SECOND EDITION

MATHEMATICAL MINDSETS

UNLEASHING STUDENTS' POTENTIAL THROUGH CREATIVE MATHEMATICS, INSPIRING MESSAGES AND INNOVATIVE TEACHING

JO BOALER

FOREWORD BY CAROL DWECK

JOSSEY-BASS
A Wiley Brand

Table of Contents

List of Tables

List of Illustrations

Chapter 9

JO BOALER
FOREWORD BY CAROL DWECK

MATHEMATICAL MINDSETS

UNLEASHING STUDENTS' POTENTIAL Through Creative Mathematics, Inspiring Messages and INNOVATIVE TEACHING

Second Edition

JOSSEY-BASS™
A Wiley Brand

Published by Jossey-Bass A Wiley Brand
111 River St
Hoboken, New Jersey 07030
www.josseybass.com

Jossey-Bass books and products are available through most bookstores. To contact Jossey-Bass directly call our Customer Care Department within the U.S. at 800-956-7739, outside the U.S. at 317-572-3986, or fax 317-572-4002.

Wiley publishes in a variety of print and electronic formats and by print-on-demand. Some material included with standard print versions of this book may not be included in e-books or in print-on-demand. If this book refers to media such as a CD or DVD that is not included in the version you purchased, you may download this material at http://booksupport.wiley.com. For more information about Wiley products, visit www.wiley.com.

Library of Congress Cataloging-in-Publication Data is Available:

ISBN: 9781119823063 (paperback)
ISBN: 9781119823087 (ePDF)
ISBN: 9781119823070 (ePub)

Cover Design: Wiley
Cover Image: © SilverCircle/Shutterstock
Author Photo: Cathy Williams

SECOND EDITION

FOREWORD

One of my former Stanford students teaches fourth grade in the South Bronx, an area of New York City with many underserved, underachieving minority students. Her students invariably believe they are bad at math, and if you looked at their past performance, you might be tempted to think so too. And yet, after one year in her class, her fourth graders became the #1 fourth-grade class in the state of New York: 100% of them passed the state math test, with 90% of them earning the top score. And this is just one of many examples of how all students can learn math.

When people think that some kids just can't do math, that success in math is reserved for only certain kids, thought of as “smart,” or that it's just too late for kids who haven't had the right background, then they can easily accept that many students fail math and hate math. In fact, we have found that many teachers actually console their students by telling them not to worry about doing poorly in math because not everyone can excel in it. These adult enablers —parents and teachers alike—allow kids to give up on math before they've barely gotten started. No wonder more than a few students simply dismiss their own poor performance by declaring: “I'm not a math person.”

Where do parents, teachers, and students get the idea that math is just for some people? New research shows that this idea is deeply embedded in the field of mathematics. Researchers polled scholars (at American universities) in a range of disciplines. They asked them how much they thought that success in their field depended on fixed, innate ability that cannot be taught, as opposed to hard work, dedication, and learning. Of all the STEM fields (science, technology, engineering, and math), math scholars were the most extreme in emphasizing fixed, innate ability (Leslie, Cimpian, Meyer, & Freeland, 2015). Other researchers are finding that many math instructors begin their courses by referring to students who have the aptitude and those who do not. One college instructor, on the first day of an introductory college course, was heard to say, “If it's not easy for you, you don't belong here” (Murphy, Garcia, & Zirkel, in prep). If this message is passed down from generation to generation, no wonder students are afraid of math. And no wonder they conclude they're not math people when it doesn't come easily.

But when we begin to see evidence that most students (and maybe almost all students) are capable of excelling in and enjoying math, as the following chapters show, it is no longer acceptable that so many students fail math and hate math. So what can we do to make math learning happen for all students? How can we help teachers and children believe that math ability can be developed, and then show teachers how to teach math in a way that brings this belief to life? That's what this book is about.

