SECOND EDITION

MATHEMATICAL MINDSETS

UNLEASHING STUDENTS' POTENTIAL THROUGH CREATIVE MATHEMATICS, INSPIRING MESSAGES AND INNOVATIVE TEACHING

JO BOALER

FOREWORD BY CAROL DWECK

JOSSEY-BASS™
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Published by Jossey-Bass A Wiley Brand
111 River St
Hoboken, New Jersey 07030
www.josseybass.com

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Library of Congress Cataloging-in-Publication Data is Available:

ISBN: 9781119823063 (paperback)
ISBN: 9781119823087 (ePDF)
ISBN: 9781119823070 (ePub)

Cover Design: Wiley
Cover Image: © SilverCircle/Shutterstock
Author Photo: Cathy Williams

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Foreword

One of my former Stanford students teaches fourth grade in the South Bronx, an area of New York City with many underserved, underachieving minority students. Her students invariably believe they are bad at math, and if you looked at their past performance, you might be tempted to think so too. And yet, after one year in her class, her fourth graders became the #1 fourth-grade class in the state of New York: 100% of them passed the state math test, with 90% of them earning the top score. And this is just one of many examples of how all students can learn math.

When people think that some kids just can't do math, that success in math is reserved for only certain kids, thought of as "smart," or that it's just too late for kids who haven't had the right background, then they can easily accept that many students fail math and hate math. In fact, we have found that many teachers actually console their students by telling them not to worry about doing poorly in math because not everyone can excel in it. These adult enablers—parents and teachers alike—allow kids to give up on math before they've barely gotten started. No wonder more than a few students simply dismiss their own poor performance by declaring: "I'm not a math person."

Where do parents, teachers, and students get the idea that math is just for some people? New research shows that this idea is deeply embedded in the field of mathematics. Researchers polled scholars (at American universities) in a range of disciplines. They asked them how much they thought that success in their field depended on fixed, innate ability that cannot be taught, as opposed to hard work, dedication, and learning. Of all the STEM fields (science, technology, engineering, and math), math scholars were the most extreme in emphasizing fixed, innate ability (Leslie, Cimpian, Meyer, & Freeland, 2015). Other researchers are finding that many math instructors begin their courses by referring to students who have the aptitude and those who do not. One college instructor, on the first day of an introductory college course, was heard to say, "If it's not easy for you, you don't belong here" (Murphy, Garcia, & Zirkel, in prep). If this message is passed down from generation to generation, no wonder students are afraid of math. And no wonder they conclude they're not math people when it doesn't come easily.

But when we begin to see evidence that most students (and maybe almost all students) are capable of excelling in and enjoying math, as the following chapters show, it is no longer acceptable that so many students fail math and hate math. So what can we do to make math learning happen for all students? How can we help teachers and children believe that math ability can be developed, and then show teachers how to teach math in a way that brings this belief to life? That's what this book is about.

In this unique and wonderful book, Jo Boaler distills her years of experience and her powerful wisdom to show teachers exactly how to present math work, structure math problems, guide students through them, and give feedback in a way that helps students toward a "growth mindset" and keeps them there. Boaler is one of those rare and remarkable educators who not only know the secret of great teaching but also know how to give that gift to others. Thousands of teachers have learned from her, and here's what they say:

> "Throughout my schooling years . . . I was left feeling stupid and incapable of doing [math] . . . I cannot tell you the relief I now have that I can learn math myself, and I can teach students that they can too."
>
> "[You have] helped me think about the transition to common core and how to help my students develop a love and curiosity for math."
>
> "I was searching for a process of learning math that would change the attitude of students from dislike to enjoy . . . this was the change I needed."

Imagine your students joyfully immersed in really hard math problems. Imagine them begging to have their mistakes discussed in front of the class. Imagine them saying, "I *am* a math person!" This utopian vision is happening in classrooms around the world, and as you follow the advice in this book, you may well see it happening in your classroom too.

Carol Dweck
Professor of psychology and author of
Mindset: The New Psychology of Success

Preface to the Second Edition

It has been six years since I wrote the first edition of *Mathematical Mindsets*, and what an amazing six years it has been! As I write this we are emerging from a global pandemic that has stretched the capacity of teachers like nothing else before. It was incredible to see the work and creativity of teachers who continued to make mathematics a beautiful and exciting subject, even when teaching online. My personal mission in writing books like this and sharing resources on our website, youcubed.org, is always to support the work of teachers. I was thrilled that the first edition of *Mathematical Mindsets* became an international best seller, was translated into eight languages, and was appreciated by so many people. When my publishers asked about writing an updated edition, I reflected on the new knowledge and ideas that have been created over the last six years and decided it was time.

One of my reasons for writing the first edition of *Mathematical Mindsets* was to share the neuroscience that was emerging at the time and that I knew was really important for educators, students, and their families. That neuroscientific evidence is just as important today, and new studies have now been published that I have shared in this edition—translated into usable ideas for teachers. I am fortunate to work directly with neuroscientists at Stanford, and this book shares some of our joint work that we have completed since the first edition was written—which included a foray into teaching first graders! As a former secondary mathematics teacher, I found it really interesting and fun to observe the learning of first-grade students.

At the time of writing the first edition of *Mathematical Mindsets*, Youcubed was just beginning. I had recently completed my first online course and received many emails from teachers asking what was next. Cathy Williams and I decided to create a website, youcubed.org, never imagining we would be celebrating 50 million visits to the site six years later. I love that teachers who regularly use our resources call themselves Youcubians, and I have had the pleasure of meeting and working with many of them over the last few years. When we meet for professional development workshops, we always, as Cathy says, have a maths party!

Since the first edition was written, I have also had the opportunity to think really carefully about the creation of equitable group work. I know that good teachers are always concerned about creating opportunities for students to work together and for group discussions to be equitable—with all students involved and included. My recommendations for group work are a little more developed in this edition, particularly after teaching middle school students myself in our Youcubed summer camps in California. This edition shares some of the learnings from that camp, as well as the camps that were taught across the U.S. and in Scotland and Brazil, by Youcubians. The results of those, which were carefully researched, were incredible.

