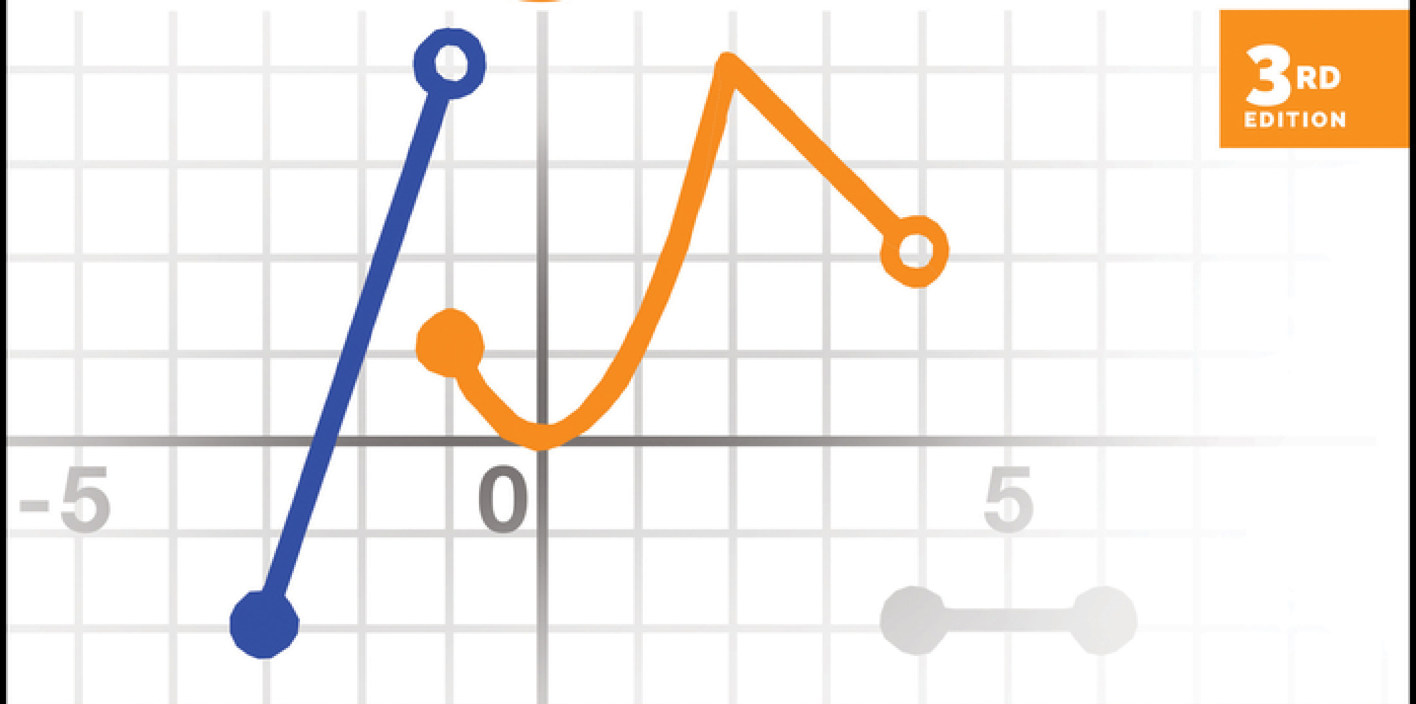
A SELF-TEACHING GUIDE

Practical Algebra

3RD EDITION



Bobson WONG
Larisa BUKALOV
and Steve SLAVIN

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A Self-Teaching Guide

Third Edition

Bobson Wong
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Writing a book is hard. Writing a book while teaching full-time during a pandemic is even harder. Fortunately, many people helped make this edition of *Practical Algebra* a reality. Our students' mathematical struggles and joys over the years inspired us to write this book. Conversations with our colleagues at Bayside High School and Math for America helped us develop many of the ideas and techniques we describe. Bayside students Juliana Campopiano and Queena Yue helped us proofread the text. The team at Desmos designed a powerful online graphing tool that we used to create the graphs in this book. The staff at John Wiley & Sons (especially Pete Gaughan, Christine O'Connor, Riley Harding, Julie Kerr, and Mackenzie Thompson) have been especially patient and supportive. Larry Ferlazzo introduced us to publishing math books, opening up countless opportunities. Finally, our spouses and children deserve special mention for tolerating our conversations about this book, peppering us with mathematical questions over the years, and helping to keep our work in perspective.

INTRODUCTION

What is algebra? You may associate it with solving equations such as $2x + 7 = 19$. However, both the history of algebra and the way that it's taught today show that algebra is much more. For thousands of years, people solved algebraic problems without symbols such as x and $+$. By the 9th century, people including the Persian mathematician Muḥammad ibn Mūsā al-Khwārizmī had popularized the idea of using an *algorithm* (a set of well-defined instructions) to determine unknown quantities. In fact, the word *algebra* comes from the Arab word *al-jabr*, meaning "the reduction," from the title of al-Khwārizmī's most famous mathematical text, *Kitāb al-jabr wa al-muqābalah*. Symbolic notation didn't become widespread until European mathematicians such as François Viète and René Descartes developed them in the 16th and 17th centuries. Nowadays, algebra courses include not just equations but also functions (the special rules that define mathematical relationships) and real-world modeling with statistics. In short, today's algebra students must know how to understand word problems, make and interpret graphs, create and solve equations, and draw appropriate conclusions from data.