In this unique and wonderful book, Jo Boaler distills her years of experience and her powerful wisdom to show teachers exactly how to present math work, structure math problems, guide students through them, and give feedback in a way that helps students toward a “growth mindset” and keeps them there. Boaler is one of those rare and remarkable educators who not only know the secret of great teaching but also know how to give that gift to others. Thousands of teachers have learned from her, and here's what they say:

> "Throughout my schooling years … I was left feeling stupid and incapable of doing [math] … I cannot tell you the relief I now have that I can learn math myself, and I can teach students that they can too."

> "[You have] helped me think about the transition to common core and how to help my students develop a love and curiosity for math."

> "I was searching for a process of learning math that would change the attitude of students from dislike to enjoy … this was the change I needed."

Imagine your students joyfully immersed in really hard math problems. Imagine them begging to have their mistakes discussed in front of the class. Imagine them saying, "I *am* a math person!" This utopian vision is happening in classrooms around the world, and as you follow the advice in this book, you may well see it happening in your classroom too.

Carol Dweck
Professor of psychology and author of
Mindset: The New Psychology of Success

PREFACE TO THE SECOND EDITION

It has been six years since I wrote the first edition of *Mathematical Mindsets*, and what an amazing six years it has been! As I write this we are emerging from a global pandemic that has stretched the capacity of teachers like nothing else before. It was incredible to see the work and creativity of teachers who continued to make mathematics a beautiful and exciting subject, even when teaching online. My personal mission in writing books like this and sharing resources on our website, youcubed.org, is always to support the work of teachers. I was thrilled that the first edition of *Mathematical Mindsets* became an international best seller, was translated into eight languages, and was appreciated by so many people. When my publishers asked about writing an updated edition, I reflected on the new knowledge and ideas that have been created over the last six years and decided it was time.

One of my reasons for writing the first edition of *Mathematical Mindsets* was to share the neuroscience that was emerging at the time and that I knew was really important for educators, students, and their families. That neuroscientific evidence is just as important today, and new studies have now been published that I have shared in this edition—translated into usable ideas for teachers. I am fortunate to work directly with neuroscientists at Stanford, and this book shares some of our joint work that we have completed since the first edition was written—which included a foray into teaching first graders! As a former secondary mathematics teacher, I found it really interesting and fun to observe the learning of first-grade students.

At the time of writing the first edition of *Mathematical Mindsets*, Youcubed was just beginning. I had recently completed my first online course and received many emails from teachers asking what was next. Cathy Williams and I decided to create a website, youcubed.org, never imagining we would be celebrating 50 million visits to the site six years later. I love that teachers who regularly use our resources call themselves Youcubians, and I have had the pleasure of meeting and working with many of them over the last few years. When we meet for professional development workshops, we always, as Cathy says, have a maths party!

Since the first edition was written, I have also had the opportunity to think really carefully about the creation of equitable group work. I know that good teachers are always concerned about creating opportunities for students to work together and for group discussions to be equitable—with all students involved and included. My recommendations for group work are a little more developed in this edition, particularly after teaching middle school students myself in our Youcubed summer camps in California. This edition shares some of the learnings from that camp, as well as the camps that were taught across the U.S. and in Scotland and Brazil, by Youcubians. The results of those, which were carefully researched, were incredible.

I started my own career as a mathematics teacher in inner-city London, teaching students ages 11–18 in public schools that were extremely culturally diverse. This rekindled my fascination in students' learning of mathematics—an interest that I had developed when I was a student in school. I decided to study for a master's degree at King's College, London, to learn more about research in mathematics education. I took the courses in the evenings after teaching at Haverstock school during the day. One of my clearest memories of those years was of traveling on the tube across London in the evenings, at the end of a busy day of teaching, moving from the north to the south of London, where King's College is located. It was during those years that I first worked with Sir Paul Black—a wonderful person who created “assessment for learning” and would later become my PhD advisor.

