Studies in Systems, Decision and Control 430

Lei Zou
Zidong Wang
Jinling Liang

Communication-Protocol-Based Filtering and Control of Networked Systems

Springer

Studies in Systems, Decision and Control

Volume 430

The series "Studies in Systems, Decision and Control" (SSDC) covers both new developments and advances, as well as the state of the art, in the various areas of broadly perceived systems, decision making and control–quickly, up to date and with a high quality. The intent is to cover the theory, applications, and perspectives on the state of the art and future developments relevant to systems, decision making, control, complex processes and related areas, as embedded in the fields of engineering, computer science, physics, economics, social and life sciences, as well as the paradigms and methodologies behind them. The series contains monographs, textbooks, lecture notes and edited volumes in systems, decision making and control spanning the areas of Cyber-Physical Systems, Autonomous Systems, Sensor Networks, Control Systems, Energy Systems, Automotive Systems, Biological Systems, Vehicular Networking and Connected Vehicles, Aerospace Systems, Automation, Manufacturing, Smart Grids, Nonlinear Systems, Power Systems, Robotics, Social Systems, Economic Systems and other. Of particular value to both the contributors and the readership are the short publication timeframe and the world-wide distribution and exposure which enable both a wide and rapid dissemination of research output.

Indexed by SCOPUS, DBLP, WTI Frankfurt eG, zbMATH, SCImago.

All books published in the series are submitted for consideration in Web of Science.

More information about this series at https://link.springer.com/bookseries/13304

Lei Zou · Zidong Wang · Jinling Liang

Communication-Protocol-Based Filtering and Control of Networked Systems

Springer

Lei Zou
College of Information Science
and Technology
Donghua University
Shanghai, China

Zidong Wang
Department of Computer Science
Brunel University London
London, Middlesex, UK

Jinling Liang
School of Mathematics
Southeast University
Nanjing, Jiangsu, China

ISSN 2198-4182 ISSN 2198-4190 (electronic)
Studies in Systems, Decision and Control
ISBN 978-3-030-97511-1 ISBN 978-3-030-97512-8 (eBook)
https://doi.org/10.1007/978-3-030-97512-8

MATLAB is a registered trademark of The MathWorks, Inc. See https://www.mathworks.com/trademarks for a list of additional trademarks.

Mathematics Subject Classification: 34H05, 37N35, 49J15, 58E25, 93Bxx, 93Exx

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

This book is dedicated to the Dream Dynasty, consisting of a group of diligent people who have enjoyed intensive research into analysis and synthesis of networked systems ...

Preface

In recent years, theoretical and practical research on networked systems (NSs) has attracted growing attention due primarily to their successful applications in an extensive range of fields. In the NS, signal transmissions among system components (e.g. controllers, sensors, filters and actuators) are implemented through a shared communication network, thereby improving the system reliability and reducing the maintenance cost. In such a network-based communication, signal transmissions might be failed due to data collisions if multiple network nodes try to access the channel simultaneously. Accordingly, various communication agreements are introduced to restrain the network accesses for all the network nodes. These agreements are referred to communication protocols. The utilization of communication protocols has a great impact on signal transmissions, which further complicates the dynamical behaviors of NSs. In this case, the traditional filtering and control schemes can no longer guarantee satisfactory performance for NSs subject to communication protocols. Consequently, it is of practical significance to establish new techniques for the filtering and control of NSs subject to communication protocols.

This book is concerned with the communication-protocol-based filtering and control problems for several classes of discrete-time NSs. Particularly, the communication protocols under consideration contain the Round-Robin (RR) protocol, Try-Once-Discard (TOD) protocol and Random Access (RA) protocol. The content of this book can be divided into two parts, where the first part (Chaps. 2–7) studies the filter design methodologies and the second part (Chaps. 8–10) investigates the controller design methodologies. These results provide a framework dealing with the controller/filter design, stability analysis and performance analysis for different NSs (e.g. nonlinear systems, time-varying systems, time-delay systems, complex networks and multi-agent systems) subject to different communication protocols. Some techniques including the backward Riccati difference equations, minimum mean square error estimation theory, mathematical induction method, linear matrix inequalities and optimization approaches are employed to handle the filtering and control issues with specific performance requirements.