I started my own career as a mathematics teacher in inner-city London, teaching students ages 11–18 in public schools that were extremely culturally diverse. This rekindled my fascination in students' learning of mathematics—an interest that I had developed when I was a student in school. I decided to study for a master's degree at King's College, London, to learn more about research in mathematics education. I took the courses in the evenings after teaching at Haverstock school during the day. One of my clearest memories of those years was of traveling on the tube across London in the evenings, at the end of a busy day of teaching, moving from the north to the south of London, where King's College is located. It was during those years that I first worked with Sir Paul Black—a wonderful person who created "assessment for learning" and would later become my PhD advisor.

People who have worked with me or who have taken one of my online courses know that I use the word "maths," not "math." I do this partly to stay true to my British origins but also because I prefer the plural term. Mathematics was originally shortened to maths because it is short for mathematic**s**, a plural noun. Mathematics was chosen to be plural to reflect all the many parts of mathematics and ways of being mathematical. Math, to me, sounds more singular and narrow, and "do the math" usually means do a calculation. To me, "maths" helps keep the idea of a multidimensional and varied subject—including all of the different mathematic**s**. But when writing a book, I wanted the ideas to flow for readers, so I have chosen to use the word "math"—even though I am saying "maths" in my head when I write it!

Following the conclusion of my master's degree, I decided to study for a PhD, also at King's College, London University. The main component of a PhD in England is a research study. I chose to design a four-year study to help answer a question that had been debated across England for years: What is the best way to teach mathematics? I chose to contrast two approaches, one that is typical in classrooms across the world, in which a teacher explains methods and then students work through textbook questions, versus an approach in which teachers engaged students in more open tasks and projects. I particularly wanted to investigate the mathematical relationships students developed through these different teaching methods. To compare the effectiveness of these approaches, I collected multiple forms of evidence over three years, and later chapters of this book share some of that evidence. It was that study, which won the award for the best PhD in education in England, that caught the attention of the search committee at Stanford and that resulted in my moving from London to California, where I now live and work.

As a professor in the Graduate School of Education at Stanford, I am fortunate in being able to learn from multiple people, including the teachers and students I work with, the Youcubed team, and the mathematicians, scientists, engineers, and neuroscientists with whom I collaborate. This enables me to keep in touch with the latest research on learning and the brain, and the innovative ideas that this amazing group of people produces. I teach new teachers, Stanford undergraduates, and doctoral students, and I work regularly with experienced teachers. All of these different opportunities have helped me develop the ideas in this book, and I am very grateful to all the people I have learned from—especially the teachers who are highlighted throughout this book. Whether you are an avid Youcubian or this is your first introduction to these ideas, I hope you enjoy this revised edition of *Mathematical Mindsets* and that the ideas will help you unlock the limitless potential of your students. As always, I love to hear about the work you are doing with these ideas, and encourage you to keep in touch—in person or on social media (@joboaler). Viva la Maths Revolution!

—Jo

Introduction: The Power of Mindset

I remember clearly the fall afternoon that I sat down with my dean in her office, waiting for what would turn out to be a very important meeting. I had only recently returned to Stanford University from England where I was the Marie Curie Professor for Mathematics Education.

I was still getting used to the change from the grey cloudy skies that seemed to be my constant companion during the three years I was on the Sussex coast in England to the sunshine that shines down on Stanford's campus almost continuously. I walked into the dean's office that day with some anticipation, as I was going to meet Carol Dweck for the first time. I was a little nervous to meet the famous researcher whose books on mindset had revolutionized people's lives, across continents, and whose work had moved governments, schools, parents, and even leading sports teams to approach life and learning differently.

Carol and her research teams have collected data over many years that support a clear finding—that everyone has a mindset, a core belief about how they learn (Dweck, 2006b). People with a growth mindset are those who believe that smartness increases with hard work, whereas those with a fixed mindset believe that you can learn things but you can't change your basic level of intelligence. Mindsets are critically important because research has shown that they lead to different learning behaviors, which in turn create different learning outcomes for students. When people change their mindsets and start to believe that they can learn to high levels, they change their learning pathways (Blackwell, Trzesniewski, & Dweck, 2007) and achieve at higher levels, as I will share in this book.

In our conversation that day, I asked Carol if she had thought about working with mathematics teachers, as well as students, because I knew that mindset interventions given to students help them, but math teachers have the potential to deeply impact students' learning in a sustained way over time. Carol responded enthusiastically and agreed with me that math was the subject most in need of a mindset makeover. That was the first of what would become many enjoyable conversations and collaborations over the next four years, which now include our working together on shared research projects with math teachers and presenting our research and ideas to them in workshops. My work on mindset and math over recent years has helped me develop a deep appreciation of the need to teach students about mindset *inside* mathematics, rather than in general. Students have such strong and often negative ideas about math that they can develop a growth mindset about everything else in their life but still believe that you can either achieve highly in math or you can't. To change these damaging beliefs, students need to develop *mathematical mindsets*, and this book will teach you ways to encourage them.

The fixed mindsets that many people hold about mathematics often combine with other negative beliefs about mathematics, to devastating effect. This is why it is so important to share with learners the new knowledge we have of mathematics and learning that I set out in this book.

Over the last few years, I have taught and shared a number of online courses: a free course for students and parents (www.youcubed.org/online-student-course/), which has now been taken by approximately half a million people; and three teacher courses, sharing ways to teach mathematics using the ideas that bring about equitable and high achievement. I always interact with people inside the courses, which makes me realize how many people have been traumatized by mathematics. Not only did I find out how widespread the trauma is, but the evidence I collected showed that the trauma is fueled by incorrect beliefs about mathematics and intelligence. Math trauma and math anxiety are kept alive within people because these incorrect beliefs are so widespread that they permeate society in countries across the world.