Not surprisingly, algebra makes many people nervous. Maybe you recall endless drills and elaborate procedures from years ago. Perhaps you're a middle school or high school student who's intimidated by the high level of abstract reasoning that's required. If so, you're not alone. We understand how you feel! For many years, we've taught all levels of high school math, so we have a lot of experience working with diverse learners. This book contains concrete strategies that help our students succeed. We strongly believe that people can get better at math if they have access to the right tools.

We wrote this book as a general introduction to algebra. We assume that you're familiar with basic arithmetic (adding, subtracting, multiplying, and dividing numbers) and fractions. If you're *not* comfortable with these topics, don't worry—we briefly review them in Chapters 1 and 2. Even if you *are* comfortable with them, we suggest that you look through these chapters anyway. We explain why these ideas work and how they're related to the algebraic ideas we discuss later on.

Each chapter in this book is divided into sections, with model examples and tips. At the end of each section, you'll find several exercises to help you practice and apply your skills. These exercises include what we call Questions to Think About (open-ended questions designed to help you think about important concepts) as well as dozens of word problems. Each chapter has a test with multiple-choice and open-ended questions. The solutions to all exercises and chapter tests are located at the end of each chapter.

As you work through this book, you'll see some important ideas about algebra that we emphasize:

- **Algebra is a language.** We believe that many people find algebra intimidating because the words and symbols we use, such as polynomial, a_n, and $f(x)$, literally look like a different language. In addition, we don't just *write* math, we also *read* and *speak* it. In the Reading and Writing Tips, we discuss how to write and pronounce mathematical symbols as well as how to use them in context. We also include a glossary of mathematical terms and symbols in the back of the book.
- **Algebra should make sense.** We believe that algebra should be taught in a way that makes sense. In our experience, part of the reason why so many people suffer from math anxiety is that they see it as a collection of disjointed and confusing tricks. Throughout this book, we use techniques (such as the area model for multiplication) that relate to other mathematical topics, such as geometry and statistics. By making these connections, you can extend what you learned in one situation to another context, which will strengthen your mathematical skills and boost your confidence!
- **Algebra requires pictures.** As we taught during the pandemic, we had to adjust our instruction. We couldn't be with our students in person, so they often had to teach themselves more independently. Incorporating graphs, tables, diagrams, and other images into our teaching helped our students make sense of math. Since this book is a *self-teaching* guide, we've included many visual strategies throughout this book.
- **Algebra requires technology.** Calculators, computers, and other technology aren't just shortcuts for menial computations. They are now required for today's complex modeling tasks. Using technology helps us to see patterns more efficiently. Since each of these tools has vastly different user instructions, we don't include specific instructions for each device. Instead, we include Technology Tips that apply *no matter what device you're using*.
- **Algebra is a human endeavor.** We believe that algebra should not be perceived as a set of rigid rules developed by a select group of people. In fact, as we note throughout this book, many mathematical concepts were developed in different cultures around the world over thousands of years. (We mention some of the more interesting stories in the Did You Know? callouts.) In addition, we recognize that making mistakes is a natural part of doing math. In the Watch Out! callouts, we point out many of the common errors that we've seen students make over the years so that you can avoid them!

We hope that as you work through this book, you'll find that algebra can be less intimidating and more meaningful than you originally thought.

— *Bobson Wong and Larisa Bukalov*

1 BASIC CONCEPTS

In this chapter, we review some of the concepts that students are typically expected to know before learning algebra. Although we don't have the space to fully develop these concepts, we point out some common mistakes and other important points that you should keep in mind. Even if you think that you know these topics, we recommend that you work through this chapter.

1.1 Addition, Subtraction, Multiplication, Division

Throughout this book, we use visual models to represent mathematical ideas. One important model is a **number line**, a line on which each point represents exactly one number. The numbers always increase from left to right. To show the scale, numbers are marked off at equal intervals. We draw an arrow at the end to indicate that the numbers extend infinitely in that direction.

Positive numbers, which we indicate with a + in front of the number, are numbers greater than 0. **Negative numbers**, which we indicate with a − in front of the number, are numbers less than 0. The word **sign** refers to the property of being positive or negative. The term **signed numbers** refers to numbers and their signs. Numbers that don't have a sign in front of them are understood to be positive.

On a horizontal number line (Figure 1.1), positive numbers lie to the right of 0, and negative numbers lie to the left of 0:

Did You Know?

The idea of positive numbers, negative numbers, and 0 may seem obvious to us now, but they actually developed around the world over thousands of years. By the 3rd century BCE, the Chinese were using counting rods of different colors to represent positive and negative numbers in their calculations. The 7th-century Indian mathematician Brahmagupta described rules in terms of "fortunes" (positive numbers) and "debts" (negative numbers). Ancient societies understood the concept of nothing ("we have *no* water"), but many cultures, such as the Egyptians, Romans, and Greeks, created complex mathematics without 0. The use of 0 didn't fully develop until the 5th century CE in India.

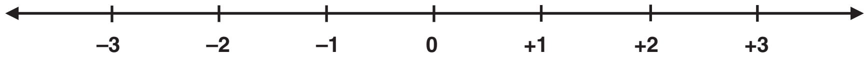


Figure 1.1 Number line

The **absolute value** of a number is its distance from 0 on a number line. Since the absolute value represents distance, it is always positive (unless we're talking about 0, which has an absolute value of 0). We use vertical bars to indicate absolute value. We read $|+2|$ as "the absolute value of positive two." For example, $|+15|$ is equal to 15, $|-15|$ is equal to 15, and $|0|$ is equal to 0. Two numbers that are the same distance from 0 on the number line but have different signs, such as +2 and −2, are **opposites**. Zero is an exception—the opposite of 0 is itself.

