

Vienna Circle Institute Library

Jan von Plato

# Chapters from Gödel's Unfinished Book on Foundational Research in Mathematics



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## Volume 6

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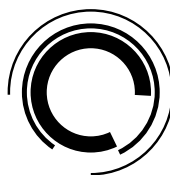
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## PREFACE

The surfacing of Gödel's part of a planned joint book with Arend Heyting on foundations of mathematics has been one of the pleasant surprises among the papers that Gödel left behind. I have had the good fortune to lead since 2018 a research project dedicated to their study and wish here to thank Tim Lethen for his precious help with the reading of difficult shorthand passages, and Maria Hämeen-Anttila for having prepared the list of Gödel's library loans from the times he worked on the book project.

The Kurt Gödel Papers that this book explores are kept at the Firestone Library of Princeton University. A finding aid with details about their contents is found at the end of the fifth volume of Gödel's *Collected Works*. The papers were divided by their cataloguer John Dawson into archival boxes and within boxes into folders. Folders can have a third division into documents, with a running document numbering system. The papers have been mainly accessed through a microfilm that is publicly available, but also directly in Princeton. References to specific pages of notebooks usually require the use of the reel and frame numbers of the microfilm and that is how the sources are mostly identified in this book. The shorthand manuscript sources are described in detail in Part I, Section 4.1 of this book. These descriptions together with the frame and page numberings in Parts II–III allow the interested reader to identify the source texts with the precision of a notebook page.

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## DEDICATION

This little book was happily recovered from the enormous wealth of materials in Kurt Gödel's papers, in what turned out to be a real detective work. I wish to recollect a decisive circumstance here: The main reason why I was able to learn the Gabelsberger shorthand script in which the pages to follow were written was my exposure to the German language at a proper age, from seven on. Relatively short as this exposure was, it happened at the right time and was quite particular, and the person responsible for it was very special. I recollect with affection Wilhelm Wiczerkowski, my teacher at the German school of Helsinki. To me he appeared just like any other adult – they seemed to be all of one and the same indistinct age – but now I know that he was about thirty at the time and working on his doctoral thesis. The thesis was that bilingualism is not a handicap or a risk that could lead to defects in one's personal development, as the prevailing doctrine had been, but instead an asset. My class was his test object, together with a comparison class in Germany. Here are some of his reflections:

The thing that makes a difference between instruction given in the German language, and the instruction of a foreign language is [...] in the first place the human encounter between the teacher and his foreign-language pupils. The teacher represents, for the child, and the growing young person, the foreign language circle as well as another way of being. Pupils usually don't make any difference between this foreign way and the person who leads his life accordingly. The deep impressions that come from a personal contact have, without doubt, a long-lasting effect on the attitudes acquired.

After the early career as a school teacher, Wilhelm Wiczerkowski became an esteemed scholar and university professor with highly gifted children as his specialty. I want to dedicate this little volume to his memory, knowing precisely what he would have said had he been confronted with anything of the kind, namely: "Gut Jan, sehr gut!"

## CONTENTS

PART I: GÖDEL'S "OWN BOOK ON FOUNDATIONS" .....	1
1. A Gödel puzzle .....	3
2. The <i>Ergebnisse</i> book project .....	4
3. Gödel's reading of the logical literature .....	11
4. Gödel's manuscript for the <i>Ergebnisse</i> book series .....	13
4.1. Description of the sources .....	13
4.2. Summary of contents of Gödel's chapters .....	15
PART II: OWN BOOK (FOUNDATIONS) .....	19
1. Introduction .....	21
2. Logicism .....	24
3. Antinomies .....	34
4. Clear version, from the beginning to the antinomies .....	61
5. The epistemological standpoint of the logicians .....	68
6. Logical calculus .....	70
7. Metamathematics .....	106
8. General metamathematics (Princeton) .....	108
PART III: GÖDEL'S READING NOTES .....	113
1. Editorial remarks .....	115
2. The untitled notebook .....	117
3. The <i>Altes Excerptenheft</i> .....	141
Gödel's references .....	196
Index of names for Gödel's chapters .....	211
Index of names for Gödel's notebooks of excerpts .....	212