I first became aware of the extent of math trauma in the days after I released my first book for parents and teachers, titled *What's Math Got to Do with It?* in the United States and *The Elephant in the Classroom* in the United Kingdom. That book details the teaching and parenting changes we need to make for mathematics to be more enjoyable and achievable. After the book was released, I was invited onto numerous different radio shows, on both sides of the Atlantic, to chat with the hosts about mathematics learning. These varied from breakfast show chats to a 20-minute, in-depth discussion with a very thoughtful PBS host and a spot on a much-loved British radio show called *Women's Hour*. Talking with radio hosts was a really interesting experience. I started most of the conversations talking about the changes we need to make, pointing out that math is traumatic for many people. This statement seemed to relax the hosts and caused many of them to open up and share with me their own stories of math trauma. Many of the interviews then turned into what seemed like therapy sessions, as the highly accomplished and knowledgeable professionals shared their various tales of math trauma, usually triggered by something a single math teacher had said or done. I still remember Kitty Dunne in Wisconsin telling me that the name of her algebra book was "burned" into her brain, revealing the strength of the negative associations she held on to. Jane Garvey at the BBC, an amazing woman for whom I have complete admiration, told me that she was so scared of mathematics that she had been fearful of interviewing me, and she had already told her two daughters that she was terrible at math in school (something you should never do, as I will discuss later). This level of intensity of negative emotion around mathematics is not uncommon. Mathematics, more than any other subject, has the power to crush students' spirits, and many adults do not move on from mathematics experiences in school if they are negative. When students get the idea they cannot do math, they often maintain a negative relationship with mathematics throughout the rest of their lives.

Mathematics trauma does not reside only in people in the arts or entertainment professions. The release of my books led to meetings with some incredible people, one of the most interesting of whom was Dr. Vivien Perry. Vivien is a top scientist in England; she was recently awarded an OBE, the greatest honor bestowed in England, given by the queen. Her list of accomplishments is long, including being the vice chair of council for University College, London; a member of the medical research council; and a presenter of BBC TV science programs. Surprisingly perhaps, with Vivien's scientific career, she talks publicly and openly about a crippling fear of mathematics. Vivien has shared with me that she is so scared of mathematics that she cannot work out percentages when she needs to complete tax documents at home. In the months before I left the United Kingdom and returned to Stanford University, I presented at the Royal Institution in London. This was a great honor, to present at one of Britain's oldest and most respected institutions that has the worthy goal of bringing scientific work to the public. Every year in Britain the Christmas Lectures, founded by Michael Faraday in 1825, are aired on TV, given by eminent scientists who share their work with the public. I had asked Vivien to introduce me at the Royal Institution, and during that introduction she shared with the audience that when she was a child she had been made to stand in the corner by her mathematics teacher, Mrs. Glass, for not being able to recite her seven times table. She then went on to make the audience laugh by telling them that when she shared this story on the BBC, six women called the BBC action line and asked—was it Mrs. Glass of Boxbury School? Vivien shared that indeed it was.

Fortunately, such harsh teaching practices are almost extinct, and I continue to be inspired by the devotion and commitment of most mathematics teachers I work with. But we know that negative and damaging messages are still handed out to students every day—messages that are not intended to harm, but that we know can start students on a damaging and lasting mathematics pathway. Such pathways can be reversed, at any time, but for many they are not, and they affect every future experience of mathematics that people have. Changing the messages that students receive about mathematics is not, sadly, as simple as just changing the words teachers and parents use, although words are very important. Students also receive and absorb many indirect messages about mathematics through many aspects of math teaching, such as the questions they work on in math class, the feedback they get, the ways they are grouped, and other aspects of mathematics teaching and help that we will consider together in this book.

Vivien is convinced that she has a brain condition, called dyscalculia, that stops her from being successful with math. But we now know that one experience or message can change everything for students (Cohen & Garcia, 2014), and it seems very likely that Vivien's negative math experiences were at the root of the math anxiety she now struggles with daily. Vivien—fortunately for the many who have benefited from her work—was able to be successful despite her mathematics experiences, even in a quantitative field, but most people are not so fortunate, and the early damaging experiences they have with mathematics close doors for them for the rest of their lives.

Taking math courses matters. Research studies have established that the more math classes students take, the higher their earnings ten years later, with advanced math courses predicting an increase in salary as high as 19.5% ten years after high school (Rose & Betts, 2004). Research has also found that students who take advanced math classes learn ways of working and thinking—especially learning to reason and be logical—that make them more productive in their jobs. Students taking advanced math learn how to approach mathematical situations so that once they are employed, they are promoted to more demanding and more highly paid positions than those who did not take mathematics to advanced levels (Rose & Betts, 2004). In my study of schools in England, I found that students were advanced in their jobs, ending up with higher-paid employment, because they learned mathematics through a project-based approach in high school that I will discuss in later chapters (Boaler, 2005; Boaler & Selling, 2017).

We all know that math trauma exists and is debilitating for people; numerous books have been devoted to the subject of math anxiety and ways to help people overcome it (Tobias, 1978). It would be hard to overstate the number of people who walk on our planet who have been harmed by bad math teaching, but the negative ideas that prevail about math do not come only from harmful teaching practices. They come from one idea, which is very strong, permeates many societies, and is at the root of math failure and underachievement: that only some people can be good at math. That single belief—that math is a "gift" that some people have and others don't—is responsible for much of the widespread math failure in the world.

So where does that damaging idea—an idea that notably is absent in countries such as China and Japan that top the world in math achievement—come from? When my two daughters were younger, I had the dubious pleasure of catching regular glimpses of "tweenie" TV programs. This was very enlightening—and worrying—as a day did not seem to go by without mathematics being portrayed in a negative light. Math is conveyed as a really hard subject that is uninteresting, inaccessible, and only for "nerds"; it is not for cool, engaging people, and it is not for girls. It is no wonder that so many children in schools disengage from math and believe they cannot do well.

