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Abstract

Generalized probabilistic theories (GPTs) allow us to write quantum theory in a purely operational language and enable us to formulate other, vastly different theories. As it turns out, there is no canonical way to integrate the notion of subsystems within the framework of convex operational theories. Sections can be seen as generalization of subsystems and describe situations where not all possible observables can be implemented. We discuss the mathematical foundations of GPTs using the language of Archimedean order unit spaces and investigate the algebraic nature of sections. This includes an analysis of the category theoretic structure and the transformation properties of the state space. Since the Hilbert space formulation of quantum mechanics uses tensor products to describe subsystems, we show how one can interpret the tensor product as a special type of a section. In addition we apply this concept to quantum theory and compare it with the formulation in the algebraic approach. Afterwards we give a complete characterization of low dimensional sections of arbitrary quantum systems using the theory of matrix pencils. In addition, we combine the notion of sections with the dynamics in a GPT and consider the implications for quantum theory. As an application we introduce Spekkens' toy model, a hidden variable theory which mimics many of the features of quantum theory. We show that this model cannot be obtained as a section of a qubit system, but emerges naturally as a section of the Kadison representation what can be interpreted as an eight-level quantum system.

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