Trends in Mathematics

Vera Viana
Helena Mena Matos
João Pedro Xavier
Editors

Polyhedra and Beyond

Contributions from Geometrias'19, Porto, Portugal, September 05-07

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Trends in Mathematics

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Foreword

Leonardo da Vinci's approximately 60 illustrations for the book *The Divine Proportion* presented polyhedra for the very first time in history so spectacularly that they seemed to jump out of the paper as three-dimensional representations. 1800 years after Plato and Archimedes, they caused a true "polyhedra mania": Nuremberg artists Albrecht Durer, Wenzel Jamnitzer, and Lorenz Stoer drew polyhedra extensively; Dutchmen Simon Stevin and Claes Pietersz van Deventer published about them; Johannes Kepler rediscovered the complete list of regular polyhedra from the time of the Greeks and added two new ones in 1619 by including pentagrams. In the seventeenth century, the interest in the artistic presentation of polyhedra declined although their mathematical study continued. In 1809, Frenchman Louis Poinsot discovered two new solids by allowing intersecting faces. Englishman John Flinders Petrie added three infinite regular polyhedra in 1926, in collaboration with Canadian Donald Coxeter. American architect Buckminster Fuller caused an artistic revival, and his designs were so influential that the 1996 Nobel Prize winners in chemistry who discovered the C60 molecule gave it the name *Buckminsterfullerene*, although the shape of that molecule corresponds to the truncated icosahedron, already known to Archimedes. 2011 Chemistry Nobel Prize winner Daniel Shechtman likes to emphasize the divine proportion in his quasicrystals. In Europe, the work of Dutch artist Maurits Cornelis Escher led to a revival of the artistic study of polyhedra, and later Belgian Luc Tuymans and Danish Icelander Olafur Eliasson also represented them. Eventually, polyhedra conquered the whole world as they even inspired versatile Chinese artist Ai Weiwei, for instance. Thus, artistically and scientifically the polyhedral topic surely still is of interest. And perhaps even more than before, as modern 3D-software brings it within reach of any motivated computer enthusiast.

This was amply illustrated at the Geometrias'19 Conference in Porto, of which this book collects some highlights (not necessarily in order of their presentation at the conference). Some contributions were developed further into full papers, for the purpose of their inclusion in this book. The participants still hold vivid memories of the presentation on synthetic methods for constructing polyhedra, where one could actually see a snub cube being created on the screen, as the result of a kind of equilibrium process. Anyone who ever tried to draw a snub cube using even the most sophisticated 3D-software, quickly experiences this is impossible, thus confirming the fact that it can't be constructed in a finite number of steps with lines and circular arcs. Historical aspects are emphasized by a contribution on small stellated dodecahedrons in Genoa, Italy, and that is quite unusual, as Kepler-Poinsot solids do not seem so popular. A paper on confocal quadratic surfaces gives a more theoretical *intermezzo*, while regular participants of geometry-related conferences are probably happy to see a continuation of the work on concave deltahedral rings. This contrasts with the considerations on the gyroid, which is probably new to most readers. Admittedly, double-layered polyhedra are beautiful—even if one doesn't grasp what they are about! Geodesic structures can't be omitted in any self-respecting conference on polyhedra, while Vittorio Giorgini's organic structures are, regrettably, less well-known. Even Gaudí can be a topic, when combined with John Pickering's form-finding method. Several talks and workshops about pedagogical aspects were on the conference program too, during the conference, and the introduction to solid tessellations, included in this book, is but one of them. This variety of subjects of the Porto Geometrias'19 Conference, presented in an open exchange, created a pleasant ambience that will hopefully filter through these selected papers.

Dirk Huylebrouck holds a PhD in mathematics from the University of Ghent, Belgium. He lectured in Congo and Burundi for 12 years, interrupted by assignments in Portugal and at Maryland University Europe. Next, he taught at the Faculty of Architecture of the KU Leuven (Belgium) and edited the column *The Mathematical Tourist* in the journal *The Mathematical Intelligencer*. Author of seven books in popular mathematics in Dutch, his first, *Africa + Mathematics*, has already been translated in English (2019, Springer).

Ostend, Belgium
08 December 2021

Dirk Huylebrouck

Preface

Aproged, the Portuguese Geometry and Drawing Teachers Association, invited us to organize its 5th international conference and thus, *Geometrias'19: Polyhedra and Beyond* was held in the Department of Mathematics of the Faculty of Sciences, University of Porto, in September 5–7, 2019. The aim of this conference was to bring together international experts, scholars, researchers, and students from diverse backgrounds to engage in interdisciplinary discussions on theoretical research and practical studies on polyhedra and geometrical structures under development in different fields of knowledge and institutions. The *Geometrias'19: Book of Abstracts*[1], published in its outcome, summarized the essence of this Conference, offering a clear testimony of how the atmosphere of dialog and shared knowledge created renewed mutual interests between the participants, encouraging new synergies.

This book reflects a selection of the investigations presented during *Geometrias'19* that were developed into full papers, so some contributions contain materials somehow beyond the results presented in the talks, addressing different subjects and explorations of polyhedral theory within architecture, computer science, mathematics, and structural design, broadly construed.

For their contribution to the accomplishment of this book, we are especially grateful to the Scientific Committee members and additional reviewers for their commitment in revisiting these studies, and to all the authors for the development of their research and their openness during the reviewing procedures. An additional appreciation to our Foreword's Author, for such an inspirational input.

We thank you all for your contributions and for understanding the time it took us to achieve this publication, of which we are very proud of.

Porto, Portugal

Vera Viana
Helena Mena Matos
João Pedro Xavier

[1] Viana, V. (Ed.). (2019). *Geometrias'19: Book of Abstracts*. Porto: Aproged. https://doi.org/10.24840/978-989-98926-8-2

Acknowledgments

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We are grateful for the opportunity of working with each of the scientific reviewers mentioned below without whose contribution this book would not have been possible.

Vera Viana, Helena Mena Matos, and João Pedro Xavier

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