# PART I: GÖDEL'S "OWN BOOK ON FOUNDATIONS"



## 1. A Gödel puzzle

Soon after his astonishing work on incompleteness had been published in 1931, young Kurt Gödel was invited to write together with Arend Heyting a concise overview on foundational research in mathematics. He never delivered his planned part, and that of Heyting got published separately in 1934. It has been often assumed that Gödel simply had not made much progress with the project. The truth, instead, is that he wanted to start by reading practically all of the extant literature on the topics that were allotted to him, namely logicism and the antinomies, and logical calculus and metamathematics. It was an approach that took too much time, and others lost patience at a point where he had practically finished two chapters and planned in detail a third. Thereafter he entertained for a couple of years the idea of “an own book on foundations.”

Gödel’s great amount of work with the book project did not go lost. It was the basis of his superb, comprehensive knowledge of the logical literature that in turn led him to new discoveries, such as his 1933 work on special cases of the decision problem of predicate logic, and many others that he wrote up but didn’t publish. His own views and preferences in foundational matters were informed by alternative ones, works forgotten today that he studied carefully. The thought behind seems to have been, always with an eye for the positive aspects of what others had to present: Could it be that an important or even decisive insight is hidden somewhere in these dusty pages?

For reasons that will be soon explained, the recovery of Gödel’s lost chapters on foundational research in mathematics has been quite some puzzle solving in an almost literal sense. To fit his chapters within the bounds set by the publisher, he had to cut out parts of text and write passages in a more compact form. On the other hand, the perfectionist that he was he kept adding materials. Add to this that except for the first part of the first chapter, all others were just first versions. Still, in the end, jumping back and forth, sometimes several pages, and left and right and up and down on notebook page openings, and adding parts from loose sheets, all the bits fit together and coherent, continuous versions of Gödel’s chapters miraculously

emerged.

The Heyting-Gödel book was meant as a compact introduction to the then current situation in foundational research. Gödel's part was packed with information that would be best read by a fellow logician rather than a newcomer to the field, something that he seems to have realised himself as one letter to the book series editor suggests. Today, when we look at his chapters in a historical perspective and with an interest focused on Gödel himself, the wealth of details just makes it all the more interesting. They show how he perceived the work of others. The two notebooks given in Part III give additional details, many of them about authors completely forgotten today, that did not find their way to the finished chapters.

## 2. The *Ergebnisse* book project

The Gödel papers contain shorthand versions of an introduction and of two chapters for a small book on the foundations of mathematics, the latter on logicism and the antinomies and on logical calculus. These are found in a notebook that begins with a heavily underlined title "Manuskript für Ergebnisse." There are in addition two plans for one more chapter on metamathematics, found in different places among the papers but clearly identifiable as belonging here.

Gödel had agreed in August 1931 to write together with Arend Heyting a short book on foundational research in mathematics, with publication in Springer's *Ergebnisse der Mathematik und ihrer Grenzgebiete* series. The length of the reports in this series was set as between five and seven printed sheets, i.e., 80 to 112 pages. The Heyting-Gödel book was listed in announcements in 1932 with the title *Mathematische Grundlagenforschung*. Andrei Kolmogorov's famous *Grundbegriffe der Wahrscheinlichkeitsrechnung* of 1933 was another "Heft" in these lists.

Gödel wrote up a reading list for his part, over 130 items, and also more detailed summaries in the notebook *Altes Excerptenheft I 1931-*. A meticulous collector of each and every form of printed or written thing, he kept his library loan requests that can be found among the Gödel microfilm collection. They give us dates for when

he read what from the items found in these two notebooks.