People who have worked with me or who have taken one of my online courses know that I use the word “maths,” not “math.” I do this partly to stay true to my British origins but also because I prefer the plural term. Mathematics was originally shortened to maths because it is short for mathematic**s**, a plural noun. Mathematics was chosen to be plural to reflect all the many parts of mathematics and ways of being mathematical. Math, to me, sounds more singular and narrow, and “do the math” usually means do a calculation. To me, “maths” helps keep the idea of a multidimensional and varied subject—including all of the different mathematic**s**. But when writing a book, I wanted the ideas to flow for readers, so I have chosen to use the word “math”—even though I am saying “maths” in my head when I write it!

Following the conclusion of my master's degree, I decided to study for a PhD, also at King's College, London University. The main component of a PhD in England is a research study. I chose to design a four-year study to help answer a question that had been debated across England for years: What is the best way to teach mathematics? I chose to contrast two approaches, one that is typical in classrooms across the world, in which a teacher explains methods and then students work through textbook questions, versus an approach in which teachers engaged students in more open tasks and projects. I particularly wanted to investigate the mathematical relationships students developed through these different teaching methods. To compare the effectiveness of these approaches, I collected multiple forms of evidence over three years, and later chapters of this book share some of that evidence. It was that study, which won the award for the best PhD in education in England, that caught the attention of the search committee at Stanford and that resulted in my moving from London to California, where I now live and work.

As a professor in the Graduate School of Education at Stanford, I am fortunate in being able to learn from multiple people, including the teachers and students I work with, the Youcubed team, and the mathematicians, scientists, engineers, and neuroscientists with whom I collaborate. This enables me to keep in touch with the latest research on learning and the brain, and the innovative ideas that this amazing group of people produces. I teach new teachers, Stanford undergraduates, and doctoral students, and I work regularly with experienced teachers. All of these different opportunities have helped me develop the ideas in this book, and I am very grateful to all the people I have learned from—especially the teachers who are highlighted throughout this book. Whether you are an avid Youcubian or this is your first introduction to these ideas, I hope you enjoy this revised edition of *Mathematical Mindsets* and that the ideas will help you unlock the limitless potential of your students. As always, I love to hear about the work you are doing with these ideas, and encourage you to keep in touch—in person or on social media (@joboaler). Viva la Maths Revolution!

—Jo

INTRODUCTION: THE POWER OF MINDSET

I remember clearly the fall afternoon that I sat down with my dean in her office, waiting for what would turn out to be a very important meeting. I had only recently returned to Stanford University from England where I was the Marie Curie Professor for Mathematics Education.

I was still getting used to the change from the grey cloudy skies that seemed to be my constant companion during the three years I was on the Sussex coast in England to the sunshine that shines down on Stanford's campus almost continuously. I walked into the dean's office that day with some anticipation, as I was going to meet Carol Dweck for the first time. I was a little nervous to meet the famous researcher whose books on mindset had revolutionized people's lives, across continents, and whose work had moved governments, schools, parents, and even leading sports teams to approach life and learning differently.

Carol and her research teams have collected data over many years that support a clear finding—that everyone has a mindset, a core belief about how they learn (Dweck, 2006b). People with a growth mindset are those who believe that smartness increases with hard work, whereas those with a fixed mindset believe that you can learn things but you can't change your basic level of intelligence. Mindsets are critically important because research has shown that they lead to different learning behaviors, which in turn create different learning outcomes for students. When people change their mindsets and start to believe that they can learn to high levels, they change their learning pathways (Blackwell, Trzesniewski, & Dweck,

2007) and achieve at higher levels, as I will share in this book.

In our conversation that day, I asked Carol if she had thought about working with mathematics teachers, as well as students, because I knew that mindset interventions given to students help them, but math teachers have the potential to deeply impact students' learning in a sustained way over time. Carol responded enthusiastically and agreed with me that math was the subject most in need of a mindset makeover. That was the first of what would become many enjoyable con- versations and collaborations over the next four years, which now include our working together on shared research projects with math teachers and presenting our research and ideas to them in workshops. My work on mindset and math over recent years has helped me develop a deep appreciation of the need to teach students about mindset *inside* mathematics, rather than in general. Students have such strong and often negative ideas about math that they can develop a growth mindset about everything else in their life but still believe that you can either achieve highly in math or you can't. To change these damaging beliefs, students need to develop *mathematical mindsets*, and this book will teach you ways to encourage them.