In math, we have four basic **operations** (mathematical processes performed on quantities to get a result): addition, subtraction, multiplication, and division. When we combine quantities with operations, we make an **expression**, such as $5 + 3$ and $|+15| - 4$.

Watch Out!

We use the + and – symbols to represent *both* addition and subtraction *and* the sign of a number.

- When + and – represent the sign of a number (which only occurs *before* a number), we read + as "positive" and – as "negative." We *never* put a space between the symbol and the number, so "negative 5" would be written –5, never – 5.
- When + and – represent addition or subtraction (which only occurs *between* two numbers), we read + as "plus" and – as "minus," and we put 1 space before and after the symbol. For example, 4 + 5, which is read as "4 plus 5," means 5 *is added to* 4 to get a sum of 9.

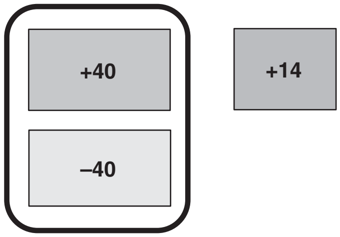
The + and – symbols can represent both operations and signs in the same mathematical sentence. For example, +5 – –3 is read "positive 5 minus negative 3," not "plus 5 minus minus 3." Sometimes, we put parentheses around signed numbers to separate them from the addition or subtraction symbols, so we write +5 – –3 as (+5) – (–3). The parentheses are not pronounced.

You may recall working with number lines in elementary school. In this book, we also use squares to model signed numbers because they enable us to represent far more complicated ideas that we need to work with in algebra. To represent +1, we use a square whose area is +1. To represent −1, we use a square whose area is −1. (Don't worry about what a square with a negative area actually "means"—it's just a model!) A square with area +1 and a square with area −1 have a total area of 0. We call this pair a **zero pair**. We can group zero pairs into rectangles (think of them as "jumbo packs" of +1 or −1 squares) and use them to add signed numbers, as shown in Example 1.1:

Example 1.1 Evaluate −40 + 54.

Solution: When we **evaluate** an expression, we perform mathematical calculations to get a single number.

$-40 + 54$	
$= -40 + 40 + 14$	Split +54 into +40 and +14.
$= (-40 + 40) + 14$	Group the −40 and +40 together to make 40 zero pairs.
$= 14$	The remainder is 14, the final answer.



In this example, we use the = symbol (which is called an equal sign and read "equals" or "is equal to"). The equal sign means that the expression on its left has the same value as the expression on its right. A mathematical statement containing an equal sign is called an **equation**. To make your work easier to read, do one part of the calculation at a time and write each step on a different line, starting each line with the equal sign.

Watch Out!

One common mistake when writing several equations on one line is to ignore the meaning of the equal sign. For example, when evaluating 2 + 3 + 4, some students write: 2 + 3 = 5 + 4 = 9. This "run-on" equation implies that 2 + 3, 5 + 4, and 9 are all equal, which isn't what we meant! Instead, write the following:

$2 + 3 + 4$	
$= 5 + 4$	The sum of 2 and 3 is 5.
$= 9$	The sum of 5 and 4 is 9.

How to Add Signed Numbers

1. Determine the number with the larger absolute value.
2. Form zero pairs with the number with the smaller absolute value.
3. The remainder is the final answer, called the **sum**.

Addition and subtraction undo each other. For example, $5 + 3 - 3$ equals 5. More formally, we say that addition and subtraction are **inverse operations**. This means that when we apply inverse operations on a number, the result is the original number. We can think of subtraction in terms of addition.

How to Subtract Signed Numbers

- To subtract a *positive* number, add a negative number with the same absolute value, so $5 - 3 = 5 + (-3)$. The result, called the **difference**, is 2. (This models real-world behavior—adding debt lowers your net worth.)
- To subtract a *negative* number, add a positive number with the same absolute value, so $5 - (-3)$ is the same as $5 + 3$. The difference is 8. (This also models real-world behavior—removing debt raises your net worth.)

Example 1.2 illustrates how these rules work.

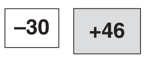
Example 1.2 Evaluate $(-30) - (-46)$.

Solution:

$-30 - (-46)$

1. From -30, we remove 46 negative unit squares. Since we don't have any more negative unit squares, add 46 zero pairs (46 negative and 46 positive unit squares) and remove the 46 negative unit squares.

$= -30 + 46$

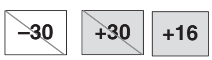


2. After removing the 46 negative unit squares, we have 30 negative and 46 positive unit squares.

$= -30 + 30 + 16$

3. To determine what we have left in step 2, we separate the +46 into 30 positive and 16 positive unit squares (since $46 - 30$ equals 16).

$= 16$



4. The 30 negative and 30 positive unit squares make 30 zero pairs, which add up to 0, leaving 16 positive unit squares.

Technology Tip

Many calculators have different buttons for subtraction and negative numbers. Often, the subtraction button is located next to the buttons for addition, multiplication, and division. To change the sign of an entry, they have a button labeled +/- or (–), where the - symbol on the button is shorter than the – symbol. Some calculators will return an error if you try to use the subtraction button to change the sign of a number, so be careful! In contrast, most software applications and mathematical websites don't differentiate between the negative and subtraction symbols, so entering $5 - -3$ will result in the correct answer of 8.

When we multiply numbers, we add groups of the same size.