Gödel's letters to Heyting, published in volume *V* of his *Collected Works*, show that he had written three quarters of his part by October 1932. In September 1933, he promised to deliver his chapters by the beginning of 1934. At the end of that month, he was to leave for Princeton, where he then stayed until early summer 1934, and wrote from there in January that he would deliver his part by July 1934. Series editor Otto Neugebauer decided at this point to publish Heyting's part separately, with the title:

*Mathematische Grundlagenforschung:  
Intuitionismus, Beweistheorie*

Gödel's correspondence with Hans Hahn, his professor and mentor in Vienna, contains two items that relate to this phase of the book project. Hahn reminds him in a letter of 22 December 1933 about the report and writes again on 2 March 1934 about the "urgent request" for the manuscript on the part of Neugebauer, then that "it seems hopeless that you should finish it during your stay in America."

Gödel's notes give the impression that he, in turn, planned to publish also his part as a separate booklet. Indeed, Heyting's part has two main chapters, about thirty pages each on "intuitionism" and on "axiomatics and proof theory," and some additional material, and Gödel's would have had similarly three chapters. At the end of his introduction, Heyting writes that he does not deal with logicism, problems of pure logic, and metamathematics – exactly the chapters in Gödel's part – and adds that "a separate report is planned on these for this collection." Gödel's title would have been, correspondingly:

*Mathematische Grundlagenforschung:  
Logizismus und Antinomien, Logikkalkül, Metamathematik*

Charles Parsons' introduction to the Gödel-Heyting correspondence gives a discussion of their book project, with details drawn also from letters between the two and Neugebauer. He suggests that Gödel may have been held back "by exceptional meticulousness" which indeed is the case.

Gödel's notes for his chapters give a somewhat bewildering im-

pression. He would write, as was his habit, on the left side of a notebook opening with the right side reserved for additions. A great number of pages have long cancellations at left, blackened to unreadability in what seems quite some work in itself. Often a phrase has just begun, to be interrupted by a “black hole” up to half a page in length, then to continue as if nothing had intervened.

The explanation of the striking cancellations is as follows: The first opening of the *Heft* has the title “Manuskript für Ergebnisse” at right, in a typical mix of longhand and shorthand, and the left side contains the calculations:

$$1 p = 44 \times 21 \text{ Silben} = 924$$

$$\text{Heyting } 40 \times 19 = 760$$

Gödel had estimated the page length of the *Ergebnisse* book format, by calculating the number of syllables per line times 44 lines, and similarly for Heyting’s typewritten manuscript. The number of lines and pages for Heyting’s part results in 63 pages for his two chapters and two shorter sections. To remain within his own space limits, Gödel had to cut down the text, which he did by cancelling at left and rewriting in a shorter form at right – or let’s say that that would be the typical pattern. At places, he keeps adding. With the two chapters written down, it must have become clear to Gödel that his part, together with that of Heyting, would not fit within the *Ergebnisse* overall limit of 80–112 pages.

The notebook pages show Gödel’s own estimates of page length, always written in the lower right corner of the right-hand side of a notebook opening. The numbers for the first chapter on logicism begin with:

$$1, 2, 2\frac{3}{4}, 3\frac{1}{4}, 4\frac{3}{4}, 5\frac{7}{8}, 6\frac{1}{4}, 7\frac{3}{4} \dots$$

These numbers and similar ones all over in the two chapters on logicism and logical calculus are a clear witness of a basically finished text, with length calculated to one eighth of a page. I say basically, because not all things match, in particular in the chapter on logical calculus, but they could have been fixed at the stage of writing a fair version, something Gödel had time to do only for the first section of the first chapter. I hope this conclusion is warranted by the

transcription and translation that I offer. Tim Lethen has provided invaluable help with difficult passages, especially the tiny additions between lines that can be extremely hard to read. Uncertain readings of words have a question mark attached? or ?? in case I did not want to make any guesses. Gödel's own question marks, usually suppressed in shorthand writing, are printed in the German style with a space ? ahead, and boldface type.

