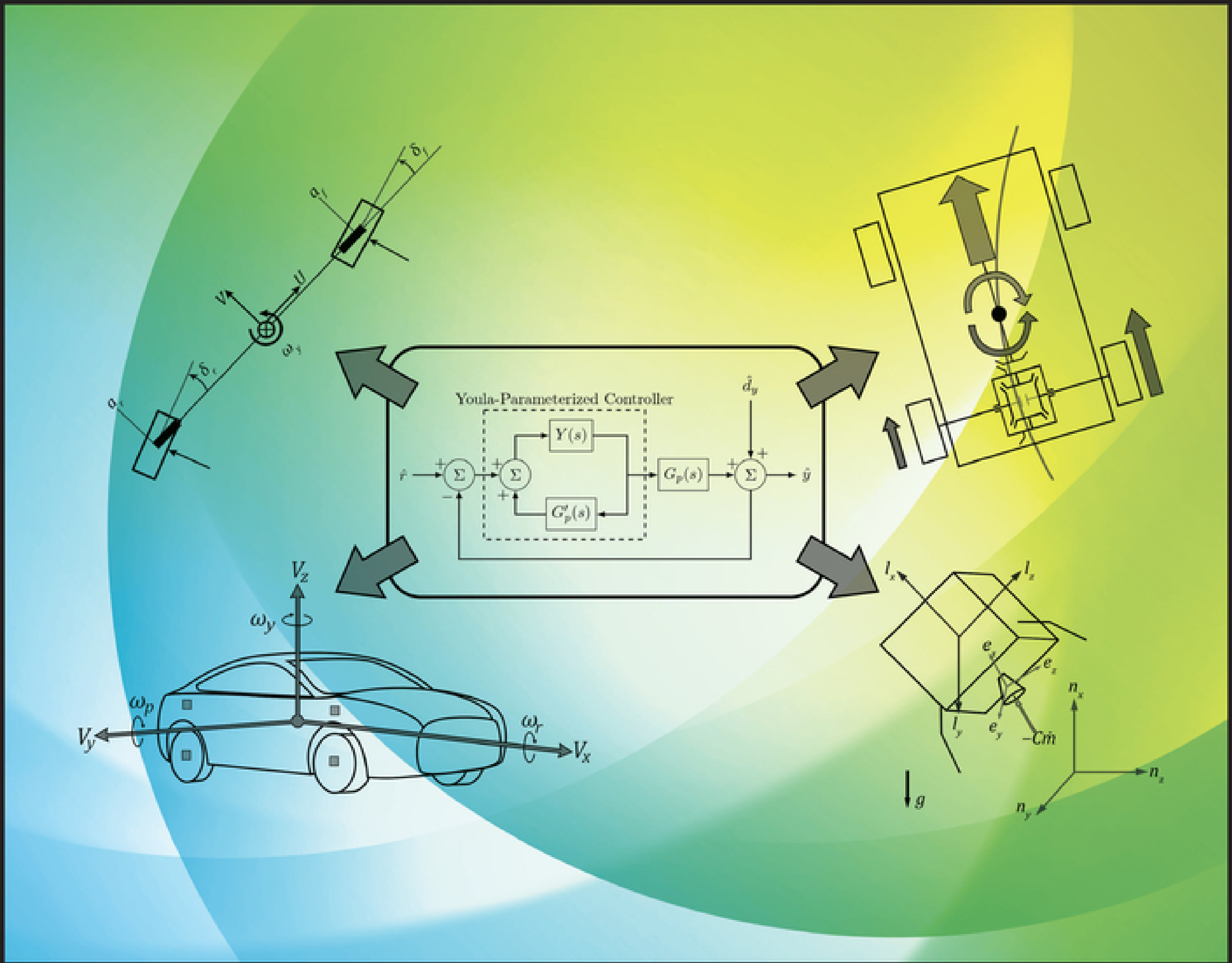

Robust Control

Youla Parameterization Approach

Farhad Assadian and Kevin R. Mallon

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To Professor John Brewer

Contents

Preface

In the past several decades, there have been many published control system design books in classical (PID), modern (state-space), and neoclassical and optimal robust control. Then, why is it necessary to publish yet another control system design book?

