M.A. Gordon R.L. Sorochenko

Radio Recombination Lines

Their Physics and Astronomical Applications





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Radio Recombination Lines

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Radio Recombination Lines

Their Physics and Astronomical Applications

by

M.A. Gordon *and*

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To our children and grandchildren, to their children, and to all of the next generations, who may benefit from a deeper understanding of the universe in which they live.

Preface

Recombination lines at radio wavelengths have been – and still are – a powerful tool for modern astronomy. For more than 30 years, they have allowed astronomers to probe the gases from which stars form. They have even been detected in the Sun.

In addition, observations of these spectral lines facilitate basic research into the atom, in forms and environments that can only exist in the huge dimensions and extreme conditions of cosmic laboratories.

We intend this book to serve as a tourist's guide to the world of Radio Recombination Lines. It contains three divisions: a history of their discovery, the physics of how they form and how their voyage to us influences their spectral profiles, and a description of their many astronomical contributions to date. The appendix includes supplementary calculations that may be useful to some astronomers. This material also includes tables of line frequencies from 12 MHz to 30 THz ($\lambda = 10 \,\mu\text{m}$) as well as FORTRAN computer code to calculate the fine-structure components of the lines, to evaluate radial matrix integrals, and to calculate the departure coefficients of hydrogen in a cosmic environment. It also describes how to convert observational to astrophysical units. The text includes extensive references to the literature to assist readers who want more details.

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Chapter 1 Introduction

Abstract This chapter describes the early theory and initial detection of radio recombination lines from astronomical objects. The focus is historical.

1.1 The Cosmos as a Laboratory

The history of science shows many close connections between physics and astronomy. It is well known that a number of physical laws evolved from a base of astronomical observations. For example, Kepler observed and, later Newton derived, the laws of gravitation while studying the motion of planets and their satellites. The existence of thermonuclear energy was solidly established when it explained the energy balance of the Sun and stars. The anomalous shift of Mercury's perihelion showed us that Newton's gravitational theory was incomplete; this observation helped lead Einstein to the more comprehensive theory of General Relativity.

The cosmos is a wonderful laboratory. There, physicists find that matter can have very high and very low temperatures. It can have ultrahigh and ultralow densities. It can occupy huge volumes. It can exist in states impossible to duplicate in a terrestrial laboratory – states that are not always in dynamical or thermal equilibrium. This extreme diversity of matter in the cosmos is one of the reasons that astronomical observations and astrophysical studies are so valuable.

1.2 Spectral Lines in Astronomy

Low-density cosmic matter gives us a unique opportunity to study elementary processes in atoms and molecules by means of the phenomenon of spectral lines. This is important for physics. It is worthwhile to remember that the first spectral lines – the Fraunhofer lines – were first detected in astronomic objects, in the spectra of the Sun and the stars. These observations stimulated the development of laboratory spectroscopy.

Emission of spectral lines from cosmic objects became an essential tool in astronomy. The frequency of each line is unique and identifies the atom, ion, or molecule emitting that radiation. Knowing the line frequency through laboratory measurements or through calculations, astronomers can determine the velocity shift of the line and, by local kinematics and by the Hubble law, estimate the distance of the emitting region. The line intensities are related to the number of atoms along the line of sight within the telescope's field of view. The line widths are produced by a combination of the motion of the emitting atoms, of perturbations to the radiation induced by magnetic fields, and by the difficulty that the photons experienced passing through the medium. In this way, the line shapes are the record of what the photons experienced when they were created and in their voyage to us.

The opportunities to investigate spectral lines in astronomy broadened considerably with the extension of astronomical observations into the radio regime, now known as "radio astronomy." One enormous advantage of the radio regime relative to the optical was that the spectral window could be shifted from high to low frequencies, thereby obtaining high spectral resolution at easily managed frequencies. Called "superheterodyne" conversion, this process was developed in the early 1900s to enhance radio receivers for communications. Implementing this technique in the optical regime involves solving difficult physics problems. At present, only limited applications exist.

Spectral lines from a great number of cosmic atoms and molecules are now available throughout the electromagnetic spectrum. In this book, we consider a special class of these spectral lines, namely, spectral lines resulting from transitions between highly excited atomic levels. Conceptually, these lines appear after the recombination of ions and electrons to form atoms, leaving the electrons in levels with high principal quantum numbers n. These newly bound electrons jump downward from level to level much like going down a flight of stairs, losing energy in each jump by radiating it away in the form of a spectral line. When these lines appear in radio regime, they are called "radio recombination lines" (RRLs).