The idea that only some people can do math is embedded deep in the American and British psyche. Math is special in this way, and people have ideas about math that they don't have about any other subject. Many people will say that math is different because it is a subject of right and wrong answers, but this is incorrect, and part of the change we need to see in mathematics is acknowledgment of the creative and interpretive nature of mathematics. Mathematics is a very broad and multidimensional subject that requires reasoning, creativity, connection making, and interpretation of methods; it is a set of ideas that helps illuminate the world; and it is constantly changing. Math problems should encourage and acknowledge the different ways in which people see mathematics and the different pathways they take to solve problems. When these changes happen, students engage with math more deeply and well.

Another misconception about mathematics that is pervasive and damaging—and wrong—is the idea that people who can do math are the smartest or cleverest people. This makes math failure particularly crushing for students, as they interpret it as meaning that they are not smart. We need to dispel this myth. The combined weight of all the different wrong ideas about math that prevail in society is devastating for many children—they believe that mathematics ability is a sign of intelligence and that math is a gift, and if they don't have that gift then they are not only bad at math but they are unintelligent and unlikely to ever do well in life.

As I write this book, it is clear that the world is developing a great appreciation for and understanding of the importance of mindset. Carol Dweck's book has been translated into more than 20 languages (Dweck, 2006b), and interest in the impact of mindset continues to grow. But a dangerous misconception exists about mindset, which is the idea that you can instill a growth mindset in students by sharing positive messages while still teaching in a fixed way—with math questions that have one answer and one valued method. Teachers are critical in changing students' ideas about mathematics, and how they can do so is the subject of this book. The ideas I share with teachers and parents and set out in this book include paying attention to the math questions and tasks that students work on, the ways teachers and parents encourage or grade students, the forms of grouping used in classrooms, the ways mistakes are dealt with, the norms developed in classrooms, the math messages we can give to students, and the strategies students learn for approaching math—in essence, the whole of the mathematics teaching and learning experience. I am excited to share this new knowledge with you, and I am confident that it will help you and anyone you work with on mathematics.

In the next chapter I will set out some of the fascinating and important ideas that have emerged from research in recent years; in the eight chapters that follow, I will focus on the strategies that can be used in math classrooms and homes to implement the ideas I share in these first two chapters. I strongly recommend reading all of the chapters, skipping to the strategies will not be helpful if the underlying ideas are not well understood.

Ever since we started youcubed.org and I released different online classes, I have received thousands of letters, emails, and other messages from people sharing with me the changes they had made in their classrooms and homes and the impact this has had on the students (see also Boaler, 2019). Relatively small changes in teaching and parenting can change students' mathematical pathways, because the new knowledge we have on the brain, mindset, and mathematics learning is truly revolutionary. This book is about the creation of *mathematical mindsets* through a new kind of teaching and parenting that is, at its heart, about growth, innovation, creativity, and the fulfillment of mathematics potential. Thank you for joining me, and for taking steps on a pathway that could change your and your students' relationships with mathematics forever.

CHAPTER 1

The Brain and Mathematics Learning

In the last few decades we have seen the emergence of technologies that have given researchers new access into the workings of the mind and brain. Now scientists can study children and adults working on math and watch their brain activity; they can look at brain growth and brain degeneration, and they can see the impact of different emotional conditions upon brain activity. One area that has emerged in recent years and stunned scientists concerns "brain plasticity." It used to be believed that the brains people were born with couldn't really be changed, but this idea has now been resoundingly disproved. Study after study has shown the incredible capacity of brains to grow and change within a really short period (Abiola & Dhindsa, 2011; Maguire, Woollett, & Spiers, 2006; Woollett & Maguire, 2011).

When we learn a new idea, one of three things happens in the brain (see Figure 1.1). The first possibility is that you start a new brain pathway. The more deeply you learn, the stronger the pathway becomes. The second possibility is that you strengthen a pathway you already had, and the third possibility is that you make connections between pathways. This brain development is taking place all the time, and the pathways you build, strengthen, or connect were not in your brain at birth; they are created by your learning experiences.

I wish all students knew this—when you are teaching them math, you are changing their brains! Neuroscientist Norman Doidge (2007) likes to share with his audiences that every day you wake up, your brain is different from the day before—that is the extent of brain growth and change that occur every day. If you learn something deeply, you form lasting brain pathways that you can revisit and use, but if you visit an idea only once or in a superficial way, the pathway can "wash away" like a path made in the sand. These brain connections form when learning takes place, but learning does not occur only in classrooms or when reading books; as we all know, we are forming brain connections when we have conversations, play games, or build with toys, and in the course of many, many other experiences.

The first research on what became known as neuroplasticity, which shocked the scientific world, came from studies of "Black Cab" drivers in London. I am from England, and I have traveled in taxicabs in London many times. I still have fond

FIGURE 1.2 The Black Cab of London
Source: Peter Fuchs/Shutterstock.

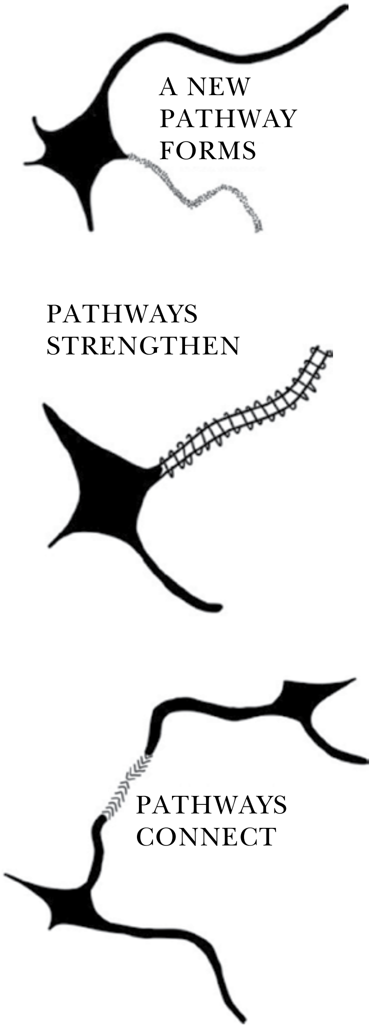

FIGURE 1.1 Brains form, develop, and connect pathways.

memories of the exciting day trips my family and I took to London when I was a child, from our home a few hours away. As an adult I studied and worked at King's College, London University, and had many more opportunities for trips around London in taxis. A number of different taxis work in the London area, but the queen bee of taxis in London is the Black Cab (see Figure 1.2).