To clearly answer this question, I need to say a few words about my own background. Before becoming a Professor in System Dynamics and Control, I have been a control engineer with both energy and automotive industries for over 30 years. In these industries, feedback is still a novelty and majority of feedback control technique is around designing PID controllers. Control theory has drastically advanced and yet with these advancements, the gap between industrial application of controls and theoretical controls has been increased. Majority of the current control publications are very much math heavy and make the assumptions that all practitioners and students have the right background to fully appreciate these theoretical development. Most importantly, in all these developments, the beauty of control design at times is not clearly addressed and hence, large number of the students and the practitioners are not still have a good understanding of why feedback control is needed.

In this book, we try to close this gap by illustrating that feedback control could be elegantly designed in the frequency domain using Youla parameterization approach. It is the system bandwidth which elegantly defines a trade-off between performance and robustness. The father of quantitative feedback theory (QFT), Issac Horowitz, said that the bandwidth is like an open window during a hot summer night. If the window is kept widely open, then one will let the flies in and if the window is kept shut, then one will limit the summer night breeze. This simple and elegant metaphor defines the trade-off or balance between performance (speed) and noise rejection capability in feedback control design.

Youla parameterization technique is not new; the technique was first developed in the 1970s [1] and addressed in several other existing textbooks, for instance [2, 3]. However, in this book, we will provide deeper insights with many practical applications in utilizing this technique in both single input single output (SISO) and multiple input multiple output (MIMO) frequency domain feedback control design. In addition, an estimation technique using Youla parameterization and controller output observer both for SISO and MIMO plants, for the first time, is introduced. Although, the emphasis of these two parts is on frequency domain, we will discuss optimal robust control approaches such as H_2/LQG and H_∞ using state-space approaches in Chapter 5 of this book. We will provide comparative results through practical examples between the optimal robust control methods and the Youla parameterization approach.

We have to apologize for the level of mathematical rigor provided in this book. It goes without saying that control system design is a subject which requires mastery of basic mathematical concepts. We have tried hard to avoid theorem and proof format when introducing mathematical concepts and provided the level mathematics to give the readers the necessary background for understanding and appreciating the control theoretical concepts discussed in this book.

This book is divided into two parts: Part I covers SISO control and estimation design using the Youla parameterization approach, while Part II covers MIMO control and estimation design using Youla parameterization approach.

Part I of this book includes eight chapters:

In Chapter 1, a review of the Laplace transform is given. In this chapter, we tried to avoid many details in introducing the Laplace transform and concentrated on providing the properties of the Laplace transform. It is these properties will be utilized throughout this book for developing the main concepts.

In Chapter 2, the response of linear time-invariant (LTI) dynamic systems is provided. A review of stability of LTI dynamic systems is given and frequency response including Bode and Nyquist diagrams are discussed.

In Chapter 3, we provide a discussion of feedback principals, a review of well-posedness and internal stability is given, and the role of Youla parameter and interpolation conditions in assuring internal stability is provided. Complementary sensitivity transfer function T and sensitivity transfer function S are derived, and modeling uncertainty is discussed.

In Chapter 4, we provide the fundamentals of feedback design using Youla parameterization. We discuss algebraic and analytical constraints in feedback design and we reveal how Youla parameter could be utilized in designing a feedback control by considering the constraints and trade-offs in SISO control design.

In Chapter 5, a review of signal and system size (norms) is provided. linear fractional transformation (LFT) platform and its use in control design are discussed including uncertainty modeling with the use of this platform. A brief review of state-space method including concepts such as controllability and observability are discussed.

In Chapter 6, we discuss the Mixed Sensitivity Method and H_∞ method for SISO robust control design. In this chapter, we provide insights to the frequency domain solution of H_∞ control and selection of the weighting filters in shaping complementary sensitivity, sensitivity, and Youla transfer functions.