The study of RRLs has revealed a number of surprising new concepts for physics and astronomy. For example, in ultralow-density regions of the interstellar medium (ISM), an atom can exist with electrons in very high quantum levels – up to $n \approx 1,000$ and, correspondingly, with huge diameters approaching 0.1 mm. We can observe the spectral lines from these giant atoms over a wide range of radio waves, from millimeter to decameter wavelengths. Because interstellar atoms are sensitive to variations in gas densities and temperatures in any region, their RRL emission sends us information about the structure of their cosmic environments. And, as we shall see, the basic physics underlying these atomic lines are easy to understand.

1.3 The Bohr Atom

To understand the early searches for RRLs, we first need to discuss a basic physics model known today as the "Bohr atom." This model explains atomic emission lines in a simple way.

Line radiation caused by transitions between atomic levels was detected about 100 years ago. These lines were grouped into series such as the then well-known Lyman, Balmer, and Paschen line series emitted by hydrogen in the ultraviolet (UV), visible, and infrared (IR) wavelength ranges. Physicists soon found empirically that the frequencies ν of the lines in these series could be represented by a simple formula:

$$\nu_{n_2 \to n_1} = Rc \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2} \right), \quad n_2 > n_1 > 0,$$
(1.1)

where n_1 and n_2 are positive integers, c is the speed of light, and the constant R was called the Rydberg constant. Each line series could be fitted by choosing a value for n_1 and then sequentially entering values for n_2 . For example, $n_1 = 1$ would give the Lyman series; $n_1 = 2$, the Balmer series; $n_1 = 3$, the Paschen series; and so on. Examination of (1.1) shows that the lines of each series become closer together as n_2 increases, forming a "series limit" when $n_2 \Rightarrow \infty$ of

$$\nu_{n_2=\infty\to n_1} = \frac{Rc}{n_1^2} \tag{1.2}$$

beyond which the lines become a continuum clearly visible on the spectral plates.

What are the physics behind these empirical formulas? From these observations, Bohr (1913) developed his quantum theory of the atom – a mathematically simple theory that explained most of the series of atomic lines known at that time.

In this theory, Bohr postulated that atoms have discrete stationary energy levels; in other words, these energy levels are "quantized" rather than continuous. One can imagine a set of orbits of electrons circulating around the nucleus at quantized radii. Introducing discrete quantum numbers for angular momentum, Bohr assumed that only those orbits can exist for which the angular momentum L is a multiple of $h/2\pi$, i.e., described by following expression:

$$L = n \frac{h}{2\pi}, \quad n > 0 \tag{1.3}$$

$$= 1.0545919 \times 10^{-27} n \quad \text{erg sec}, \tag{1.4}$$

where h is a Planck's constant and n is any positive integer. Bohr's formulation allowed orbits of discrete diameters 2a given by

1 Introduction

$$2a = \frac{n^2 h^2}{2\pi^2 m Z e^2} \tag{1.5}$$

 $= 1.05835 \times 10^{-8} n^2 \quad \text{cm}, \tag{1.6}$

where m is the mass of the electron, e is the electronic charge in ESU, and Ze is the charge of the nucleus. Equation (1.6) indicates that the sizes of orbits as well as atom's sizes increase as n^2 . Setting n = 1 and Z = 1 produces the radius of the first orbit of hydrogen, known as the "Bohr radius,"

$$a_0 = \frac{h^2}{4\pi^2 m e^2} \tag{1.7}$$

often used as a parameter in equations involving atomic physics.

Classical electrodynamics predicted orbital diameters by equating the electrical attraction between each electron and the nucleus to the centripetal acceleration:

$$\frac{Ze^2}{a^2} = \frac{mv^2}{a}, \quad \text{or} \tag{1.8}$$

$$2a = \frac{2Ze^2}{mv^2},\tag{1.9}$$

so that every orbital diameter would be allowed depending upon the orbital speed v or the kinematic energy mv^2 of the electron.