How to Multiply Signed Numbers

- Multiply the absolute values of the **factors** (the numbers being multiplied).
- If we multiply two numbers with *different* signs, the result (called the **product**) is negative.
- If we multiply two numbers with the *same* sign, the result is positive.

We write the multiplication of 3 times 2 using one of these methods:

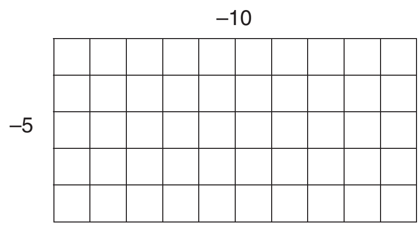
- with $\times$ between the numbers, as in 3×2
- with $\cdot$ between the numbers, as in $3 \cdot 2$
- with parentheses around one or both numbers, as in (3)(2), 3(2), or (3)2

We recommend not using the $\times$ symbol in algebra because it can easily be mistaken for the letter x, which has a special meaning that we discuss in Chapter 3.

Since the area of a rectangle is the product of its length and width, then we can use rectangles to represent multiplication. This idea dates back thousands of years to ancient Mesopotamia, Greece, and the Middle East. Unfortunately, we can't realistically show the difference between positive and negative dimensions with a rectangle, so we label the dimensions with the appropriate signed numbers and use the multiplication rules that we described above to find the correct sign of the product.

Example 1.3 Represent (−10)(−5) using a rectangle and evaluate the result.

Solution: We can represent this as a rectangle whose dimensions are −10 and −5:



NOTE: We can also think of this as removing 5 groups of −10, which results in a net increase of 50.

Here are some special cases of multiplication:

- **Any number multiplied by 0 equals 0.** For example, 4 groups of 0 is still 0.
- **A number multiplied by 1 equals itself.** We can explain this conceptually by noticing that 1 group of 4 is just that number, so $4(1) = 4$.
- **A number multiplied by −1 equals its opposite.** For example, $4(-1) = -4$ and $-4(-1) = 4$.

When we multiply a number by itself several times, we say that we raise it to a power. For example, we say that 2(2)(2)(2) equals 2^4, which we read as "two to the fourth power" or "two to the fourth." In this case, 2 is called the **base** (the number being multiplied) and 4 is the **power** or **exponent** (the number of times the base is being multiplied). The exponent is written above and to the right of the base. The term power refers to both the number 16 (what 2^4 equals) as well as the exponent 4.

Here are some special cases for powers:

- A number raised to the first power is equal to the number, so $2^1 = 2$.
- A number raised to the second power is **squared**, so 4^2 can be read as "four squared," "four to the second power," or "four to the second." (We get this term from the formula for the area of a square, which is the length of its edge multiplied by itself.)
- A number raised to the third power is **cubed**, so 4^3 can be read as "four cubed," "four to the third power," or "four to the third." (We get this term from the formula for the volume of a cube, which is the length of its edge multiplied by itself three times.)

A positive number raised to a positive power is always positive. We can surround the base with parentheses, so $(3)^4$, $(+3)^4$, and 3^4 all represent the same quantity.

When we raise negative numbers to a power, we always surround the base with parentheses, so we write $(-3)(-3)(-3)(-3)$ as $(-3)^4$. If we raise a negative number to powers that are counting numbers, we see an interesting pattern in the signs:

- $(-3)^1 = -3$
- $(-3)^2 = (-3)(-3) = +9$
- $(-3)^3 = (-3)(-3)(-3) = -27$
- $(-3)^4 = (-3)(-3)(-3)(-3) = +81$
- $(-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

We summarize this pattern as follows:

- A negative number raised to an odd power is negative.
- A negative number raised to an even power is positive.

Reading and Writing Tip

We have no easy way to express in words the difference between numbers like -3^4 and $(-3)^4$, since both can be pronounced as "negative 3 to the fourth power." We find that people pronounce $(-3)^4$ as "the quantity negative 3 to the fourth power," "parentheses negative 3 to the fourth power," or "negative 3 *(pause)* to the fourth power." This is an example of a situation where mathematical symbols can communicate ideas more clearly and succinctly than words. Pay careful attention to how mathematical symbols are written. In the same way that a missing comma can completely change the meaning of a sentence, missing parentheses can give you a different answer!

When we divide numbers, we separate into groups of equal size. Multiplication and division are inverse operations.

How to Divide Signed Numbers

- Divide the absolute values of the number that we divide (called the **dividend**) and the number that we divide by (called the **divisor**).
- If we divide two numbers with different signs, the result (called the **quotient**) is negative.
- If we divide two numbers with the same sign, the result is positive.

Division is often associated with fractions. A **fraction** is a quantity consisting of one number (called the **numerator**) divided by a nonzero number (called the **denominator**).

We write division using one of these methods:

- Using the ÷ symbol (called the division symbol) between the two numbers, such as $8 \div 2$
- Using the / symbol between the two numbers, all written on the same line, such as 8/2
- Writing one number on top of the other and separating the two with a **fraction bar** (sometimes called a **vinculum**), such as $\frac{8}{2}$

All of these division examples are read as "8 divided by 2." In this book, we prefer using the fraction bar to represent division. It provides the clearest separation of the quantities in division and minimizes the use of parentheses in more complicated mathematical statements.