In Chapter 7, we discuss how Youla parameterization could be utilized in deriving a state and input estimation method. In this chapter, an introduction to Luenberger observer and Kalman filtering is provided including a discussion about the Controlled Output Observer approach. A step-by-step transformation approach from Luenberger observer to Youla parameterization estimation using the Controller Output Observer framework is developed.

In Chapter 8, we present two automotive examples, one for control example and another for estimation. The control example demonstrates yaw stability control with an active differential. The estimation example consists of vehicle yaw rate and sideslip estimation using the Youla controller output observer (YCOO) method. In the control example, we reveal the differences between using SISO robust control design using the standard Matlab toolbox and Youla parameterization hand computation technique. In the estimation example, we compare the result of YCOO to both linear and nonlinear Kalman filter method.

Part II of this book includes nine chapters.

In Chapter 9, we provide an overview of different multivariable feedback control methods and the importance of Youla parameterization approach in designing multivariable feedback control system. In the same chapter, we introduce multivariable transfer function matrix, Schur complement, and Rosenbrock's system matrix including its application for gaining insight into poles and zeros of a transfer function matrix.

In Chapter 10, Matrix Fractional Description, unimodular matrices, and the concept of polynomial matrices' equivalency with several insightful examples are introduced. A discussion on Smith canonical form and Smith-McMillan form of a transfer function matrix including several examples is provided. The idea of coprimeness and several approaches for transforming a transfer function matrix to its equivalent stat-space form are discussed.

In Chapter 11, we discuss the eigenvalue problem and matrix diagonalization using eigenvectors and eigenvalues. The diagonalization issues using eigenvalues and eigenvectors are discussed, and the use of singular value decomposition as a general method for transfer function matrices diagonalization is reviewed.

In Chapter 12, we discuss MIMO feedback principals. We introduce MIMO return ratio and return difference, complementary sensitivity and sensitivity transfer function matrices, and Youla transfer function matrix. Concepts such as Nyquist stability, well-posedness, and internal stability for MIMO feedback systems are discussed. Some of the properties of singular value decomposition are reviewed and their implications on the limit of MIMO feedback performance are revealed.

In Chapter 13, we introduce the concept of MIMO feedback control design using Youla parameterization. We illustrate the simplicity of this technique by showing that for low-order transfer function matrices, the MIMO control design could be easily performed by hand.

In Chapter 14, a review of vector signal and MIMO system size (norms) is provided. Linear fractional transformation (LFT) platform is revisited, and the uncertainty modeling for MIMO feedback systems with the use of this platform is discussed.

In Chapter 15, we utilize the norms discussed in the previous chapter to derive MIMO controllers. We discuss optimal control methods. We review the derivation of linear quadratic regulator (LQR) and linear quadratic Gaussian (LQG) and discuss the equivalency between H_2 and LQG optimal control. We provide the state-space solutions to H_2 and H_∞ optimal robust control.

In Chapter 16, we revisit the system from Chapter 7 and discuss how Youla parameterization could be utilized in deriving state and input estimation method for MIMO systems. A step-by-step design approach for MIMO Youla parameterization estimation using the Controller Output Observer framework is developed.

In Chapter 17, we present three MIMO control examples and one MIMO estimation example. The control examples include active suspension control, advanced engine speed control for hybrid vehicles, and robust control for the powered descent of a multibody lunar landing system. The estimation example revisits the vehicle yaw rate and sideslip estimation example of Chapter 8, transforming the SISO problem into a MIMO one through the introduction of rear wheel steering. In the control examples, we reveal the differences between using MIMO robust control design using the standard Matlab toolbox and MIMO YCOO hand computation technique. In the estimation example, we compare the result of MIMO YCOO with both linear and nonlinear Kalman filter method.

University of California, Davis
July, 2017

Farhad Assadian
Kevin R. Mallon

Acknowledgments

My sincere thanks and appreciation to Professor John Brewer who introduced me to the concept of Youla parameterization in 1990s. Unfortunately, Professor Brewer passed way on 17 October 2003 at the age of 65. His enthusiasms and passion for teaching control system design to his students was exemplary. He shall always be remembered as one of the best teachers by his students.