The compendious framework and description of this book are given as follows. Chapter 1 introduces the recent advances on communication-protocol-based filtering and control problems of NSs and the outline of this book. Chapter 2 studies the ultimately bounded filtering problem for time-delay complex networks under the effects of RR protocol. Chapter 3 is concerned with the finite-horizon $\mathscr{H}_\infty$ filtering issue of nonlinear time-varying systems with high-rate communication network and the RA protocol scheduling. Chapter 4 extends the results in Chap. 3 to the finite-horizon $\mathscr{H}_\infty$ fault estimation issue subject to the RA protocol. Chapter 5 handles the communication-protocol-based set-membership filtering problem for time-varying systems with mixed-time-delays. Chapter 6 deals with the recursive filtering problem for time-varying systems subject to the RA protocol scheduling. Chapter 7 addresses the ultimately bounded filtering issue of communication-based train control systems subject to the p-persistent CSMA protocol. In Chap. 8, we consider the finite-horizon $\mathscr{H}_\infty$ control problem subject to the RA protocol. Chapter 9 discusses the ultimately bounded control problem of nonlinear systems subject to the TOD protocol scheduling and uniform quantization effects. In Chap. 10, the finite-horizon $\mathscr{H}_\infty$ consensus control problem is investigated for time-varying multi-agent systems subject to the RA protocol. Chapter 11 provides the conclusion of this book and suggests some possible research topics related to the results proposed in this book.

This book is a research monograph whose intended audience is graduate and postgraduate students as well as researchers. The background required of the reader is the knowledge of basic Lyapunov stability theory, basic stochastic process, basic optimal estimation theory and matrix theory.

Shanghai, China — Lei Zou
London, UK — Zidong Wang
Nanjing, China — Jinling Liang

Acknowledgements

We would like to express our sincere thanks to those who have been devoted to the research in this book. Particular thanks are given to Professor Huijun Gao from Harbin Institute of Technology of China, Professor Donghua Zhou from Shandong University of Science and Technology of China, Professor Xiaohui Liu from Brunel University London of the U.K. and Professor Qing-Long Han from Swinburne University of Technology of Australia. We also extend our thanks to many colleagues who have offered support and encouragement throughout this research effort. In particular, we would like to acknowledge the contributions and friendly support from Guoliang Wei, Bo Shen, Hongli Dong, Xiao He, Lifeng Ma, Derui Ding, Jun Hu, Yurong Liu, Liang Hu, Yang Liu, Nianyin Zeng, Sunjie Zhang, Qinyuan Liu, Fan Wang, Wenying Xu and Hang Geng. Last but not the least, we are especially grateful to our families for their never-ending understanding, unfailing encouragement and never-ending support when it was most required.

The writing of this book was supported in part by the National Natural Science Foundation of China under Grants 61703245, 61873148, 61933007, 61873058 and 61673141, the China Postdoctoral Science Foundation under Grant 2018T110702, the European Union's Horizon 2020 Research and Innovation Programme under Grant 820776 (INTEGRADDE), the Royal Society of the UK and the Alexander von Humboldt Foundation of Germany. The support of these organizations is much acknowledged.