The fixed mindsets that many people hold about mathematics often combine with other negative beliefs about mathematics, to devastating effect. This is why it is so important to share with learners the new knowledge we have of mathematics and learning that I set out in this book.

Over the last few years, I have taught and shared a number of online courses: a free course for students and parents (www.youcubed.org/online-student-course/), which has now been taken by approximately half a million people; and three teacher courses, sharing ways to teach mathematics using the ideas that bring about equitable and high achievement. I always interact with people inside the courses, which makes me realize how many people have been traumatized by mathematics. Not only did I find out how widespread the trauma is, but the evidence I collected showed that the trauma is fueled by incorrect beliefs about mathematics and intelligence. Math trauma and math anxiety are kept alive within people because these incorrect beliefs are so widespread that they permeate society in countries across the world.

I first became aware of the extent of math trauma in the days after I released my first book for parents and teachers, titled *What's Math Got to Do with It?* in the United States and *The Elephant in the Classroom* in the United Kingdom. That book details the teaching and parenting changes we need to make for mathematics to be more enjoyable and achievable. After the book was released, I was invited onto numerous different radio shows, on both sides of the Atlantic, to chat with the hosts about mathematics learning. These varied from breakfast show chats to a 20-minute, in-depth discussion with a very thoughtful PBS host and a spot on a much-loved British radio show called *Women's Hour*. Talking with radio hosts was a really interesting experience. I started most of the conversations talking about the changes we need to make, pointing out that math is traumatic for many people. This statement seemed to relax the hosts and caused many of them to open up and share with me their own stories of math trauma. Many of the interviews then turned into what seemed like therapy sessions, as the highly accomplished and knowledgeable professionals shared their various tales of math trauma, usually triggered by something a single math teacher had said or done. I still remember Kitty Dunne in Wisconsin telling me that the name of her algebra book was “burned” into her brain, revealing the strength of the negative associations she held on to. Jane Garvey at the BBC, an amazing woman for whom I have complete admiration, told me that she was so scared of mathematics that she had been fearful of interviewing me, and she had already told her two daughters that she was terrible at math in school (something you should never do, as I will discuss later). This level of intensity of negative emotion around mathematics is not uncommon. Mathematics, more than any other subject, has the power to crush students’ spirits, and many adults do not move on from mathematics experiences in school if they are negative. When students get the idea they cannot do math, they often maintain a negative relationship with mathematics throughout the rest of their lives.

Mathematics trauma does not reside only in people in the arts or entertainment professions. The release of my books led to meetings with some incredible people, one of the most interesting of whom was Dr. Vivien Perry. Vivien is a top scientist in England; she was recently awarded an OBE, the greatest honor bestowed in England, given by the queen. Her list of accomplishments is long, including being the vice chair of council for University College, London; a member of the medical research council; and a presenter of BBC TV science programs. Surprisingly perhaps, with Vivien's scientific career, she talks publicly and openly about a crippling fear of mathematics. Vivien has shared with me that she is so scared of mathematics that she cannot work out percentages when she needs to complete tax documents at home. In the months before I left the United Kingdom and returned to Stanford University, I presented at the Royal Institution in London. This was a great honor, to present at one of Britain's oldest and most respected institutions that has the worthy goal of bringing scientific work to the public. Every year in Britain the

Christmas Lectures, founded by Michael Faraday in 1825, are aired on TV, given by eminent scientists who share their work with the public. I had asked Vivien to introduce me at the Royal Institution, and during that introduction she shared with the audience that when she was a child she had been made to stand in the corner by her mathematics teacher, Mrs. Glass, for not being able to recite her seven times table. She then went on to make the audience laugh by telling them that when she shared this story on the BBC, six women called the BBC action line and asked—was it Mrs. Glass of Boxbury School? Vivien shared that indeed it was.

