1.10}$$

The problem falls into three cases:

$\zeta > 1$ overdamped
$\zeta < 1$ underdamped
$\zeta = 1$ critically damped.

The first case leads to two real roots, and no oscillation is possible. The second case gives two complex roots, which means that (damped) oscillation occurs. The third case is a transition case between the two other. Subsections 1.1.2–1.1.4 deal with each case in detail.

1.1.2 The Overdamped Oscillator ($\zeta > 1$)

Both roots in Equation (1.10) are real, distinct and negative. The motion is called overdamped because introducing this into Equation (1.4) gives a sum of decaying exponential functions:

$$u(t) = B_1 e^{(-\zeta+\sqrt{\zeta^2-1})\omega_0 t} + B_2 e^{(-\zeta-\sqrt{\zeta^2-1})\omega_0 t} \tag{1.11}$$

The movement of such a system is illustrated in Figure 1.2. Using the above solution and applying the initial conditions u_0 and v_{x0} we get for B_i:

$$B_{1/2} = \pm\frac{u_0\omega_0(\zeta \pm \sqrt{\zeta^2-1}) + v_{x0}}{2\omega_0\sqrt{\zeta^2-1}} \tag{1.12}$$

1.1.3 The Underdamped Oscillator ($\zeta < 1$)

Here, the roots are complex conjugates and the solution of Equation (1.10) becomes:

$$u(t) = e^{-\zeta\omega_0 t}\left(B_1 e^{j(1-\zeta^2)^{1/2}\omega_0 t} + B_2 e^{-j(1-\zeta^2)^{1/2}\omega_0 t}\right) \tag{1.13}$$

$$= \hat{u}_0 e^{-\zeta\omega_0 t} cos((1-\zeta)^{1/2}\omega_0 t + \phi_0) \tag{1.14}$$

The motion is oscillatory with a frequency that is lower than in the undamped configuration:

$$\omega_d = \omega_0\sqrt{1-\zeta^2} = \omega_0\gamma \tag{1.15}$$

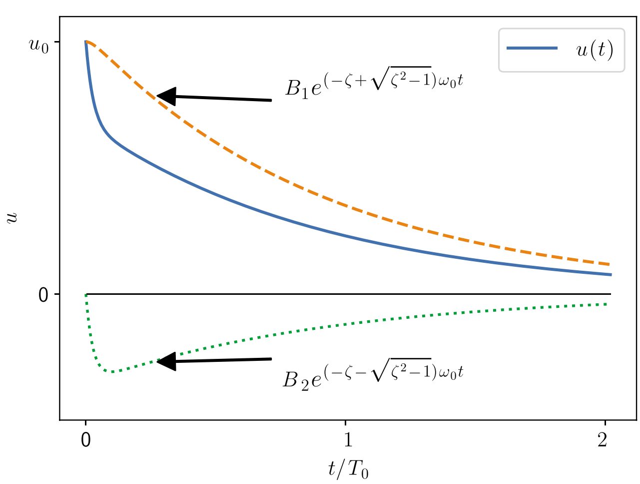

Figure 1.2 Decaying components of the overdamped oscillator. *Source:* Alexander Peiffer.

Introducing the initial conditions u_0 and v_{x0} at $t = 0$ the solution for the initial amplitude $\hat{u}_0$ and phase ϕ_0 reads as:

$$\hat{u}_0 = \frac{\sqrt{u_0^2 \omega_d^2 + (v_{x0} + \zeta \omega_0 u_0)^2}}{\omega_d} \tag{1.16}$$

$$\phi_0 = -\arctan\left(\frac{v_{x0} + \zeta \omega_0 u_0}{u_0 \omega_d}\right) \tag{1.17}$$

The damped oscillatory motion is illustrated in Figure 1.3. It shows a decreasing motion that never approaches the equilibrium.

1.1.4 The Critically Damped Oscillator ($\zeta = 1$)

The last case is a transition between both systems. There is only one root $s = -\omega_0$, and the solution in Equation (1.4) becomes:

$$u(t) = (B_1 + B_2)e^{-\omega_0 t} \tag{1.18}$$

This solution does not provide enough constants to fulfil the initial conditions, so that we need an extra term $te^{-\omega_0 t}$:

$$u(t) = (B_3 + B_4 t)e^{-\omega_0 t} \tag{1.19}$$

Introducing the initial conditions again, the constants are:

$$B_3 = u_0 \tag{1.20}$$

$$B_4 = v_{x0} + \omega_0 u_0 \tag{1.21}$$

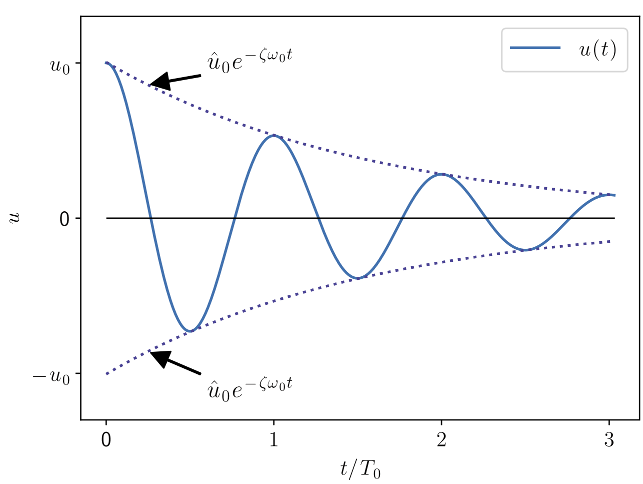

Figure 1.3 Damped, sinusoidal motion of the underdamped oscillator. *Source:* Alexander Peiffer.