For most of my rides through London in a Black Cab, I had no idea how highly qualified the drivers were. It turns out that to become a Black Cab driver in London, applicants need to study for four years or more, and during that time learn routes around 25,000 streets and 20,000 landmarks within a 25-mile radius of Charing Cross in London. Learning your way around the city of London is considerably more challenging than learning your way around most American cities, as London is not built on a grid structure and comprises thousands of interweaving, interconnected streets (see Figure 1.3).

At the end of their training period the Black Cab drivers take a test that is simply and elegantly called "The Knowledge." If you ride in a London Black Cab and ask your driver about "The Knowledge," they are usually happy to regale you with stories of the difficulty of the test and their training period. The Knowledge is known to be one of the world's most demanding courses, and applicants take the test an average of 12 times before passing.

In the early 2000s scientists chose to study London Black Cab drivers to look for brain changes as the drivers took years of complex spatial training, but the scientists were not expecting such dramatic results. Researchers found that at the end of the training period the hippocampus in the taxi drivers' brains had grown significantly (Maguire et al., 2006; Woollett & Maguire, 2011).

The hippocampus is the brain area specialized in acquiring and using spatial information (see Figure 1.4).

FIGURE 1.3 Map of London
Source: jason cox/Shutterstock.

In other studies, scientists compared the brain growth of Black Cab drivers to that of London bus drivers. Bus drivers learn only simple and singular routes, and the studies showed that they did not experience the same brain growth (Maguire et al., 2006). This confirmed the scientists' conclusion that the Black Cab drivers' unusually complex training was the reason for their dramatic brain growth. In a further study, scientists found that after Black Cab drivers retired, their hippocampus shrank back down again (Woollett & Maguire, 2011). This was not because of age but because of lack of using the brain pathways.

The studies conducted with Black Cab drivers, of which there have now been many (Maguire et al., 2006; Woollett & Maguire, 2011), showed a degree of brain flexibility, or plasticity, that stunned scientists. They had not previously thought that the extent of brain growth they measured was possible. This led to a shift in the scientific world in thinking about learning and "ability" and the possibility of the brain to change and grow.

FIGURE 1.4 The hippocampus
Source: decade3d/Shutterstock.

Around the time that the Black Cab studies were emerging, something happened that would further rock the scientific world. A nine-year old girl, Cameron Mott, had been having seizures that the doctors could not control. Her physician, Dr. George Jello, proposed something radical. He decided he should remove half of her brain, the entire left hemisphere. The operation was revolutionary—and ultimately successful. In the days following her operation, Cameron was paralyzed. Doctors expected her to be disabled for many years, as the left side of the brain controls physical movements. But as weeks and months passed, she stunned doctors by recovering function and movement that could mean only one thing—the right side of her brain was developing the connections it needed to perform the functions of the left side of the brain. Doctors attributed this to the incredible plasticity of the brain and could only conclude that the brain had, in effect, "regrown." The new brain growth had occurred faster than doctors imagined possible (http://www.today.com/id/36032653/ns/today-today_health/t/meet-girl-half-brain/#.UeGbixbfvCE).

This operation has now been performed on many different people. Christina Santhouse was eight when she underwent the operation to have half of her brain removed. Christina went on to many notable achievements, including making the honor roll at high school, earning a master's degree, and becoming a speech pathologist.

The new findings that brains can grow, adapt, and change shocked the scientific world and spawned new studies of the brain and learning, making use of ever-developing new technologies and brain scanning equipment. In one study, researchers at the National Institute for Mental Health gave people a 10-minute exercise to work on each day for three weeks. The researchers compared the brains of those receiving the training with those who did not. The results showed that the people who worked on an exercise for a few minutes each day experienced structural brain changes. The participants' brains "rewired" and grew in response to a 10-minute mental task performed daily over 15 weekdays (Karni et al., 1998).

In another study on mathematics learning in particular, Teresa Iuculano and her colleagues at Stanford's school of medicine uncovered critically important information. They brought into their labs two groups of students. One group had been diagnosed in schools as having mathematics learning disabilities; the other group comprised "regular" performers. The researchers looked at the students' brains as they worked on mathematics, using MRI scans. They found fascinating differences—the students identified as having learning disabilities had more brain regions lighting up when they worked on a mathematics question. This is counterintuitive for many people, who think that students with learning disabilities have less going on in their brains, not more. The researchers point out that success does not always come from *more* brain activity, but focused activity in certain areas. The research then became even more interesting. Both sets of students were given eight weeks of one-to-one tutoring. At the end of the eight-week period, the two sets of students had not only the same achievement but the exact same brain areas lighting up (Iuculano et al., 2015).

These and other results should prompt educators to abandon the traditional fixed ideas of the brain and learning that currently permeate schools—ideas that children are smart or dumb, quick or slow. If brains can change significantly in eight weeks, imagine what can happen in a year of maths class if students are given engaging tasks (more on that later) and receive positive messages about their potential and ability. Chapter Five will explain the nature of the very best mathematics tasks that students should be working on to experience this brain growth.