Fortunately, such harsh teaching practices are almost extinct, and I continue to be inspired by the devotion and commitment of most mathematics teachers I work with. But we know that negative and damaging messages are still handed out to students every day—messages that are not intended to harm, but that we know can start students on a damaging and lasting mathematics pathway. Such pathways can be reversed, at any time, but for many they are not, and they affect every future experience of mathematics that people have. Changing the messages that students receive about mathematics is not, sadly, as simple as just changing the words teachers and parents use, although words are very important. Students also receive and absorb many indirect messages about mathematics through many aspects of math teaching, such as the questions they work on in math class, the feedback they get, the ways they are grouped, and other aspects of mathematics teaching and help that we will consider together in this book.

Vivien is convinced that she has a brain condition, called dyscalculia, that stops her from being successful with math. But we now know that one experience or message can change everything for students (Cohen & Garcia, 2014), and it seems very likely that Vivien's negative math experiences were at the root of the math anxiety she now struggles with daily. Vivien—fortunately for the many who have benefited from her work—was able to be successful despite her mathematics experiences, even in a quantitative field, but most people are not so fortunate, and the early damaging experiences they have with mathematics close doors for them for the rest of their lives.

> Taking math courses matters. Research studies have established that the more math classes students take, the higher their earnings ten years later, with advanced math courses predicting an increase in salary as high as 19.5% ten years after high school (Rose & Betts, 2004). Research has also found that students who take advanced math classes learn ways of working and thinking—especially learning to reason and be logical—that make them more productive in their jobs. Students taking advanced math learn how to approach mathematical situations so that once they are employed, they are promoted to more demanding and more highly paid positions than those who did not take mathematics to advanced levels (Rose & Betts, 2004). In my study of schools in England, I found that students were advanced in their jobs, ending up with higher-paid employment, because they learned mathematics through a project-based approach in high school that I will discuss in later chapters (Boaler, 2005; Boaler & Selling, 2017).

We all know that math trauma exists and is debilitating for people; numerous books have been devoted to the subject of math anxiety and ways to help people overcome it (Tobias, 1978). It would be hard to overstate the number of people who walk on our planet who have been harmed by bad math teaching, but the negative ideas that prevail about math do not come only from harmful teaching practices. They come from one idea, which is very strong, permeates many societies, and is at the root of math failure and underachievement: that only some people can be good at math. That single belief—that math is a “gift” that some people have and others don't—is responsible for much of the widespread math failure in the world.

So where does that damaging idea—an idea that notably is absent in countries such as China and Japan that top the world in math achievement—come from? When my two daughters were younger, I had the dubious pleasure of catching regular glimpses of “tweenie” TV programs. This was very enlightening—and worrying—as a day did not seem to go by without mathematics being portrayed in a negative light. Math is conveyed as a really hard subject that is uninteresting, inaccessible, and only for “nerds”; it is not for cool, engaging people, and it is not for girls. It is no wonder that so many children in schools disengage from math and believe they cannot do well.

The idea that only some people can do math is embedded deep in the American and British psyche. Math is special in this way, and people have ideas about math that they don't have about any other subject. Many people will say that math is different because it is a subject of right and wrong answers, but this is incorrect, and part of the change we need to see in mathematics is acknowledgment of the creative and interpretive nature of mathematics. Mathematics is a very broad and multidimensional subject that requires reasoning, creativity, connection making, and interpretation of methods; it is a set of ideas that helps illuminate the world; and it is constantly changing. Math problems should encourage and acknowledge the different ways in which people see mathematics and the different pathways they take to solve problems. When these changes happen, students engage with math more deeply and well.

Another misconception about mathematics that is pervasive and damaging—and wrong—is the idea that people who can do math are the smartest or cleverest people. This makes math failure particularly crushing for students, as they interpret it as meaning that they are not smart. We need to dispel this myth. The combined weight of all the different wrong ideas about math that prevail in society is devastating for many children—they believe that mathematics ability is a sign of intelligence and that math is a gift, and if they don't have that gift then they are not only bad at math but they are unintelligent and unlikely to ever do well in life.