When we use the ÷ or / symbol, the dividend appears before the symbol and the divisor appears after it. When we use a fraction bar, the dividend appears above it and the divisor appears below it. Figure 1.2 shows the terms associated with division:

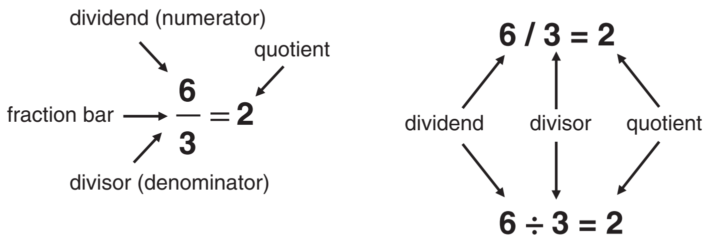


Figure 1.2 Terms associated with division.

Some special cases of division deserve special attention:

- **Any number divided by 1 equals itself.** For example, $\frac{8}{1}$, which means 8 divided into 1 group, equals 8.
- **Any number divided by 0 is meaningless.** Another way of saying this is that a fraction can never have a denominator equal to 0. For example, $\frac{8}{0}$ has no meaning since there is no number that when multiplied by 0 would give a product of 8 (this would have to be true since multiplication and division are inverse operations).
- **Any nonzero number divided by itself equals 1.** For example, $\frac{8}{8} = 1$.
- **Zero divided by a nonzero number equals 0.** The fraction $\frac{0}{8}$ equals 0.
- **The reciprocal of a number is 1 divided by that number.** The reciprocal of 8 is $\frac{1}{8}$.
- **The product of a number and its reciprocal is 1.** For example, $8\left(\frac{1}{8}\right) = 1$.

Example 1.4 Represent $\frac{+6}{-3}$ using a rectangle and evaluate the quotient.

Solution: Using a rectangle, we can think of this as dividing a rectangle that has an area of +6 and a side length of −3. Using the rules for dividing two numbers with different signs, we conclude that the quotient must be negative, so the answer is −2.

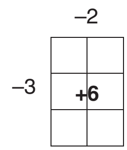


Table 1.1 summarizes the steps for operations with signed numbers:

Table 1.1 Operations with signed numbers.

Operation	Steps	Example
Addition	**1.** Determine the number with the larger absolute value.	−40 + 54 = −40 + 40 + 14
	2. Form zero pairs with the number with the smaller absolute value.	= 0 + 14
	3. The remainder is the final answer.	= 14
Subtraction	• To subtract a *positive* number, add a negative number with the same absolute value.	5 − 3 = 5 + (−3) = 2
	• To subtract a *negative* number, add a positive number with the same absolute value.	5 − (−3) = 5 + 3 = 8
Multiplication	• Multiply the absolute values of each number being multiplied.	10(2) = 20
	• If we multiply two numbers with *different* signs, the result is negative.	10(−2) = −20
	• If we multiply two numbers with the *same* sign, the result is positive.	−10(−2) = 20
Division	• Divide the absolute values of each number being multiplied.	$\frac{20}{2} = 10$
	• If we divide two numbers with *different* signs, the result is negative.	$\frac{-20}{2} = -10$
	• If we divide two numbers with the *same* sign, the result is positive.	$\frac{-20}{-2} = 10$

One final note: although understanding the rules for operations with signed numbers is important, you can always use technology to help you with these calculations.

Exercises

Write the pronunciation of each expression.

1. $-8 - (-12)$
2. $(+1) - (+3)$
3. $(+6) + (-4)$
4. $(+7)(+15)$
5. $(-2)(-32)$
6. $(-6)^4$

Evaluate each expression:

7. $|+7.5|$
8. $|-3|$
9. $|-889|$
10. $(+8) + (+5)$
11. $(+50) + (-10)$
12. $(-30) + (+20)$
13. $(-9) - (+5)$
14. $(-20) - (+30)$
15. $(-100) - (-40)$
16. $(+3)(-5)$
17. $(-3)(-7)$
18. $(-6)(+2)$
19. $(+2)^3$
20. $(+6)^2$
21. $(-7)^2$
22. $-(-1)^2$
23. $\dfrac{+15}{-3}$
24. $\dfrac{0}{+9}$
25. $\dfrac{-25}{-5}$

Questions to Think About

26. What is the difference between the words "plus" and "positive" as they are used in math?
27. What are two examples of real-life quantities that could be modeled by adding negative numbers?
28. What are two examples of real-life quantities that could be modeled by subtracting negative numbers?
29. Is $(-1{,}234{,}567{,}890{,}000{,}000)^{999}$ positive or negative? Explain.

1.2 Order of Operations

For many years, mathematicians didn't have a standardized set of rules for operations. When math education became more widespread in the 19th century, textbooks codified rules for what became known as the **order of operations**, the order in which mathematical operations should be performed. The growing popularity of computers in the last few decades has made the need for a standardized order of operations even more important.

In this book, we use the following convention (Figure 1.3) for order of operations:

1. **GROUPING:** First, evaluate everything surrounded by parentheses, brackets, fraction bars, absolute value symbols, and other grouping symbols, working from the innermost symbols outwards. To group numbers inside parentheses, we use square brackets or another set of parentheses: $2 - (3 - [5 - 1])$ or $2 - (3 - (5 - 1))$.