Contents

List of Notations

$\mathbb{R}^n$	The n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	The set of all $n \times m$ real matrices
$\mathbb{N}^+$	The set of non-negative integers
$\mathbb{N}^-$	The set of negative integers
$\mathbb{N}$	The set of integers
$\|A\|$	The spectral norm of the matrix A
$\|A\|_F$	The Frobenius norm of the matrix A
A^T	The transpose of the matrix A
A^{-1}	The inverse of the matrix A
$A^\dagger$	The Moore–Penrose pseudo inverse of the matrix A
I	An identity matrix of compatible dimension
0	A zero matrix of compatible dimension
$\mathbf{1}_n$	An n dimensional column vector with all ones
$\text{Prob}(\cdot)$	The occurrence probability of the event "·"
$\mathbb{E}\{x\}$	The expectation of the stochastic variable x
$\mathbb{E}\{x\|y\}$	The expectation of the stochastic variable x conditional on y
$\lambda_{\max}\{A\}$	The largest eigenvalue of a square matrix A
$\lambda_{\min}\{A\}$	The smallest eigenvalue of a square matrix A
$\text{diag}\{\dots\}$	The block-diagonal matrix
$l_2[0, \infty)$	The space of square summable sequences
$l_2([0, N], \mathbb{R}^n)$	The space of the square-summable n-dimensional vector functions over the interval $[0, N]$
$\text{tr}\{A\}$	The trace of a matrix A
$X > Y$	The $X - Y$ is positive definite, where X and Y are real symmetric matrices
$X \geq Y$	The $X - Y$ is positive semi-definite, where X and Y are real symmetric matrices
$\|x\|$	The Euclidean norm of a vector x
$\boldsymbol{\delta}(a)$	The Kronecker delta function that equals 1 if $a = 0$ and equals 0 otherwise

mod (a, b)	The unique non-negative remainder on division of the integer a by the positive integer b
$\lfloor a \rfloor$	The largest integer not greater than a
$\otimes$	The Kronecker product of matrices
$\circ$	The Hadamard product of matrices
"$*$"	An ellipsis for terms induced by symmetry in symmetric block matrices

Chapter 1
Introduction

In recent years, the communication-protocol-based synthesis and analysis issues have gained substantial research interest owing mainly to their significance in networked systems. The core characteristic of networked systems is the utilization of network-based communication among different system components. In such network-based communication, signal transmissions via the communication channel are implemented according to certain agreements, namely, the communication protocols. These communication protocols would give rise to certain complex yet important impact on the system performance (e.g. stability). The necessity of designing the filtering/control strategies arises naturally in situations where the effects induced by communication protocols are inevitable due to the network-based communication. As such, the communication-protocol-based filtering and control problems for different networked systems serve as very interesting, imperative yet challenging topics.

1.1 Research Background

The research on networked systems has attracted a vast amount of interest in the past several decades [1–10]. The most typical characteristic of networked systems is the utilization of network-based communication technology, under which the data exchange among different system components (e.g. controllers, sensors, filters and actuators) is implemented via the shared communication network. Different from the traditional point-to-point communication technology, such network-based communication technology possesses numerous advantages (e.g. simple installation, reduced hardwire, low cost and high reliability) [11]. Accordingly, networked systems have achieved successful applications in an extensive range of fields such as smart vehicles,

L. Zou et al., *Communication-Protocol-Based Filtering and Control of Networked Systems*, Studies in Systems, Decision and Control 430,
https://doi.org/10.1007/978-3-030-97512-8_1