An inspection of the original notebook for the *Ergebnisse* book project, kept at the Firestone Library of Princeton University, shows that the "black holes" in the text are a product of the microfilming process. To obtain a readable reproduction of the text, a high-contrast film has been used, with the described effect on the cancelled passages. They are mostly readable from the original. I have gone through them with an eye on anything that could differ significantly from what Gödel's later uncanceled formulations were.

The introduction and the chapter on logicism and the antinomies, in two sections the first of which comes even in a fair version, are reasonably finished. The chapter on logical calculus is a bit longer, written with much less changes, but unfinished at places. What little exists on the chapter on metamathematics, a list of contents and a couple of introductory pages, is found in a completely different place in the Gödel papers, with a fragment of a letter sketch to Neugebauer as an indication of where it belongs. Remarkably, there is a second start for a chapter—or perhaps a whole little book?—on general metamathematics, not just for arithmetic but for type theory and set theory, in a *Heft* Gödel wrote in Princeton in 1933/34.

Almost all of Gödel's references are indicated by a name and some space. These can be figured out from the context and the two notebooks that list and summarize his sources. I have made some suggestions in clear cases, but have left most of the gaps as they are. All the references are collected together into an extensive bibliography. With its help, those who aim at discussions of specific details should be able to determine what exact passages Gödel has had in mind.

Gödel took up the *Mathematische Grundlagenforschung* initiative and diligently gathered and studied all the pertinent sources, then started to write. Other duties intervened, he was sick for a notice-

able time, and he had to make compromises about the presentation. We shall see that the chapter on logicism, followed by his presentation of pure logic, took up all the space that was available, with no possibility to include metamathematics. With his “own book on foundations,” instead, he had the space needed, and he had the liberty to present things in the light in which he saw them. For the joint book project, he had first planned a conventional chapter on metamathematics in the style of his 1931 article, but as mentioned, in Princeton he was envisioning a chapter or book on general metamathematics, a treatment that covered type theory and set theory and displayed the implications of incompleteness on mathematics in general, not just for elementary arithmetic. That narrative was to be based on the concept of truth, exactly as his original approach to incompleteness had been, even if that crucial feature was completely suppressed in the final version of his 1931 incompleteness article.

It appears that Gödel entertained the idea of an “own book” about foundations even after his stay in Princeton. His correspondence contains shorthand drafts of letters to Neugebauer, Rudolf Carnap, and Alfred Tarski in which he suggests a new organization of the book. The drafts are somewhat sketchy, with cancellations and incomplete phrases. I shall give these letters here.

Letter sketch to series editor Neugebauer:

Esteemed Professor!

As you perhaps have heard, I have been ill last summer and fall. The loss of time thus caused as well as other circumstances make it unfortunately impossible for me to work further with the *Ergebnisse* report. If there should be, as I assume, soon a publication, I would like to propose a division of the report among several collaborators. I would like to propose as such R. Carnap (for the chapter on logicism) and A. Tarski for the one on metamathematics, whereas I would myself write a report, some 20–30 pages long, on the present state of the question of freedom from contradiction (newer as well as classical results). Considering the modest scope, it



should be possible to finish the report in a few months, in case the two gentlemen mentioned are prepared to take the task over and in case you are at all prepared to undertake the thing.

I myself had written a great part of the report on logicism, but see now, going through the manuscript, that I had perhaps kept on in too many details of theories.<sup>1</sup> The scope was therefore too strongly limited. I would therefore have to undertake a revision for which, as I saw, time is lacking at present.

with the highest respects, Kurt Gödel

Letter sketch to Carnap:

Dear Carnap,

Many thanks for the complimentary copy of the book as well as the offprints. The rumour has spread here more than once that you would come to Vienna and I hope that it perhaps really comes to that [[hard-to-read addition that mentions America]].