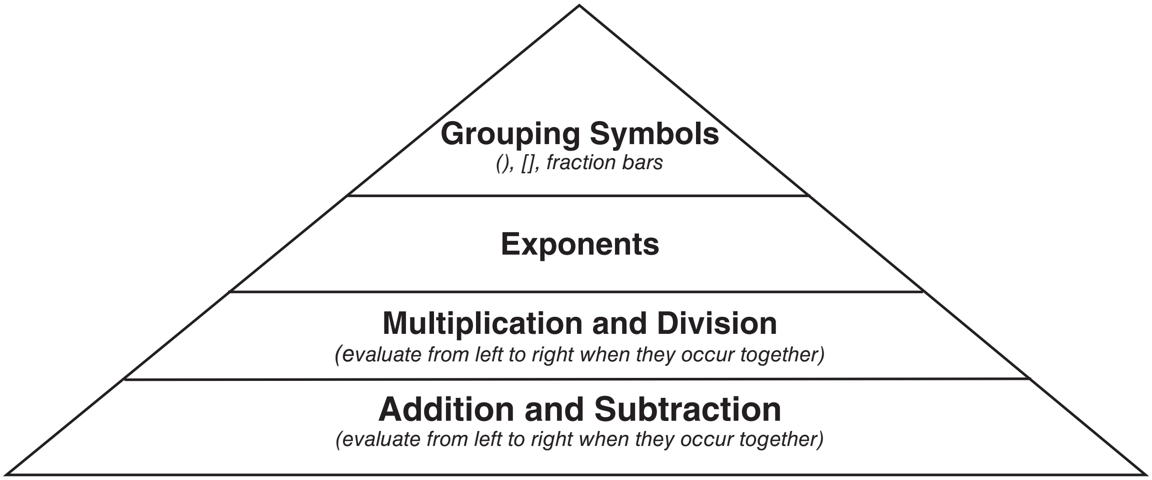


Figure 1.3 Order of operations.

2. **EXPONENTS:** Next, evaluate exponents.
3. **MULTIPLICATION/DIVISION:** Next, when multiplication and division occur together, evaluate them *left to right*.
4. **ADDITION/SUBTRACTION:** Finally, when addition and subtraction occur together, evaluate them *left to right*.

Watch Out!

Some textbooks use the mnemonic PEMDAS (which stands for Parentheses, Exponents, Multiplication, Division, Addition, Subtraction) to remember the order of operations. We recommend you avoid using PEMDAS since it implies that multiplication should be done before division and addition before subtraction. If you prefer using a mnemonic, we suggest PEMA (Parentheses, Exponents, Multiplication, Addition). Unfortunately, PEMA doesn't include division and subtraction, so you'll have to remember which operations are inverse operations (multiplication and division, addition and subtraction) and perform them left to right.

Some problems can be solved more easily using a calculator:

Example 1.5 Evaluate $2(-5)^{20}$.

Solution: The order of operations tells us that we need to evaluate the exponent first (in this case, the twentieth power) *before* the multiplication. The parentheses around -5 indicate that it, not -10 (which is the product of 2 and -5), is the base that is being raised to the twentieth power.

Multiplying −5 by itself 20 times is tedious, so we prefer using technology. To enter $2(-5)^{20}$ into a calculator, we use the exponent button (usually marked ^ or x^y), typing something like:

2	(	+/–	5	)	x^y	2	0	=

With technology, we get an answer that looks like 1.9073486328125E14. This is your device's way of displaying $1.9073486328125 \times 10^{14}$. This number is written in **scientific notation**, which consists of a number at least 1 and less than 10 that is multiplied by a power of 10. Translating this into the more familiar standard notation, this number is 190,734,863,281,250.

Just because you can solve a problem with technology doesn't mean that it's easy! Entering complicated expressions on the calculator can be quite challenging, as shown in Example 1.6:

Example 1.6 Evaluate $\frac{12-3+4(3)^2}{33-|-3(1+5)|}$.

Solution: The fraction bar acts as a grouping symbol, separating the numerator from the denominator, so we calculate each separately. In the denominator, we work with grouping symbols from the inside out.

$$\frac{12-3+4(3)^2}{33-|-3(1+5)|}$$

$= \frac{12-3+4(9)}{33-|-3(1+5)|}$ Evaluate the power in the numerator (3 is squared).

$= \frac{12-3+36}{33-|-3(1+5)|}$ Evaluate 4(9) in the numerator.

$= \frac{45}{33-|-3(1+5)|}$ Evaluate addition and subtraction from left to right.

$= \frac{45}{33-|-3(6)|}$ Add inside the parentheses in the denominator.

$= \frac{45}{33-|-18|}$ Multiply inside the absolute value symbols, which work as grouping symbols.

$= \frac{45}{33-18}$ Calculate the absolute value of −18.

$= \frac{45}{15}$ Subtract in the denominator.

$= 3$ Divide 45 by 15.

To enter this problem in the calculator, use the fraction tool if your device has one. This will put the numerator on top of the denominator, separated by a fraction bar. Doing so separates your numerator and denominator more clearly and reduces the likelihood of mistakes. Otherwise, you'd have to enter this problem on one line using many parentheses, which can be very confusing!

Technology Tip

When entering numbers into the calculator, keep the following in mind:

- Use the +/– key, not the addition and subtraction keys, to make a number positive or negative.
- Enter parentheses carefully. For example, $(3)(2 + 1)^2 = 27$, but $(3(2 + 1))^2 = 81$.
- Use the exponent key (usually marked ^ or x^y) to enter powers.
- To enter fractions, use the fraction tool if your calculator has one. (See Example 1.6.)

Exercises

Evaluate each expression.

1. $6 - 3 + 2$

2. $1 - 7 + 8$

3. $12 \div 2(3)$

4. $8 \div 2(2 + 2)$

5. $7 + 5(8)$

6. $9 + (-3)^2$

7. $2(-4)^2$

8. $4(-2)^3$

9. $9 + |1 - 5|^2$

10. $\dfrac{4 + (5 - (4 - 7))}{|-2|}$

11. $\dfrac{2^3 3^2}{5 - 1 + 4}$

12. $\left(\dfrac{5(16 + 4)}{16 - 2(3)}\right)^2$

Questions to Think About

13. Use the order of operations to explain why $3(4)^2$ is 48 and not 144.

14. Use the order of operations to explain why $5(1)^2$ is 5 and not 25.

15. Use the order of operations to explain why -4^2 is negative and not positive. (HINT: -4^2 can be rewritten as $(-1)(4)^2$.)