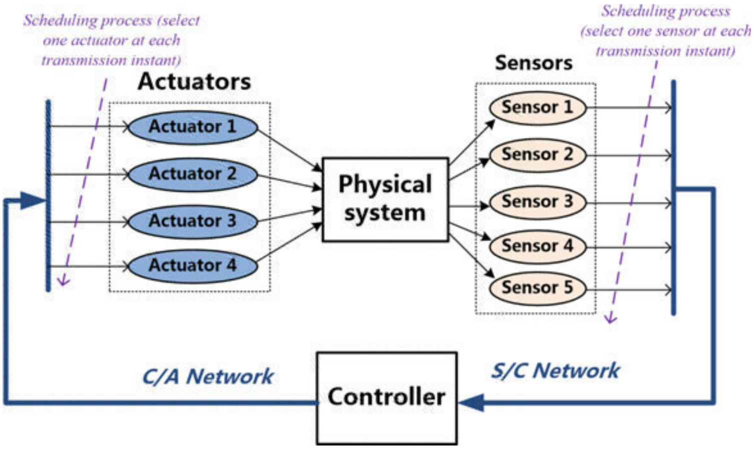

Fig. 1.1 A typical networked system with two communication networks

smart grids, environmental monitoring, industrial automation and intelligent transportation. By now, networked systems have become the focus of intensive research in signal processing and control communities. A typical networked system is shown in Fig. 1.1, where the signal transmission between the controller and plant is implemented through two communication channels: the sensor-to-controller (S/C) channel and the controller-to-actuator (C/A) channel. Nevertheless, the utilization of network-based communication would also give rise to certain special phenomena that might degrade the system performance. Such network-induced phenomena include, but are not limited to, transmission delays [12–14], packet dropouts [15–17], data packet disorder [18–20], signal quantization [21–25], communication protocol scheduling [26–28] and channel fading effects [29, 30]. As such, the synthesis and analysis issues of networked systems subject to various network-induced phenomena have attracted an ever-increasing research interest; see, for example, [31–38] and the references therein. For instance, in [39], the finite-time tracking control problem has been investigated for a type of networked system with quantized inputs. The distributed and centralized estimation issues have been studied in [40] for networked systems with missing measurements.

Among various network-induced phenomena, communication protocol scheduling is one of the most typical phenomena and would lead to a particularly significant impact on the performance of networked systems. Such a phenomenon is mainly caused by the limited communication capability of the underlying channel in the networked system. More specifically, in a typical network-based communication process, data transmissions would unavoidably suffer from data collisions in case of simultaneous multiple accesses to the shared communication network (as shown in Fig. 1.2). In order to prevent such data collisions from occurring, an effective way is to restrain the network accesses according to the so-called communication protocols by guaranteeing that only one network node is permitted to transmit its data at each

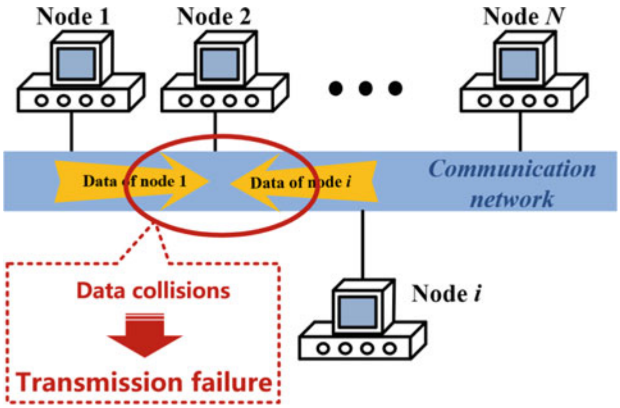

Fig. 1.2 Data collisions in case of simultaneous multiple accesses

transmission instant through the shared communication channel [41]. Communication protocols are capable of orchestrating the transmission order of all the network nodes and thereby generating certain scheduling behaviors that inevitably complicate the analysis and synthesis issues of networked systems. By now, there are three kinds of extensively investigated communication protocols in the literature; for example, the Try-Once-Discard (TOD) protocol [42], the Round-Robin (RR) protocol [27] and the Random Access (RA) protocol [30, 43].

1.2 Theoretical Frameworks

In networked systems, communication protocols are in fact a type of agreement with the aim to regulate the signal transmissions over shared channels. Accordingly, communication protocols would have a dramatic effect on the dynamical behaviors of networked systems. So far, the analysis and synthesis issues subject to different communication protocols have attracted an ever-increasing research interest. It is worth mentioning that the dynamical behaviors of networked systems subject to communication protocols are mainly affected by two aspects: the scheduling behaviors of communication protocols and the signal compensation methods. The former one determines which network node is selected to transmit its data at each transmission instant, while the latter one prescribes how to compensate the data corresponding to the network nodes that have not been selected to transmit data. Nevertheless, the so-called protocol-induced effects can be regarded as the impacts of the corresponding scheduling behavior and the signal compensation method on the networked system. Generally, there are two signal compensation methods widely adopted in practical applications: the zero-order holder (ZOH) method [44] and the zero-input (ZI) method [45].