F.A.

Introduction

"It seems that without electronic, controls nothing works."

I.1 Why Feedback Control?

Control systems are increasingly becoming an integral part of every piece of equipment that we humans use. Without control systems our lives would drastically change. From aircrafts to household appliances, control systems play a vital role in the operation of these machines. In the automotive domain, nowadays, the majority of vehicle attributes are software and control-system based. It is therefore important for everyone, especially machine operators and technical managers, to have a basic understanding of control system operation and their impact. It is also becoming significantly important that all engineers, from every specialty, should be required to take at least one good control system design class throughout their academic education.

When it comes to control system design, our emphasis in this book is on feedback control system design, particularly model-based feedback control design. We will describe what we mean by model based in the next few paragraphs. But, first let's clarify the need for feedback controls. It is interesting, especially in industry, that there is still not a good understanding about why feedback control is necessary.

Let's take a look at Figure I.1a, where a person without eyesight is trying to pick up a book, which is located on a table. From this person's point of view, there is an uncertainty about the exact position of this book. To reduce this uncertainty, the person would have to find the book by examining different locations on the table with their hands. Hence, it normally takes more effort or energy to overcome the uncertainty and locate this book. Now, imagine a person with eyesight, as in Figure I.1b. For this person, the exact location of the book is found by making a measurement using the person's eyesight and feeding back this information to the brain. Therefore, uncertainty about the position of the book is reduced by this feedback measurement. Interestingly, the effort and energy required to locate the book is also reduced when compared to the person without eyesight. We can say then that the fundamental reason for feedback in control system design is simply to reduce **uncertainty**. We will see that this uncertainty exist in many sources including the act of the measurement itself. Amazingly, the same uncertainty that gives rise to the use of feedback control itself and can result in its demise!

Another important takeaway from this discussion is that reducing uncertainty requires effort and energy and the source of this effort and energy is the actuator. We will relate the actuator effort and energy to system robustness when discussing robust control development in the following chapters.

There are several different methods that could be utilized in designing feedback controllers. Our intent in this book is to illustrate that the use of Youla parameterization technique is one of the most straightforward methods in designing model-based feedback control system, especially for single-input, single-output (SISO) control system design. We will show that control system design becomes procedural with minimal calibration effort when implementing these controllers on the actual systems. Reducing controller calibration is significantly important to industry, as reducing calibration effort directly reduces development costs. We will argue that the elegance of Youla parameterization is its simplicity and in reducing controller calibration overhead.

Imagine designing a control system for a very complex system which could be mathematically described by nonlinear partial differential equations (PDEs). What are the choices to design a controller for this complex system?

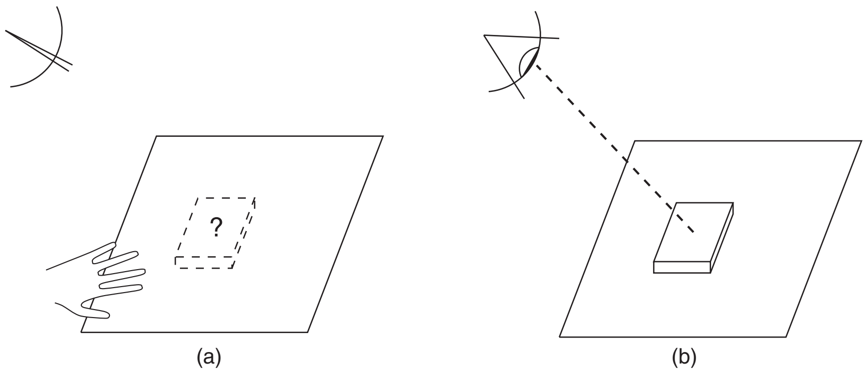

Figure I.1 There is more uncertainty about the book's location with (a) closed eyes, compared to (b) open eyes.