The new evidence from brain research tells us that everyone, with the right teaching and messages, can be successful in mathematics and that everyone can achieve at the highest levels in school. There are a few children who have severe special educational needs that make mathematics learning difficult, but for the vast majority of children—at least 95%—any level of school mathematics is within their reach. And the potential of the brain to grow and change is just as strong in children diagnosed as having special educational needs (see also Boaler & LaMar, 2019). This was clearly illustrated by a beautiful event that took place in Australia. As Nicholas Letchford was growing up, he was diagnosed as "learning disabled." In his first year of school, his parents were told that he had a "very low IQ," and teachers told his parents that "he was the worst child they had seen in twenty years of teaching." Nicholas found it difficult to focus, make connections, read, or write. But Nicholas's mother, Lois, refused to accept the labels placed on her son, and she worked with Nicholas, teaching him how to focus, connect, read, and write. In 2018, Nicholas graduated from Oxford University with a doctoral degree in applied mathematics—the highest level of achievement possible to attain (Letchford, 2018).

Parents and teachers need to know this information from neuroscience and from records of people. When I share both forms of evidence with teachers in workshops and presentations, most of them are encouraged and inspired, but not all of them. I was with a group of teachers recently, and one high school math teacher was clearly troubled by the idea. He said, "You aren't telling me, are you, that *any* of the sixth graders in my school could take calculus in twelfth grade?" When I said, "That is exactly what I am saying," I could tell he was genuinely troubled by the idea—although, to his credit, he was not rejecting it outright. Some teachers find the idea that anyone can learn math to high levels difficult to accept, especially if they have spent many years deciding who can and who can't do math and teaching them accordingly. Of course, sixth graders have had many experiences and messages since birth that have held some of them back, and some students may come to sixth grade with significantly less mathematical knowledge than others, but this doesn't mean they cannot accelerate and reach the highest levels—they can, if they receive the high-quality teaching and support that all children deserve.

I am often asked whether I am saying that everyone is born with the same brain. I am not. What I am saying is that any brain differences children are born with are nowhere near as important as the brain growth experiences they have throughout life. People hold very strong views that the way we are born determines our potential; they point to well-known people who were considered geniuses—such as Albert Einstein or Ludwig van Beethoven. But scientists now know that any brain differences present at birth are eclipsed by the learning experiences we have from birth onward (Wexler in Thompson, 2014). Every second of the day, brain pathways are forming, connecting, and strengthening, and students given encouraging messages and rich mathematics experiences are capable of anything. Brain differences can give some people a head start, but infinitesimally small numbers of people have the sort of head start that gives them advantages over time. And those people who are heralded as natural geniuses are the same people who often stress the hard work they have put in and the number of mistakes they made. Einstein, probably the most well known of those thought to be a genius, did not learn to read until he was nine and spoke often about his achievements coming from the number of mistakes he had made and the persistence he had shown. He tried hard, and when he made mistakes he tried harder. He approached work and life with the attitude of someone with a growth mindset. A lot of scientific evidence suggests that the difference between those who succeed and those who don't is not the brains they were born with, but their approach to life, the messages they receive about their potential, and the opportunities they have to learn. The very best opportunities to learn come about when students believe in themselves. For far too many students in school, their learning is hampered by the messages they have received about their own potential, making them believe they are not as good as others, that they don't have the potential of others. This book provides the information you need, whether you are a teacher or parent, to give students the self-belief they need and should have; to set them on a pathway that leads to a mathematical mindset, whatever their prior experiences. This new pathway involves a change in the way students consider themselves and also a change in the way they approach the subject of mathematics, as the rest of the book will describe.

Although I am not saying that everyone is born with the same brain, I *am* saying that there is no such thing as a "math brain" or a "math gift," as many believe. No one is born knowing math, and no one is born lacking the ability to learn math. Unfortunately, ideas of giftedness are widespread. Researchers recently investigated the extent to which college professors held ideas about giftedness in their subject, and they found something remarkable (Leslie, Cimpian, Meyer, & Freeland, 2015). Math was the subject whose professors were found to hold the most fixed ideas about who could learn. Additionally, researchers found that the more a field values giftedness, the fewer female PhDs there were in the field and that field-specific beliefs were correlated with female representation across all 30 fields they investigated. The reason that there are fewer women in fields where professors believe that only the "gifted" can achieve is that stereotypical beliefs still prevail about who really belongs, as Chapter Six describes. It is imperative for our society that we move to a more equitable and informed view of mathematics learning in our conversations and work with students. Schools should seriously consider the new science of the brain and communicate to all students that everyone can learn mathematics to high levels. This could well be the key to unlocking a different future—one in which math trauma is a thing of the past and students from all backgrounds are given access to high-quality mathematics learning opportunities.

In studies by Carol Dweck and her colleagues, about 40% of the children were found to hold a damaging fixed mindset, believing that intelligence is a gift that you either have or you don't. Another 40% of the students had a growth mindset. The remaining 20% wavered between the two mindsets (Dweck, 2006b). Students with a fixed mindset are more likely to give up easily, whereas students with a growth mindset keep going even when work is hard and are persistent, displaying what Angela Duckworth has termed "grit" (Duckworth & Quinn, 2009). In one study, seventh-grade students were given a survey to measure their mindset, then researchers followed the students over two years to monitor their mathematics achievement. The results were dramatic, as the achievement of the students with a fixed mindset stayed constant, but the achievement of those with a growth mindset went onward and upward (Blackwell et al., 2007) (see Figure 1.5).

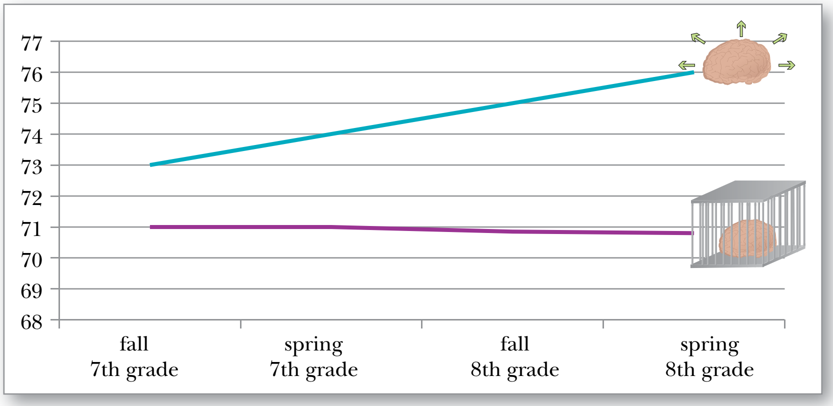

FIGURE 1.5 Students with a growth mindset outperform those with a fixed mindset in mathematics
Source: Modified from Blackwell et al., 2007.