Let us briefly introduce the protocol-induced effects by a simple example. Consider a discrete-time networked system with N network nodes labeled as $\{1, 2, \ldots, N\}$. Let ξ_k denote the chosen node which is allowed to get access to the communication channel at time instant k, $y_{i,k}$ be the signal of the ith node before transmitted at time instant k and $\bar{y}_{i,k}$ be the signal of the ith node after transmitted at time instant k. Then, the communication subject to certain communication protocol can be described by the following transmission model:

$$\bar{y}_k = \begin{cases} \daleth(\xi_k) y_k + \big(I - \daleth(\xi_k)\big)\bar{y}_{k-1}, & \text{the ZOH method} \\ \daleth(\xi_k) y_k, & \text{the ZI method,} \end{cases} \tag{1.1}$$

where

$$\daleth(\xi_k) \triangleq \mathrm{diag}\{\delta(\xi_k - 1)I, \delta(\xi_k - 2)I, \ldots, \delta(\xi_k - N)I\},$$
$$\bar{y}_k \triangleq \left[\bar{y}_{1,k}^T \; \bar{y}_{2,k}^T \; \cdots \; \bar{y}_{N,k}^T\right]^T, \quad y_k \triangleq \left[y_{1,k}^T \; y_{2,k}^T \; \cdots \; y_{N,k}^T\right]^T,$$

and $\delta(\xi_k - i) \in \{0, 1\}$ is a Kronecker delta function (i.e. $\delta(\xi_k - i) = 1$ holds if $\xi_k = i$ and $\delta(\xi_k - i) = 0$ otherwise). In this model, the scheduling behavior of the underlying communication protocol is characterized by the time-varying variable ξ_k. The main research topic of networked systems subject to communication protocols is to analyze the effects of the transmission model (1.1) on system performance and to design the corresponding controllers, filters or fault estimators according to the system dynamics and the transmission model (1.1), i.e. the analysis and synthesis problems of networked systems under the transmission model (1.1).

To date, there are three main different theoretical frameworks available in the literature dealing with the analysis and synthesis problems of networked systems subject to various communication protocols, namely, switched-system-based (SWB) framework [46], impulsive-hybrid-system-based (IHSB) framework [47] and switched-time-delay-based (STDB) framework [27]. Generally speaking, the SWB framework is always adopted to settle the discrete-time systems subject to communication protocols. Let's take the control problem of discrete-time networked system with a certain communication protocol for example. Consider a discrete-time networked system with N sensor nodes. ξ_k is the selected sensor node obtaining access to the communication network at time instant k. The networked system is characterized as follows:

$$\begin{cases} x_{k+1} = \digamma(x_k, u_k, \omega_k) \\ y_k = \gimel(x_k, \nu_k), \end{cases} \tag{1.2}$$

where x_k, y_k, u_k, ω_k and ν_k are the system state, measurement output, control input, process noise and measurement noise at time instant k, respectively. $\digamma(\cdot, \cdot, \cdot)$ and $\gimel(\cdot, \cdot)$ are two real-valued functions. Under the effects of communication protocol, the transmission model can be described by (1.1). Then, letting the control input be $u_k = \daleth(\bar{y}_k)$, the dynamics of the closed-loop system can be described by the following difference equation:

$$x_{k+1} = \begin{cases} \bar{F}_{\xi_k}(x_k, \bar{y}_{k-1}, \omega_k, \nu_k), & \text{the ZOH method,} \\ F_{\xi_k}(x_k, \omega_k, \nu_k), & \text{the ZI method.} \end{cases} \tag{1.3}$$

Obviously, the closed-loop system (1.3) can be regarded as a switched system and the switching law is determined by ξ_k which represents the scheduling behavior of the underlying communication protocol. As such, the performance analysis and controller design issues of such a networked system can be implemented based on the SWB framework. More details regarding the SWB framework can be found in [46, 48].

The IHSB framework is always employed to handle the continuous-time systems with sampled measurements and communication protocol scheduling effects. For such kinds of systems, the corresponding transmission model can be described by an impulsive system. Then, the resulted system dynamics could be represented by an impulsive switched system in which the system state would switch its value at every sampling instant. One of the representative works of the IHSB framework can be found in [49]. Both the SWB framework and IHSB framework are capable of dealing with networked systems subject to three widely studied communication protocols (i.e. the RR protocol, TOD protocol and RA protocol). Compared with such two frameworks, the STDB framework is developed to cope with the analysis and synthesis issues of networked systems with protocol scheduling and communication delays (or the delay effects induced by the sampling mechanism). In the STDB framework, the transmission model under the protocol scheduling is described by a delayed measurement model with switching parameters. Based on such a transmission model, the resulted system dynamics could be modeled by a switched time-delay system. More details regarding the STDB framework can be found in [27]. So far, a rich body of literature has appeared on the analysis and synthesis issues subject to various communication protocols based on these aforementioned frameworks; see, for example, [26, 49–53]. The relevant research on networked systems subject to communication protocols have the main focused attention on (1) the stability analysis of networked systems subject to various communication protocols, (2) the communication-protocol-based control problem of networked systems, (3) the communication-protocol-based filtering (or state estimation) problem of networked systems and (4) the communication-protocol-based fault diagnosis problem of networked systems.