As you know, I did not deliver any report on foundations for the *Ergebnisse der Mathematik*, as I was preoccupied with other things in the first place. I have made today to Neugebauer the proposal of a division between you, Tarski, and myself, in which you would take over logicism, Tarski metamathematics, and I a report on the present state of the question of freedom from contradiction. I don't know, of course, whether Neugebauer goes for this proposal, but I write anyway so that you have more time to think over whether you want to take care of this. The scope of your part would be about 30 pages.

with best wishes,

Carnap's book has a preface dated May 1934, which means it was published some time in the latter part of 1934. Gödel's help with

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<sup>1</sup> There are many changes here, with an approximative reading of what is intended.

the book project is mentioned and one can safely presume that he got the copy upon publication which would place his plans for a joint book on foundations to that time. The letter to Neugebauer mentions his illness in the summer and fall of 1934.

Carnap published in 1935 a review of Heyting's booklet in which he writes that a second booklet by Gödel is in preparation, dedicated to logicism. Carnap kept regular diaries in which further remarks can be found:<sup>2</sup> An entry of 29 June, 1935 contains that "Tarski will write alone the *Ergebnisse* booklet." Another entry, of August 29, 1937, tells that Gödel "has withdrawn his own book about foundations."

From the beginning of 1935 on, Gödel worked for half a year mainly on physics. A notebook otherwise concerned with things such as elasticity theory, thermodynamics, and quantum mechanics, begins with attempts at a formal substitution operator for handling the meaning relation between an expression and an object. There follows a plan of contents for a chapter on antinomies (reel 21, frame 662) and some attempts at handling the epistemological paradoxes formally with the substitution operator.

Gödel's little book contains detailed presentations of his early views on many matters, among them his *completeness theorem for predicate logic* and his views on *set-theoretical relativism*, the latter well before he began his own set-theoretical investigations.

A final question needs to be addressed before we go into the details of Gödel's work with the *Ergebnisse* book. It can be fairly said that both Heyting and Gödel were relative newcomers, for both had in practice just two papers published in August 1931. One can presume that in the case of Gödel, his professor Hahn had already developed great confidence in the young protege's abilities, and may have suggested Gödel to Neugebauer.

As to Gödel and Heyting, they had met each other at the September 1930 Königsberg meeting on the foundations of mathematics. Soon afterwards, Gödel became a household item in logical circles. Heyting, for his part, was the one who had succeeded in presenting intuitionism in an axiomatic form in his path-breaking article of

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<sup>2</sup> *Tagebücher 1920-1935*, C. Damböck et al., eds, Meiner Verlag 2021, and *Tagebücher 1936-1970*, Meiner Verlag, in preparation.

1930, a work that intrigued Gödel and led to many of his short, incisive articles of the early 1930s. It is clear against this background that there was a mutual esteem and trust among the two young contributors to the projected *Ergebnisse* volume on the foundations of mathematics. This impression is further confirmed by the recently discovered lecture on intuitionistic logic Gödel gave towards his *Dozentur* in Vienna, on 3 February 1933:

Brouwer holds as meaningless the claim that each proposition must be either true or false, quite independently of the possibility to establish the one or the other. Brouwer has drawn various conclusions from his claim, in more or less unsystematic ways, and his student Heyting was the first one to attack this question in an axiomatic way. Namely, he put up an axiomatic system of logic, and more precisely, of the narrowest part of logic, the propositional calculus, from which the law of excluded middle cannot be derived.

### 3. Gödel's reading of the logical literature

After agreeing to participate in the book project, Gödel made a list of literature to study, from titles to mostly short summaries. It is found in a *Heft* with no title (reel 20, frames 499 to 525) and with pages numbered 1–50. There are additional loose unnumbered pages at frames 533–536. The numbered pages 1–50 list over 130 works, with a paper published in 1932 at page 14.

The papers found in the list are directly related to Gödel's chapters. The chapter on formalism was assigned to Heyting, and indeed, none of the formalistic literature is listed but the papers are almost all about logicism, antinomies, and to a lesser extent about logical calculus. This preparatory work seems to be the beginning of Gödel's comprehensive knowledge of practically all of the existing logical literature, an aspect that has not been clear before.