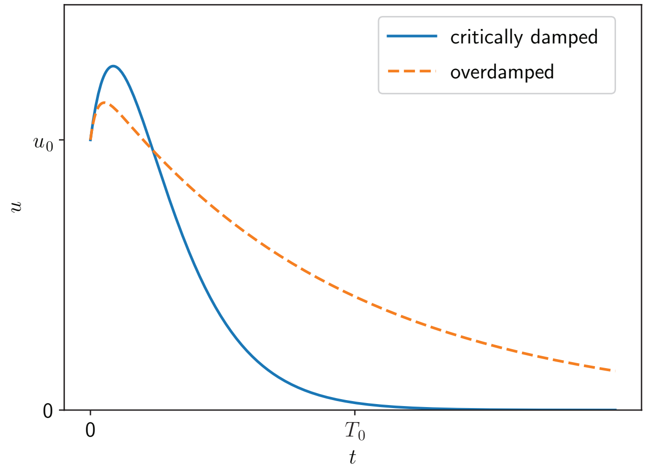

Figure 1.4 Motion of the critically damped oscillator. *Source:* Alexander Peiffer.

Critically damped systems can be of practical relevance, because the motion returns to rest in the shortest possible time, which is useful if periodic motion shall be prevented. In contrast to the overdamped oscillator the equilibrium is reached as can be seen in Figure 1.4.

Let us summarize some facts and observations about free damped oscillators:

1. Oscillation occurs only if the system is underdamped.
2. ω_d is always less than ω_0.
3. The motion will decay.
4. The frequency ω_d and the decay rate are properties of the system and independent from the initial conditions.
5. The amplitude of the damped oscillator is $\hat{u}(t) = \hat{u}_0 e^{-\beta t}$ with $\beta = \zeta\omega_0$. β is called the decay rate of the damped oscillator.

The decay rate is related to the decay time τ. This is the time interval where the amplitude decreases to e^{-1} of the initial amplitude. Thus, the decay time is:

$$\tau = \frac{1}{\beta} = \frac{1}{\zeta\omega_0} \tag{1.22}$$

1.2 Forced Harmonic Oscillator

When an external force $\hat{F}_x \cos(\omega t)$ is exciting the damped oscillator as shown in Figure 1.1 b), applying Newton's second law we get for the equation of motion:

$$m\ddot{u} + c_v\dot{u} + k_s u = \hat{F}_x \cos(\omega t) \tag{1.23}$$

This is an inhomogeneous, linear, second-order equation for u. The solution of this equation is given by a particular solution $u_P(t)$ and the solutions of the homogeneous

Equation (1.1) $u_H(t)$.

$$u(t) = u_H(t) + u_P(t) \tag{1.24}$$

Any linear combination of the homogeneous solution can be added to the particular solution because it equals zero.

1.2.1 Frequency Response

There are several methods to determine the particular solutions, like Laplace and Fourier transforms. Here, complex algebra will be used[1]. Amplitude and phase are given by a complex pointer denoted by bold italic type as depicted in Figure 1.5.

$$\hat{F}_x \cos(\omega t + \phi) = Re(\boldsymbol{F}_x e^{j\omega t}) \tag{1.25}$$

$\boldsymbol{F}_x$ is the complex amplitude of the force, and the $Re(\cdot)$ expression is usually omitted. The displacement and velocity response is then given by

$$u(t) = \boldsymbol{u}e^{j\omega t} \qquad v_x(t) = j\omega\boldsymbol{u}e^{j\omega t} = \boldsymbol{v}_x e^{j\omega t} \tag{1.26}$$

with $\boldsymbol{u}$ and $\boldsymbol{v}_x$ as complex amplitudes of the displacement and velocity, respectively. Introducing this into Equation (1.23).

$$-m\omega^2\boldsymbol{u}e^{j\omega t} + jc_v\omega\boldsymbol{u}e^{j\omega t} + k_s\boldsymbol{u}e^{j\omega t} = \boldsymbol{F}e^{j\omega t} \tag{1.27}$$

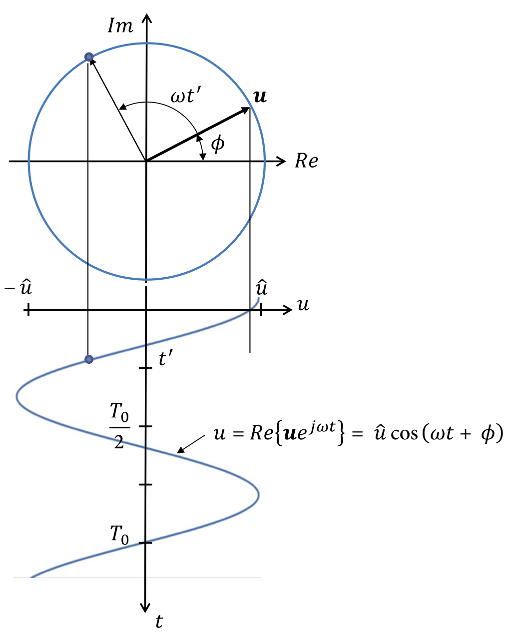

Figure 1.5 Complex pointer, amplitude and phase relationship. *Source:* Alexander Peiffer.

1 In this book the convention $e^{j\omega t}$ for the complex harmonic function is used. Literature that deals with wave propagation often use $e^{-j\omega t}$ to have positive wavenumber for positive wave propagation. However, as in every textbook in acoustics I denote the used convention on the first page to avoid confusion.