1.3 Stability Analysis Subject to Protocol Scheduling

The performance analysis issue has gained a lot of research attention for networked systems subject to protocol scheduling effects since the pioneering works [54–56].

Stability is one of the most investigated performance indices of dynamical systems [57]. The scheduling effects of communication protocols would lead to an enormous impact on the stability of networked systems [41]. In [56], the stability analysis issue has been studied for continuous-time linear time-invariant (LTI) systems subject to the effects of RR protocol and TOD protocol, respectively. Furthermore, a maximum allowable transfer interval (MATI) has been proposed to guarantee the global exponential stability (GES) of a networked system subject to the TOD protocol. In [58], an improved MATI has been obtained to guarantee the GES compared with [56] based on the hybrid system method. The input–output $\mathscr{L}_p$ stability issue has been investigated in [59] for continuous-time nonlinear systems subject to RR protocol and TOD protocol, respectively. The results have been extended to the continuous-time nonlinear systems subject to hybrid communication protocols in [60]. Based on the IHSB framework, the $\mathscr{L}_p$ stability problem has been addressed in [49] for continuous-time networked systems with communication protocol scheduling, time-varying transmission intervals and communication delays, where the trade-offs between the MATI, maximally allowable delay (MAD) and performance gains have been provided.

The study on networked systems with RA protocol has been first reported in [47], where sufficient conditions have been acquired to guarantee the $\mathscr{L}_p$ stability for continuous-time networked systems. For discrete-time networked systems, the mean-square stability has been analyzed based on the SWB framework in [51] with RR, TOD and RA protocols, respectively. Generally speaking, there are two different stochastic processes describing the scheduling behavior of the RA protocol. The first one is the independent and identically distributed (i.i.d.) sequence of random variables, which has been first introduced in [47]. The other one is the discrete-time Markov chain, which has been first adopted in [51]. The choice between such two stochastic processes is dependent on the actual communication channel. In [61], the authors have studied the exponential mean-square stability of networked systems subject to two different RA scheduling models (i.e. the i.i.d. sequence of random variables and the discrete-time Markov chain) respectively based on the IHSB framework. In [62], the $\mathscr{L}_p$ stability issue has been addressed for nonlinear networked systems with TOD protocol scheduling by using the small gain theorem.

The exponential mean-square stability analysis issue has been considered for networked systems with two kinds of RR protocols in [63], where two Markov chains have been employed to model the packet dropouts. In [64], the stability problem has been investigated based on the STDB framework for discrete-time networked systems with RR scheduling, constant communication delays and nonuniform sampling scheme. The authors have extended the results in [27], where the exponential stability has been considered for networked systems with time-varying communication delays and RR scheduling. Then, the authors have discussed the stability issues subject to TOD protocol scheduling effects for continuous-time networked systems and discrete-time networked systems in [65] and [52], respectively, where the corresponding stability criteria have been derived by using the hybrid-delayed-system-based approach. In [66], the mean-square stability has been discussed based on the STDB framework for a class of stochastic networked systems with protocol scheduling effects, where it has been shown that the TOD protocol would lead to a larger MATI compared with the RR protocol. It should be noted that the TOD protocol is designed based on the "competitive" principle. As such, it is sometimes difficult for certain network nodes to obtain sufficient opportunities in getting access to the communication channel. In this case, some "improved" TOD protocols have been developed to ensure that every node is eventually assigned with the network access opportunity within a finite window of time. In [60], a so-called constant-penalty TOD (CP-TOD) communication protocol has been introduced based on the mechanism of "silent-time" and the $\mathscr{L}_p$ stability of the networked system subject to the CP-TOD scheduling has been studied. The input-to-state stability in probability has been studied in [67] for nonlinear stochastic systems under quantization effects and communication protocols in virtue of the switched Lyapunov function method.