Gödel's notebook with the title *Altes Excerptenheft I* (1931–) is based on the initial list, with more detailed summaries and many more items, also on logical calculus. It appears to be the direct basis for the writing of Gödel's chapters for the *Ergebnisse* report. *Altes*

*Excerptenheft I* is found in reel 20, frames 236–306. The pages are continuously numbered from 1 to 123, with five more unnumbered pages that make altogether the pages 1–128, seven empty pages at the end, and three small loose notes. There follows a cover page on frame 307 and a separate lined paper page with the names in the *Excerptenheft* listed from Leibniz on page 1 to Sheffer on page 38. Four squared pages follow, a single piece that lists Frege’s works (frame 310), and the next opening (311) has the longhand title “Zahlen sind Seiten v. Altes Exc. H I” (numbers are pages of the *Altes Excerptenheft I*), and lots of names and page numbers, from 1 to 85, continued on frame 312 with more names and pages within the same range listed.

Many of the items in the two notebooks mentioned are found also in Gödel’s library slips. They give a clear idea of when he read what. The library slips have been studied and presented in a tabular form by Maria Hämeen-Anttila. Items with dates have a raised *D* in the text, to indicate that a date is found after the item in the bibliography.

The unnamed notebook contains with most items just the name, title, and place of publication. These are usually drawn over by regular wavy lines, apparently to indicate that the item was cleared in some way or other, as irrelevant or of little importance, or as something on which a summary or remarks are found in the second notebook. Some items, notably those on Ramsey’s and Russell’s works, have long summaries.

The style of writing of the two notebooks is clearly different from that of the book chapters. In particular, the sentences are typically incomplete, say, the object is clear from the context and is left out, the verb is left to easy guesswork, articles are suppressed, etc. A summary can be a series of brief remarks in the said style. At places, there are formal developments, but most of the formal work is just a recapitulation of the item’s content.

The writing of *Altes Excerptenheft I* from page 96 on is markedly different in style. The hand is lighter and more regular, the result very neat in comparison, and the page numbering style clearly different. The last numbered page is 123, but the appearance of the rest is similar to the one from page 96 on. The last page with writ-

ing is 128 that begins with: Stellen die für mein Russell Artikel von Wichtigkeit sind (passages that are important for my work on Russell). The conclusion is that the items from page 96 on are from the early 1940's:

Pages 96–109 have summaries of Russell's works with numbered items. Pages 110–114 concern Poincaré's 1906 article in the *Revue de métaphysique et morale*, 24 numbered items.

There follows about one page on Leibniz, and a summary of Frege's *Begriffsschrift* on pages 115–120 in 30 numbered items, usually passages from Frege slightly paraphrased.

There is in reel 32 a list under the title "Alte Literatur Grundlagen" (old literature foundations), with items 1–249. Item 68 has the date 1934, 70 is from 1937, and 90 has the explanation "Vortrag Gentzen 21./IX.1937, soll erscheinen ...Mai 1938," a reference to Gentzen's talk at a conference in Paris. The beginning of this list may be relevant for Gödel's book project.

The two notebooks are given in Part III. There are at places formal details that are not included. Their reading would require a study of the source on which Gödel is making notes. With such a background, the interested reader will likely have little difficulty in going through them directly in the form they are reproduced in the microfilm edition of Gödel's papers.

## 4. Gödel's manuscript for the *Ergebnisse* book series

### 4.1. Description of the sources

The notebook is in reel 24 and begins with inside cover 378L and 378R that has: Manusk. für *Ergebnisse*, "für" in shorthand.

Loose pages 379L–381L discuss decidable formulas in predicate logic (379L), then type theory and logicism in an introductory manner (379R–381L). There are two adjacent pages with a red circle, not photographed in the microfilm. I photographed them in Princeton in August 2019. All of these pages can be placed in the text, thanks to special signs Gödel had drawn on them, arrows, coloured circles etc.