- **Direct Control of Nonlinear PDEs**: As far as the authors of this book are aware, there are no methods which directly address the control design of nonlinear PDEs, especially the open loop unstable kinds. In addition, if there were control methodologies to address nonlinear PDEs directly, quantifying model errors and uncertainties would have been an extremely difficult task. The quote by George Box "Essentially, all models are wrong, but some are useful" should be kept in mind when complex systems are modeled using nonlinear PDEs.
- **Indirect Control by Model Reduction of the Nonlinear PDEs to linear time-invariant (LTI) PDEs**: There are methods which address the control of LTI PDEs, such as boundary control of LTI PDEs using backstepping control method [4], or robust control of infinite dimensional LTI using H_∞ control [5].

It should be noted that at times, linearization of particular nonlinear systems results in loss of controllability. We will not address these systems in this book.

We will use model reduction techniques tp go one step further than indirect control by reducing the original complex model to LTI ordinary differential equations (ODEs). Here stands the trade-off between complexity and uncertainty: As the complex model is reduced to a simpler model, naturally the complexity is reduced. However, uncertainty is increased due to loss of information. Therefore, the entire idea of robust control is to quantify these uncertainties in the control design phase.

There are generally two objectives in designing a feedback control system: performance and robustness. "Performance" indicates the ability of the control system to respond quickly and accurately under ideal conditions. "Robustness" indicates the ability of the control system to respond accurately in the face of disturbances, noise, or other sources of uncertainty. Generally (but not always), there is a trade-off between robustness and performance: Speeding up the response requires sacrificing robustness, while increasing robustness requires sacrificing the nominal performance. This concept – the trade-off between robustness and performance – is illustrated in Figure I.2.

More specifically, the conflicting measures are:

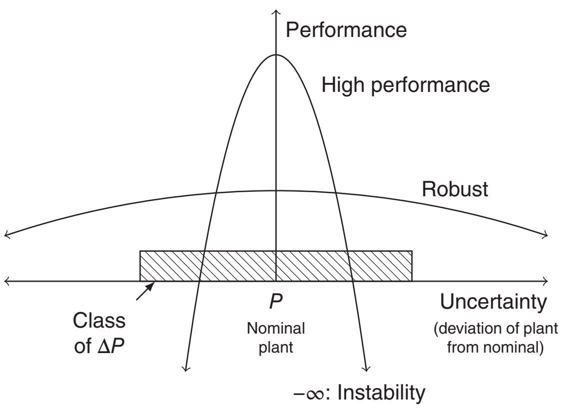

Figure I.2 Robustness and performance.

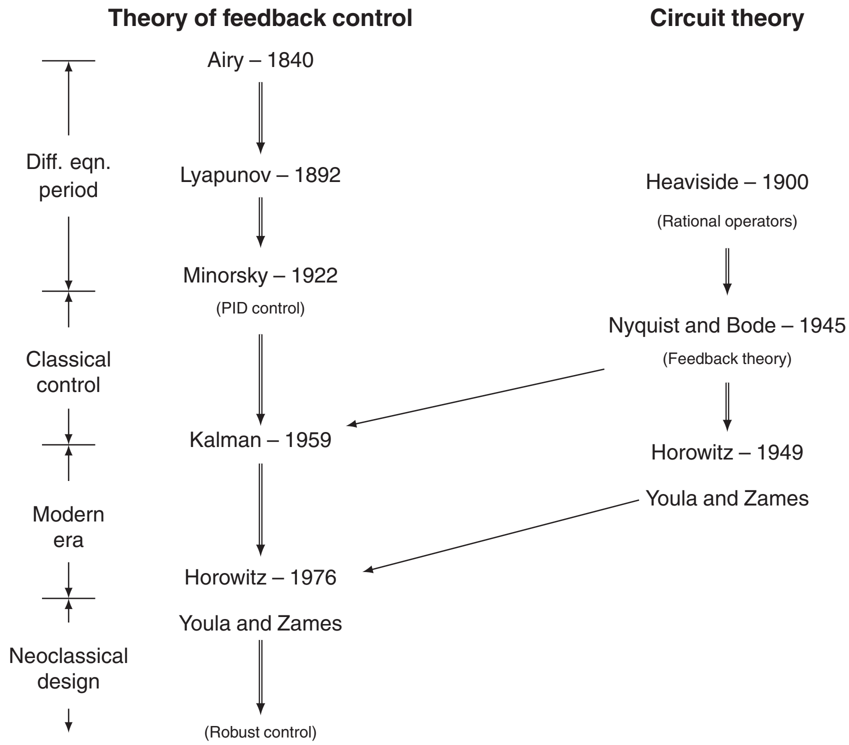

Figure I.3 Brief history of feedback control.