1.4 Communication-Protocol-Based Filtering and Control

The filtering and control problems are two fundamental research topics in industrial automation community. In order to evaluate the control and filtering performance, various control and filtering methods have been developed. These control and filtering schemes can be categorized into several groups according to the considered systems and noises as shown in Table 1.1.

As discussed in Sect. 1.2, the closed-loop system dynamics of a networked system is largely dependent on the protocol-induced effects. As such, the controller/filter design of a networked system should take the protocol-induced effects into consideration in order to achieve the desired performance. In this section, we would like to review the communication-protocol-based control and filtering problems for different systems.

1.4.1 Communication-Protocol-Based Filtering and Control of Linear Time-Invariant Systems

The control problem of linear time-invariant (LTI) system is a hot research topic that has attracted quite a lot of attention [34, 68–70]. The protocol-induced effects lead to an enormous impact on the closed-loop system. Consider a typical linear time-invariant system of the following form:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + E\omega_k \\ \quad\; y_k = Cx_k + Dv_k. \end{cases}$$

Let the control input be $u_k = K\bar{y}_k$. Then, based on the transmission model (1.1), the dynamics of the closed-loop system subject to the ZOH method can be described as follows:

Table 1.1 The control and filtering methods for different systems with different noises

Noises	Systems				
	Control problems				
	Linear time-invariant systems	Nonlinear systems	Uncertain systems	Time-varying systems	Multi-agent systems
Energy-bounded noises	$\mathscr{H}_\infty$ control, energy-to-peak control			Finite-horizon $\mathscr{H}_\infty$ control	$\mathscr{H}_\infty$ consensus control
Norm-bounded noises	Ultimate-bounded control, l_2 control			Ultimate-bounded control	Bounded consensus control
Stochastic noises	LQG control, variance-constrained control			Error-constrained control	Mean-square consensus control
Noises	Systems				
	Filtering problems				
	Linear time-invariant systems	Nonlinear systems	Uncertain systems	Time-varying systems	Networked systems over sensor networks
Energy-bounded noises	$\mathscr{H}_\infty$ filtering, energy-to-peak filtering			Finite-horizon $\mathscr{H}_\infty$ filtering	Distributed $\mathscr{H}_\infty$ filtering
Norm-bounded noises	Ultimate-bounded filtering, l_2 filtering			Set-membership filtering	Distributed ultimate-bounded filtering
Stochastic noises	Variance-constrained filtering			Recursive filtering	Distributed recursive filtering

$$\begin{bmatrix} x_{k+1} \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} A + BK\daleth(\xi_k)C & BK\big(I - \daleth(\xi_k)\big) \\ \daleth(\xi_k)C & I - \daleth(\xi_k) \end{bmatrix} \begin{bmatrix} x_k \\ \bar{y}_{k-1} \end{bmatrix} + \begin{bmatrix} E & BK\daleth(\xi_k)D \\ 0 & \daleth(\xi_k)D \end{bmatrix} \begin{bmatrix} \omega_k \\ \nu_k \end{bmatrix}.$$

The dynamics of the closed-loop system subject to the ZI method can be described as follows:

$$x_{k+1} = \big(A + BK\daleth(\xi_k)C\big)x_k + E\omega_k + BK\daleth(\xi_k)D\nu_k.$$

Obviously, the above two difference equations are in fact linear time-invariant systems with certain switching behaviors. The main purpose of the communication-protocol-based control problem of time-invariant systems is to design the controller parameters subject to such switching systems. In [71], a co-design strategy of TOD protocol scheduling and controller has been developed for a class of LTI systems, where the desired controller parameter has been acquired by solving a set of matrix inequalities. The results have been extended to the co-design problem of TOD protocol and controller for linear time-delay networked systems in [72], where the desired