In other studies, researchers have shown that students' (and adults') mindsets can change from fixed to growth, and when that happens their learning approach becomes significantly more positive and successful (Blackwell et al., 2007). We also have new evidence, that I review in Chapter Two, that students with a growth mindset have more positive brain activity when they make mistakes, with more brain regions lighting up and more attention to and correcting of errors (Moser, Schroder, Heeter, Moran, & Lee, 2011).

I didn't need more evidence of the importance of helping students—and adults—develop a growth mindset in relation to math in particular, but recently I found myself sitting with the Program for International Student Assessment (PISA) team at the Organisation for Economic Co-operation and Development (OECD) in Paris, exploring with them their incredible data set of 13 million students worldwide. The PISA team gives international tests every four years, and the results are reported in news outlets across the globe. The test scores often start alarm bells ringing around the United States, and for good reason. In the latest tests, the United States ranked 36th out of 65 OECD countries in math performance (OECD, 2018)—a result that speaks, as do many others, to the incredible need to reform mathematics teaching and learning in the United States. But the PISA team not only administer math tests; they also survey students to collect their ideas and beliefs about mathematics and their mindsets. I was invited to work with the PISA team after some of the group took one of my online classes (www.youcubed.org/resource/online-courses-for-teachers/). One of them was Pablo Zoido, a soft-spoken Spaniard who thinks deeply about math learning and has considerable expertise in working with giant data sets. Pablo, was, at the time, an analyst for PISA, and as he and I explored the data, we saw something amazing—that the highest-achieving students in the world are those with a growth mindset, and they outrank the other students by the equivalent of more than a year of mathematics (see Figure 1.6).

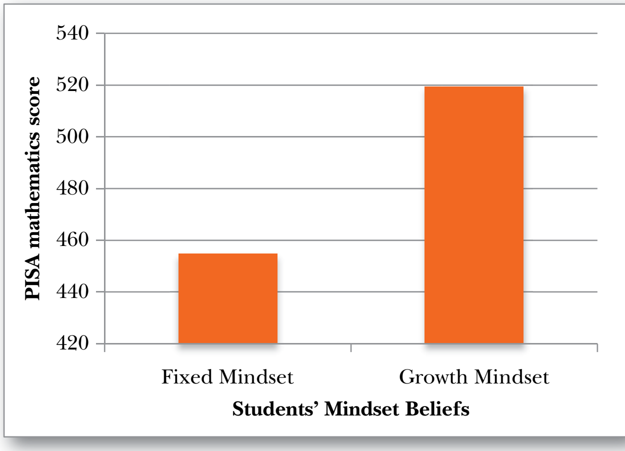

FIGURE 1.6 Mindset and mathematics
Source: Based on PISA, 2012.

The fixed mindset thinking that is so damaging—a mindset in which students believe they either are smart or are not—cuts across the achievement spectrum, and some of the students most damaged by these beliefs are high-achieving girls (Dweck, 2006a). It turns out that even believing you *are* smart—one of the fixed mindset messages—is damaging, as students with this fixed mindset are less willing to try more challenging work or subjects because they are afraid of slipping up and no longer being seen as smart. Students with a growth mindset take on hard work, and they view mistakes as a challenge and motivation to do more. The high incidence of fixed mindset thinking among girls is one reason that girls opt out of STEM subjects—science, technology, mathematics, and engineering. This not only reduces their own life chances but also impoverishes the STEM disciplines that need the thinking and perspectives that girls and women bring (Boaler, 2014a).

One reason so many students in the United States have fixed mindsets is the praise they are given by parents and teachers. When students are given fixed praise—for example, being told they are smart when they do something well—they may feel good at first, but when they fail or struggle later (and everyone does), they think that means they are not so smart after all. In an important study, researchers found that the praise parents gave their babies between birth and age three predicted their mindsets five years later (Gunderson et al., 2013). The impact of the praise students receive can be so strong that it affects their behavior immediately. In one of Carol Dweck's studies, researchers asked 400 fifth graders to take an easy short test, on which almost all performed well. Half the children were then praised for "being really smart." The other half were complimented on "having worked really hard." The children were then asked to take a second test and choose between one that was pretty simple, that they would do well on, or one that was more challenging, that they might make mistakes on. Ninety percent of those who were praised for their effort chose the harder test. Of those praised for being smart, the majority chose the easy test (Mueller & Dweck, 1998).

Praise feels good, but when people are praised for who they are as a person ("You are so smart") rather than what they did ("That is amazing thinking and creativity"), they get the idea that they have a fixed amount of ability. Telling students they are smart sets them up for problems later. As students go through school and life, failing at many tasks—which, again, is perfectly natural—they evaluate themselves, deciding how smart or not smart this means they really are. Instead of praising students for being smart, or any other personal attribute, it's better to say things like: "It is great that you have learned that," and "You have thought really deeply about this."

Our education systems have been pervaded with the traditional notion that some students are not developmentally ready for certain levels of mathematics. A group of high school math teachers in a school I recently encountered had, shockingly, written to the school board arguing that some students could never pass algebra 2. They particularly cited minority students from low-income homes; they argued that these students could not learn algebra unless the teachers watered down the curriculum. Such deficit and racist thinking needs to be banished from schools. The letter written by the teachers was published in local newspapers and ended up being used in the state legislature as an example of the need for charter schools (Noguchi, 2012). The letter shocked many people, but unfortunately this idea that some students cannot learn high-level mathematics is shared by many. Deficit thinking can take all sorts of forms and is sometimes used with genuine concern for students—many people believe there is a developmental stage students must go through before they are ready for certain mathematics topics. But these ideas are also outdated, as students are as ready as the experiences they have had, and if students are not ready, they can easily become so with the right experiences, high expectations from others, and a growth mindset. There is no preordained pace at which students need to learn mathematics, meaning it is *not* true that if they have not attained a certain age or emotional maturity they cannot learn some mathematics. Students may be unready for some mathematics because they still need to learn some foundational, prerequisite mathematics they have not yet learned, but *not* because their brain cannot develop the connections because of their age or maturity. When students need new connections, they can learn them.