1.3 Sets and Properties of Numbers

In math, we work with different groups, or sets, of numbers (Figure 1.4):

- **Counting numbers** are the numbers we use to count: 1, 2, 3, 4, and so on.
- **Whole numbers** are the counting numbers and zero: 0, 1, 2, 3, 4, and so on.

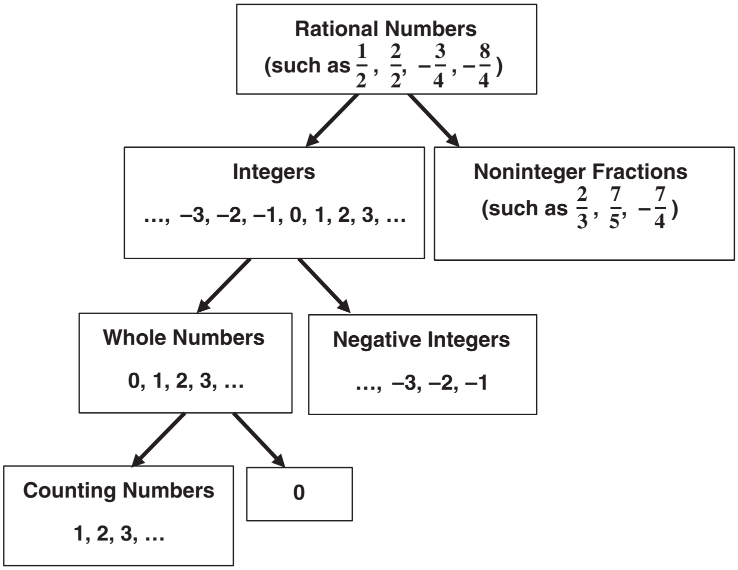


Figure 1.4 Sets of numbers.

- **Integers** are the whole numbers and their opposites: ... , −3, −2, −1, 0, 1, 2, 3, ...
- **Rational numbers** are numbers that can be expressed as an integer divided by a nonzero integer.

Example 1.7 Is every integer a whole number? Explain.

Solution: No. Whole numbers are the counting numbers and 0 (0, 1, 2, 3, ...). Integers are the whole numbers and their opposites (... , −3, −2, −1, 0, 1, 2, 3, ...), and negative integers are *not* whole numbers.

Example 1.8 Is every whole number a rational number? Explain.

Solution: The whole numbers are the counting numbers and 0: 0, 1, 2, 3, and so on. The rational numbers are numbers that can be represented as an integer divided by a nonzero integer. Every whole number can be represented as itself divided by 1. Thus, every whole number is a rational number.

Table 1.2 summarizes important properties of numbers, some of which we have already mentioned:

In this table, *a*, *b*, and *c* are **variables**—letters or other symbols that represent quantities that can change in value. We will discuss another important property that relates to addition and multiplication in Chapter 3.

Table 1.2 Properties of Numbers

Property	Description	Symbols	Example
Commutative property of addition	Numbers may be added in any order without changing the result.	$a + b = b + a$	$3 + 4 = 4 + 3$
Commutative property of multiplication	Numbers may be multiplied in any order without changing the result.	$a(b) = b(a)$	$4(3) = 3(4)$
Associative property of addition	Numbers may be grouped in any way for addition without changing the result.	$a + (b + c) = (a + b) + c$	$3 + (4 + 5) = (3 + 4) + 5$
Associative property of multiplication	Numbers may be grouped in any way for multiplication without changing the result.	$a(bc) = (ab)c$	$3(4 \cdot 5) = (3 \cdot 4)5$
Additive inverse property	A number added to its opposite equals 0.	$a + (-a) = 0$	$4 + (-4) = 0$
Additive identity property	A number added to 0 is unchanged.	$a + 0 = a$	$4 + 0 = 4$
Multiplicative inverse property	A number multiplied by its reciprocal equals 1.	$a\left(\frac{1}{a}\right) = 1$	$4\left(\frac{1}{4}\right) = 1$
Multiplicative identity property	A number multiplied by 1 is unchanged.	$a(1) = a$	$4(1) = 4$

Exercises

Determine whether the following statements are true or false. Explain your answer.

1. Every integer is a counting number.
2. Every whole number is an integer.
3. Zero is a whole number but not a counting number.
4. Every counting number is positive.
5. Every whole number is positive.
6. Every integer is either positive or negative.
7. Every rational number is an integer.
8. The number 5 is a rational number.

9. Every whole number is rational.

10. Every rational number can be written as a fraction.

11. Every whole number can be written as a fraction.

12. Every quotient is a rational number.

Questions to Think About

13. Explain why subtracting two counting numbers does not always result in a counting number.

14. How are fractions different from rational numbers?

15. Is subtraction commutative? Explain your answer.

CHAPTER 1 TEST

1. Which number is a counting number?

   (A) 0 (B) −1 (C) 1 (D) $\frac{1}{2}$

2. Which operation is the inverse of multiplication?

   (A) addition (B) subtraction (C) squaring (D) division

3. If a positive number is multiplied by a negative number, the result

   (A) is always positive.
   (C) can be positive or negative.
   (B) is always negative.
   (D) is always 0.