- Stability (is the controller robustly stable?)
- Command tracking and disturbance rejection
- Sensor noise rejection
- Actuator effort and saturation.

The history of control systems theory and the path to robust control is briefly outlined in Figure I.3.

I.2 Why Youla Parameterization?

In the prior section, we briefly discussed the benefits of feedback control and various options for model-based control development. In the next few paragraph, we will elaborate more on the benefits of feedback control and the utility of Youla parameterization in feedback control design. This section presumes some knowledge of the Laplace transform, transfer functions, and block diagram algebra.

When dealing with model-based controls, the starting point is a mathematical description of the system. Consider the simple system shown in Figure I.4, where P is the plant model and Q is the control.,

In order to enforce perfect target following, where $y(s) = r(s)$ and s is the Laplace operator, we have to select the controller $Q(s) = P^{-1}(s)$. This very simple example illustrates that plan invertability is very important in control system design. We will elaborate on this point when designing feedback controllers using Youla parameterization method. Let's assume for a moment that this plant is invertible. What happens when we implement this simple controller on an actual plant?

In order to capture the reality a bit better, we added a disturbance signal at the output of the plant, as illustrated in Figure I.5. This disturbance signal could, for example, model a wind gust acting on a vehicle while driving. Based on the

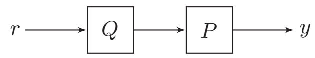

Figure I.4 System model.

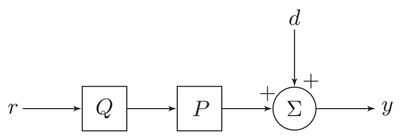

Figure I.5 System model with output disturbance.

previous feedforward controller, we can write the following equations.

$$y(s) = d(s) + P(s)Q(s)r(s) \tag{1}$$

but

$$Q(s) = P^{-1}(s) \tag{2}$$

so therefore,

$$y(s) = d(s) + r(s) \tag{3}$$

So, we are here stating the obvious point that if there is a disturbance or a source of uncertainty, the feedforward controller is not capable of coping with this. As a result, the target following in a feedforward loop deteriorates when compared to the situation when perfect knowledge of the plant was available. The output y no longer follows the commanded input r.

Let's now add a feedback to Figure I.5 as illustrated in Figure I.6.

The output $y(s)$ can be written as a function of the target signal, $r(s)$, and disturbance signal, $d(s)$, as follows.

$$y(s) = \frac{P(s)Q(s)}{1 + P(s)Q(s)} r(s) + \frac{1}{1 + P(s)Q(s)} d(s) \tag{4}$$

Two important points should be evident from this equation. First, we cannot simply set $Q(s) = P^{-1}(s)$, as we did in the feedforward case, if we are designing for good target following and disturbance rejection. Second, if we set $P(s)Q(s) \gg 1$, then we will recover the target following of the feedforward controller by rejecting or minimizing the disturbance or uncertainty effect. There are other possible disturbances and uncertainties to consider, but the value of feedback controls in the presence of uncertainties has been made clear.

In the next few paragraphs, we provide a very basic introduction to Youla parameterization. The feedback control loop is revisited in Figure I.7. However, this time, we replaced the dynamic model of the plant $P(s)$ with a generalized symbol $G_p(s)$ and replaced the feedforward controller, $Q(s)$ with a generalized symbol for feedback controller $G_c(s)$.