For many of us, appreciating the importance of mathematical mindsets and developing the perspective and strategies to change students' mindsets involves some careful thinking about our own learning and relationship with mathematics. Many of the elementary teachers I have worked with, some of whom took my online class, have told me that the ideas I gave them on the brain, on potential, and on growth mindsets has been life-changing for them. It caused them to develop a growth mindset in mathematics, to approach mathematics with confidence and enthusiasm and to pass that on to their students. This is often particularly important for elementary teachers, because many have, at some point in their own learning, been told *they* cannot do mathematics or that mathematics is not for them. Many teach mathematics with their own fear of the subject. The research I shared with them helped banish that fear and put them on a different mathematical journey. In an important study, Sian Beilock and colleagues found that the extent of negative emotions elementary teachers held about mathematics predicted the achievement of girls in their classes, but not boys (Beilock, Gunderson, Ramirez, & Levine, 2009). This gender difference probably comes about because girls identify with their female teachers, particularly in elementary school. Girls quickly pick up on teachers' negative messages about math—the sort that are often given out of kindness, such as: "I know this is really hard, but let's try and do it" or "I was bad at math at school" or "I never liked math." This study also highlights the link between the messages teachers give and the achievement of their students.

* * *

Neuroscience has also made huge contributions to our understanding of mathematics learning in other areas, beyond neuroplasticity. I am fortunate in being able to work with neuroscientists at Stanford, and two I have collaborated with in recent years have been instrumental in this work: Vinod Menon and Lang Chen. They have studied the interacting networks in the brain, particularly focusing on the ways the brain learns mathematics. They have found that whenever we work on mathematics, there are five different brain areas involved; two of them are visual pathways. The dorsal visual pathway is the main brain area for representing quantity. These researchers, and others, have also found that communication between the different brain areas enhances learning and achievement (see also Park & Brannon, 2013). For example, if a student sees a calculation with numbers, as well as a visual representation, this will encourage brain connections—communication between brain areas.

Other incredible neuroscience has shown that fingers are particularly critical to mathematics learning (Boaler & Chen, 2016). The work with first graders I mentioned in the preface was a project that involved educators (myself and my Youcubed team), neuroscientists, and engineers, building on the research on the importance of fingers to mathematics learning. We created a robotic device that allowed students to answer questions with their fingers, and feel vibrations in their fingers, to really help them associate fingers with numbers. It was a fascinating experimental study which showed that the students who worked with the robotic finger device learned significantly more mathematics over a short period of time. But finger work does not need to be so high-tech or to involve robotics. We share paper resources on youcubed.org that can be similarly helpful for students to develop what is known as "finger perception" (see www.youcubed.org/resource/visual-mathematics/).

We can learn mathematics with numbers, but we can also learn through words, visuals, models, algorithms, tables, and graphs, and from moving and touching. Experiencing mathematics in these multidimensional ways encourages brain communication and understanding. Lang Chen illustrates the different brain areas involved in mathematical thinking in Figure 1.7.

As I shall explain in later chapters, I like to create mathematical activities that give students a connected experience of mathematics, with different parts of their brains firing and communicating with one another, as they consider mathematics in different ways, as illustrated in Figure 1.8.

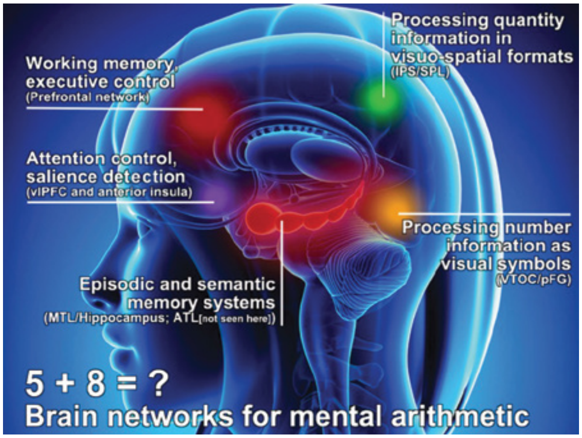

FIGURE 1.7 The brain areas involved in mathematical thought
Source: Lang Chen.

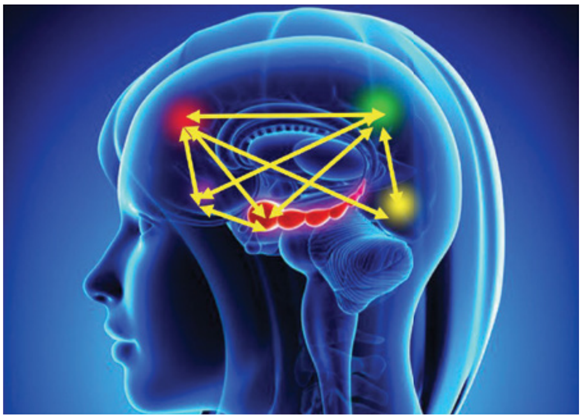

FIGURE 1.8 A connected mathematical experience
Source: pixologic/Depositphotos.

I hope that you enjoy the different ideas for helping students develop brain connections as they learn mathematics. Wherever you are on your own mathematical mindset journey, whether these ideas are new to you or you are a mathematical mindset expert, I hope that the data and ideas I share in this book will help you and your students see mathematics—any level of mathematics—as both reachable and enjoyable. In the next chapters, Two through Eight, I will share the many strategies I have collected over years of research and teaching for encouraging a mathematical mindset experience for students.