4. According to the order of operations, to calculate $1 + (3 - (6 - 4))^2$, which step must be done first?

   (A) $6 - 4$ (B) 1^2 (C) $1 + 3$ (D) $3 - 6$

5. Which number is equivalent to $(-1)^{15}$?

   (A) +1 (B) −1 (C) +15 (D) −15

6. What does the absolute value of a number represent?

   (A) the directed distance between any two points on a number line

   (B) the sum of two numbers on a number line

(C) the positive distance between any two points on a number line

(D) a number's distance from 0 on a number line

7. Which statement about $-\frac{3}{1}$ is correct?

(A) It represents a rational number and a fraction, but not an integer.

(B) It represents a rational number, a fraction, and an integer.

(C) It represents an integer and a fraction but not a rational number.

(D) It represents a fraction, but neither a rational number nor an integer.

8. If a negative number is subtracted from a positive number, which statement is correct?

(A) The result is always positive.

(B) The result is always negative.

(C) The result can be positive or 0.

(D) The result can be positive, negative, or 0.

9. Which of the following is equivalent to adding a negative number?

(A) subtracting a positive number with the same absolute value

(B) adding a positive number with the same absolute value

(C) subtracting a negative number with the same absolute value

(D) adding a positive number with a different absolute value

10. Is every integer a whole number? Explain.

11. Is 0 rational? Explain.

12. Write the pronunciation of $3^4 + |-8| - (+5)$.

13. Use rectangles to calculate $(-50) - (+40)$.

14. Calculate $(12 - (3 + 1))^2 - 8$.

15. Calculate $\frac{13 - 8 + 1}{2(6 - 7)} + 1$.

CHAPTER 1 SOLUTIONS

1.1.

1. "negative 8 minus negative 12"
2. "positive 1 minus positive 3"
3. "positive 6 plus negative 4"
4. "positive 7 times positive 15"
5. "negative 2 times negative 32"
6. "the quantity negative 6 to the fourth power"

7. 7.5	12. −10	17. +21	22. −1
8. 3	13. −14	18. −12	23. −5
9. 889	14. −50	19. 8	24. 0
10. 13	15. −60	20. 36	25. 5
11. 40	16. −15	21. 49	

26. "Plus" is an operation that shows that two numbers are being added, while "positive" is a characteristic of one number.
27. Answers may vary. Examples include adding debt (which lowers the net worth) or adding ice cubes to a drink (which lowers its temperature).
28. Answers may vary. Examples include cancelling debt (which increases the net worth) or removing ice cubes from a drink (which raises its temperature).
29. Negative. A negative number raised to an odd power is negative.

1.2.

1. 5	4. 16	7. 32	10. 6
2. 2	5. 47	8. −32	11. 9
3. 18	6. 18	9. 25	12. 100

13. The order of operations tells us to calculate powers before multiplication. We calculate 4^2, which equals 16, before multiplying it by 3 to get 48. In contrast, $(3(4))^2 = 12^2 = 144$.
14. The order of operations tells us to calculate powers before multiplication. We calculate 1^2, which equals 1, before multiplying it by 5 to get 5. In contrast, $(5(1))^2 = 5^2 = 25$.

15. The order of operations tells us to calculate powers before multiplication. We calculate 4^2, which equals 16, before multiplying it by −1 to get −16.

1.3. 1. False. Some integers are negative or 0, and every counting number is positive.

2. True. Integers are whole numbers and their opposites.

3. True. Whole numbers are the counting numbers and 0.

4. True. The counting numbers are 1, 2, 3, 4, … , all of which are greater than 0.

5. False. Zero is a whole number but is neither positive nor negative.

6. False. Zero is an integer but is neither positive nor negative.

7. False. Some rational numbers, such as $\frac{1}{2}$, are not integers ($\frac{1}{2}$ lies between the integers 0 and 1 on the number line).

8. True. The number 5 can be written as the quotient of 5 and 1, or $\frac{5}{1}$.

9. True. Every whole number can be written as the quotient of itself and 1, such as $\frac{5}{1}$.

10. True. A rational number consists of an integer divided by a nonzero integer, while a fraction consists of any quantity (not necessarily an integer) divided by another.

11. True. A whole number can be written as a fraction with a denominator of 1.

12. False. A noninteger fraction divided by an integer, such as $\frac{1}{2} \div 3$, or $\frac{\frac{1}{2}}{3}$, can be written as a fraction but is not rational (since the numerator $\frac{1}{2}$ is not an integer).

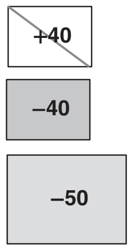
13. If the second number is larger than the first, then the result will be negative and will not be a counting number. For example, 2 − 3 is −1.

14. A rational number consists of an integer divided by a nonzero integer. A fraction consists of any quantity (not necessarily an integer) divided by a nonzero quantity.

15. Subtraction is not commutative. When the order in which numbers are subtracted is reversed, the sign of the difference is also reversed. For example, 5 − 4 = 1, but 4 − 5 = −1.

CHAPTER 1 TEST SOLUTIONS

1. (C)
2. (D)
3. (B)
4. (A)
5. (B)
6. (D)
7. (B)
8. (A)
9. (A)
10. No. Some integers are negative, and whole numbers (0, 1, 2, 3, …) cannot be negative.
11. Yes. Zero may be expressed as the quotient of 0 and a nonzero integer: $\frac{0}{1}$.
12. "Three to the fourth power plus the absolute value of negative 8 minus positive 5."
13. −90



14. 56
15. −2