The closed-loop transfer function or the complementary sensitivity transfer function, $T(s)$, can be written as a function of $G_p(s)$ and $G_c(s)$ as follows:

$$T(s) = \frac{G_p(s)G_c(s)}{1 + G_p(s)G_c(s)} \tag{5}$$

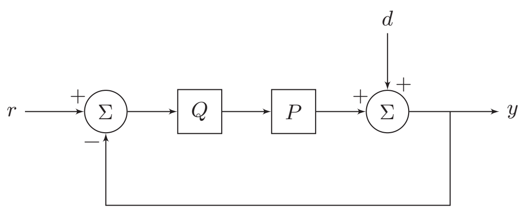

Figure I.6 System model.

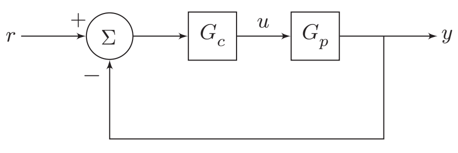

Figure I.7 Closed-loop feedback system.

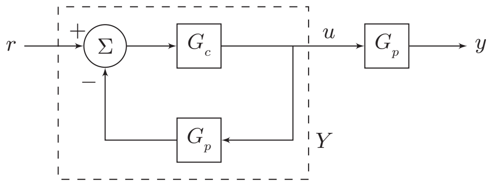

Figure I.8 Block diagram manipulation of system.

Equivalently, we define the Youla transfer function $Y(s)$ to be the transfer function from the input target, $r(s)$ to the actuator input, $u(s)$,

$$Y(s) = \frac{G_c(s)}{1 + G_p(s)G_c(s)} \tag{6}$$

At the first glance, defining the Youla transfer function seems to be trivial and not useful. However, as we discuss below and in the following chapters, this simple mapping will have important consequences both in SISO and multi-input, multi-output (MIMO) control and estimation design.

The feedback control loop presented in Figure I.8 can be easily manipulated to be drawn as a function of $Y(s)$ as follows:

It can be seen, from this manipulation, that the complementary transfer function $T(s)$ can be written as a function of $Y(s)$ as follows:

$$T(s) = Y(s)G_p(s) \tag{7}$$

The transfer function $Y(s)$ is simply equal to the original feedforward controller, $Q(s)$. In Eq. (I.7), the linearity in the free stable transfer function $Y(s)$ is what gives its practicality in hand computing or optimizing for a SISO or MIMO controller. Given a particular plant transfer function $G_p(s)$, one typically fixes a preferred closed-loop transfer function $T(s)$ as described in the following chapters, and then computes a proper and stable $Y(s)$ using (I.7).

From Eq. (I.6), the feedback controller, G_c, can be computed as follows:

$$G_c(s) = \frac{Y(s)}{1 - G_p(s)Y(s)} \tag{8}$$

The details for addressing the feedback loop internal stability are addressed in the following chapters.

Before finishing this introduction, we illustrate the utility of using $Y(s)$ and the Youla parameterization method through a very simple classical example.

Example I.1 Consider a plant with an unstable pole

$$G_p(s) = \frac{1}{s-1} \tag{9}$$

and assume the controller

$$G_c(s) = K \tag{10}$$

where K is a simple gain (proportional type) controller, as shown in Figure I.9,

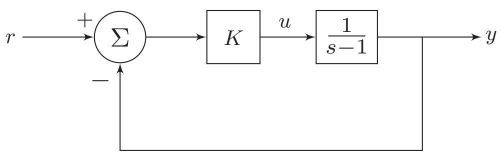

Figure I.9 Closed-loop feedback system.

The closed-loop transfer function, $T(s)$, can be written as

$$T(s) = \frac{K}{s-1+K} \tag{11}$$

The simple gain can be computed to stabilize the feedback control loop, by selecting the controller K to be $1 < K < \infty$. The root locus is illustrated in Figure I.10,

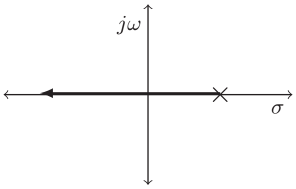

Figure I.10 Plant s-domain.

Stabilizing this plant is a simple and straightforward task. However, students who are given this simple problem, traditionally, are not given the proper reasons as of why,

1. The gain, K, could not be chosen freely without additional consequences, e.g. actuator saturation.
2. A simple gain can stabilize the plant but can not accomplish an appropriate target following, a performance measure.