As I write this book, it is clear that the world is developing a great appreciation for and understanding of the importance of mindset. Carol Dweck's book has been translated into more than 20 languages (Dweck, 2006b), and interest in the impact of mindset continues to grow. But a dangerous misconception exists about mindset, which is the idea that you can instill a growth mindset in students by sharing positive messages while still teaching in a fixed way—with math questions that have one answer and one valued method. Teachers are critical in changing students' ideas about mathematics, and how they can do so is the subject of this book. The ideas I share with teachers and parents and set out in this book include paying attention to the math questions and tasks that students work on, the ways teachers and parents encourage or grade students, the forms of grouping used in classrooms, the ways mistakes are dealt with, the norms developed in classrooms, the math messages we can give to students, and the strategies students learn for approaching math—in essence, the whole of the mathematics teaching and learning experience. I am excited to share this new knowledge with you, and I am confident that it will help you and anyone you work with on mathematics.

In the next chapter I will set out some of the fascinating and important ideas that have emerged from research in recent years; in the eight chapters that follow, I will focus on the strategies that can be used in math classrooms and homes to implement the ideas I share in these first two chapters. I strongly recommend reading all of the chapters, skipping to the strategies will not be helpful if the underlying ideas are not well understood.

Ever since we started youcubed.org and I released different online classes, I have received thousands of letters, emails, and other messages from people sharing with me the changes they had made in their classrooms and homes and the impact this has had on the students (see also Boaler, 2019). Relatively small changes in teaching and parenting can change students’ mathematical pathways, because the new knowledge we have on the brain, mindset, and mathematics learning is truly revolutionary. This book is about the creation of *mathematical mindsets* through a new kind of teaching and parenting that is, at its heart, about growth, innovation, creativity, and the fulfillment of mathematics potential. Thank you for joining me, and for taking steps on a pathway that could change your and your students’ relationships with mathematics forever.

CHAPTER 1
The Brain and Mathematics Learning

In the last few decades we have seen the emergence of technologies that have given researchers new access into the workings of the mind and brain. Now scientists can study children and adults working on math and watch their brain activity; they can look at brain growth and brain degeneration, and they can see the impact of different emotional conditions upon brain activity. One area that has emerged in recent years and stunned scientists concerns “brain plasticity.” It used to be believed that the brains people were born with couldn't really be changed, but this idea has now been resoundingly disproved. Study after study has shown the incredible capacity of brains to grow and change within a really short period (Abiola & Dhindsa, 2011; Maguire, Woollett, & Spiers, 2006; Woollett & Maguire, 2011).

When we learn a new idea, one of three things happens in the brain (see Figure 1.1). The first possibility is that you start a new brain pathway. The more deeply you learn, the stronger the pathway becomes. The second possibility is that you strengthen a pathway you already had, and the third possibility is that you make connections between pathways. This brain development is taking place all the time, and the pathways you build, strengthen, or connect were not in your brain at birth; they are created by your learning experiences.

I wish all students knew this—when you are teaching them math, you are changing their brains! Neuroscientist Norman Doidge (2007) likes to share with his audiences that every day you wake up, your brain is different from the day before—that is the extent of brain growth and change that occur every day. If you learn something deeply, you form lasting brain pathways that you can revisit and use, but if you visit an idea only once or in a superficial way, the pathway can "wash away" like a path made in the sand. These brain connections form when learning takes place, but learning does not occur only in classrooms or when reading books; as we all know, we are forming brain connections when we have conversations, play games, or build with toys, and in the course of many, many other experiences.

The first research on what became known as neuroplasticity, which shocked the scientific world, came from studies of "Black Cab" drivers in London. I am from England, and I have traveled in taxicabs in London many times. I still have fond memories of the exciting day trips my family and I took to London when I was a child, from our home a few hours away. As an adult I studied and worked at King's College, London University, and had many more opportunities for trips around London in taxis. A number of different taxis work in the London area, but the queen bee of taxis in London is the Black Cab (see Figure 1.2).