Page 381R has a detailed list of contents with the title: A. Logizismus

The title page (“Manuskript für Ergebnisse”) has a cancelled text that relates to page 378L. The latter has calculations of the length of Heyting’s chapters.

Frame 379 has somewhat oddly at left a page with Van Heijenoort style additions on top,  $\llbracket iii \rrbracket$  at left,  $\llbracket iv \rrbracket$  at right, a double bracket style known from *From Frege to Gödel*. The two missing pages with the red circle have similarly the additions  $\llbracket i \rrbracket$  and  $\llbracket ii \rrbracket$ .

The chapter on logicism has a draft and a *reine* (fair) version. The presence of two versions gives rise to some mildly interesting comparisons, for example, errors in copying, such as the word *etwa* that becomes *etwas*, etc. The fair version is written on detached pages the backsides of which contain calculations with partial differential equations. It is a topic Gödel mentions elsewhere as one on which he could give lectures.

Gödel’s preserved pages amount to some eighty printed pages. His plans for contents can be gathered from the initial pages and from the correspondence he held with Heyting, all published in volume IV of his *Collected Works*. Letters of Heyting to Gödel on 26 July 1932 and of Gödel to Heyting on 4 August 1932 give as planned contents the following:

- I Introduction
- II Logicism and the antinomies
- III Axiomatics and formalism
- IV Logical calculus and metamathematics
- V Intuitionism
- VI Other points of view
- VII Relations  $\llbracket$ between these $\rrbracket$
- VIII Mathematics and natural science

The detailed calculations about length as found on the inside cover page are as follows:

$$\begin{array}{r}
 1 \text{ p} - 44 \times 21 \text{ syllables} = 924 \\
 \text{Heyting } 40 \times 19 = 760 \qquad 6 \text{ N} \\
 \qquad \qquad \qquad \qquad \qquad \qquad 5 \text{ A} \\
 \text{Zbl} = \text{Heyting} - 2\% \\
 \text{I } 29 \text{ p H} \\
 \text{F } 24\frac{1}{2} \text{ p H}
 \end{array}$$

A + N 11 p H

$64\frac{1}{2}$  p H

63 p

Here H clearly stands for Heyting, 6 N for six pages on natural science, 5 A similarly for other points of view, Zbl for the page format used by the *Zentralblatt*, I 29 and and F  $24\frac{1}{2}$  for Heyting's two chapters on intuitionism and on formalism, and A + N 11 p H indicates that Heyting had finished the short chapters VI and VIII.  $64\frac{1}{2}$  p H is the total count of Heyting's part, and 63 p the number of pages corrected by two percent. With a maximal number of pages of only 112, Gödel had less than fifty pages at his disposal, a limit difficult to meet.

A letter of Gödel's to Heyting of 15 September 1932 tells that about half of Gödel's part is finished, the section on logicism. Another letter of 16 May 1933 tells that about three quarters were finished by October 1932. A last letter of Heyting's, of 30 September 1933, contains that Gödel had promised to deliver his part by the beginning of 1934. With the idea of a whole book of his own, Gödel started to plan an additional chapter on metamathematics.

#### 4.2. Summary of contents of Gödel's chapters

Gödel's part is divided into three chapters:

There is an introductory chapter of about 3 pages, a chapter on logicism and the antinomies of about 30 pages, and a chapter on logical calculus of about 30 pages – metamathematics not covered. There is a detailed list of contents for the chapter on logicism at frame 381R:

##### 1. Logicism and the antinomies

p  $20\frac{3}{4}$ – $23\frac{3}{4}$  not interesting

A. Logicism

1. The claim of logicism – what is logic – Russell earlier – now – Wittgenstein – Hahn

2. Necessity of a symbolism for the execution of the program – *calculus ratiocinator*

Formal system – the first such calculus Peano shows that the whole of mathematics follows from a few assumptions

3. Execution of logicism Frege Basic concepts, axioms, strict