For this simple example, let's compute the $Y(s)$ transfer function, which is equal to the ratio of the actuator input to the target input, $\frac{u(s)}{r(s)}$.

$$Y(s) = \frac{K(s-1)}{s-1+K} \tag{12}$$

It is a straightforward matter to obtain a Bode plot of this transfer function for various gains, K, as shown in Figure I.11,

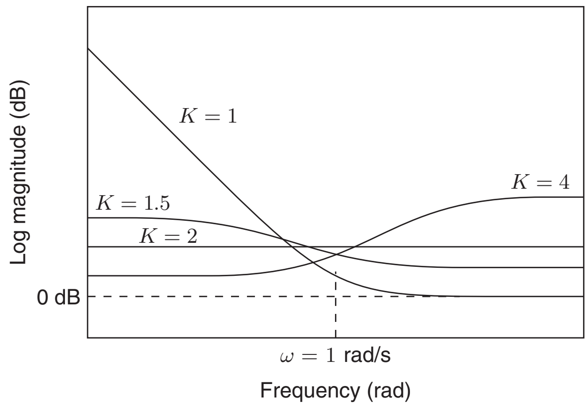

Figure I.11 Frequency response of $Y(s)$ for various gains K.

Now, it should be obvious, by looking at this plot, as we increase the gain, K, we are increasing the high-frequency demand on the actuator. More importantly, by increasing this gain, we are introducing noise to our system, as the $Y(s)$ transfer function is a high pass filter with a simple gain controller type, K.

Now, we use the Youla approach: we compute a stabilizing controller for this plant. According to this method, first, we need to select a closed-loop transfer function, $T(s)$. For this example, we select $T(s)$ to be equal to

$$T(s) = \frac{K}{\tau s + 1} \tag{13}$$

where K is some gain and is not equal to the previously computed controller gain. Preferably, we would like to have $K = 1$, to ensure good target following as the closed-loop transfer function becomes equal to 1 at steady state with this selection for K . But as we shall soon see, this won't be possible as internal stability will be compromised if we select $K = 1$, so we cannot simultaneously satisfy both target following and stability with this choice of controller, a simple gain.

Using Eq. (I.7), we can compute the $Y(s)$ transfer function as $Y(s) = \frac{T(s)}{G_p(s)}$,

$$Y(s) = \frac{K(s-1)}{\tau s + 1} \tag{14}$$

Now using Eq. (I.8), we can compute the controller, G_c,

$$G_c(s) = \frac{K(s-1)}{\tau s + (1-K)} \tag{15}$$

We are free to select K and τ controller coefficients. However, as we know from classical control, the controller G_c must not cancel the unstable pole of the plant as in this case, the internal stability of the feedback control loop will be compromised. Therefore, the coefficients K and τ must be selected as such to cancel $s - 1$ polynomial in the numerator of G_c. There is only one unique solution for the coefficients K and τ, to see this, let's rearrange Eq. (I.15) and write it as

$$G_c(s) = \frac{K}{\tau} \frac{s-1}{s + \frac{1-K}{\tau}} \tag{16}$$

By looking at this Equation, it is evident that we must select $\frac{1-K}{\tau} = -1$, to cancel the numerator polynomial, with this selection, the controller G_c can be written as $G_c = \frac{K}{\tau}$ where $K = \tau + 1$, hence,

$$G_c(s) = 1 + \frac{1}{\tau} \tag{17}$$

Again this controller is a pure gain controller, but this time, this controller is a function of τ, the first-order time constant. So, if we make τ small or the closed-loop time response fast, according to our selection of closed-loop transfer function, $T(s)$, we have a higher gain controller. In addition, when we computed this controller using the proposed method, we saw that we are unable to simultaneously satisfy both target following and stability requirements using a simple gain controller. It should be evident that the Youla method exposed several important feedback control concepts along the way, which were not obvious using the classical approach. We will discuss these points and more about the utility of the Youla approach in the next chapters.