The total energy E of an electron in a circular orbit is the sum of the electrical potential and the kinetic energy:

$$E = -\frac{Ze^2}{a} + \frac{1}{2}mv^2 \tag{1.10}$$

$$= -\frac{Ze^2}{a} + \frac{Ze^2}{2a} \tag{1.11}$$

$$= -\frac{Ze^2}{2a} \tag{1.12}$$

after substitution of (1.9). Using the quantization of orbits described by (1.6), Bohr calculated the energy E_n associated with each electronic orbit¹ n:

$$E_n = -\frac{2\pi^2 m e^4}{h^2} \frac{Z^2}{n^2}$$
(1.13)

$$= -2.17989724 \times 10^{-11} \frac{Z^2}{n^2} \quad \text{ergs.} \tag{1.14}$$

¹ A fundamental difference between the Bohr theory and classical electrodynamics is that in the Bohr theory, electrons do not radiate even though they are technically accelerating by changing direction.

Note that the energy of bound electrons must be negative. Because the energy of a photon is $h\nu$, the frequency of each atomic line would then be

$$\nu_{n_2 \to n_1} = \frac{E_{n_2} - E_{n_1}}{h}, \quad \text{or} \tag{1.15}$$

$$=\frac{2\pi^2 m Z^2 e^4}{h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
(1.16)

$$= RcZ^2 \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right), \qquad (1.17)$$

which is identical with the empirical formula given by (1.1) if the effective nuclear charge Z = 1 and the Rydberg constant is

$$R = \frac{2\pi^2 m e^4}{h^3 c}.$$
 (1.18)

Substituting into (1.18) the values of the physical constants listed in Table A.1, we derive $R = 109,737.35 \text{ cm}^{-1}$ which is close to the value of $109,675 \text{ cm}^{-1}$ obtained by Rydberg (1890) from measurements of hydrogen spectral lines. Such close agreement leaves no doubt regarding the validity of (1.14).

Later, Bohr (1914) did even better. The original theory assumed an infinitely small electronic mass orbiting the nucleus. He refined his earlier equations to use the center of mass as the centroid of the orbit and the reduced mass m_R in place of the orbiting electronic mass, so that

$$R = R_{\infty} \left(\frac{M}{M+m}\right),\tag{1.19}$$

where the coefficient

$$R_{\infty} = \frac{2\pi^2 m e^4}{ch^3} \tag{1.20}$$

and is now called the Rydberg constant for infinite mass. Section A.2.1 gives details. With this correction, the Rydberg constant for hydrogen $R_H = 109,677.57 \,\mathrm{cm}^{-1}$. The calculated and measured values of R_H now agreed within 0.002%.

There are additional refinements to the Bohr model that improve generality. These include consideration of elliptical orbits and the quantization of angular momentum. Section C.2 describes these calculations in detail.

Although our discussion has so far concentrated on hydrogen, these equations can also describe RRL spectra of multielectron atoms *and* ions. This "hydrogenic" model assumes that only one electron is in an excited level; the Z - 1 other electrons lie in or near ground levels. For neutral atoms, the net negative charge of the inner electrons would screen the positive charge of the nucleus, so that a lone outer electron would see only a single nuclear charge and Z = 1. For ions, a similar situation would obtain but with Z > 1. Table A.2 gives Rydberg constants for a few atoms² common to the cosmos.

1.3.1 Bohr Lines at Radio Wavelengths

The theory did not restrict the number of atomic levels nor the number of the line series. Bohr (1914) showed that, for large quantum numbers and for transitions from $n_2 = n + 1 \rightarrow n_1 = n$, (A.6) gives a series of line frequencies for Bohr lines of neutral hydrogen:³

$$\nu_H \approx \frac{2R_H cZ^2}{n^3}, \quad n >> 1$$

$$= 6.58 \times 10^{15} \frac{1}{n^3} \quad \text{Hz}$$
(1.22)

with an accuracy of about 2-3% depending upon the frequency. Although unrealized at the time, substituting values of, say, 100 < n < 200 into (1.22) will yield approximate frequencies for lines throughout the radio range.

1.3.2 Other Line Series

Bohr's model was a brilliant success. It not only explained the hydrogen line series observed up to the year 1913 but predicted new lines as well. However, research into spectral lines toward longer wavelengths proceeded slowly. The fourth atomic series for hydrogen with $n_1 = 4$ and the first line $\lambda_{5\rightarrow 4} = 4.05 \,\mu\text{m}$ was detected by Brackett (1922) 9 years after Bohr's theory had appeared; the fifth series with $n_1 = 5$ and the first line $\lambda_{6\rightarrow 5} = 7.46 \,\mu\text{m}$, by Pfund (1924) 11 years after; and the sixth series with $n_1 = 6$ and the first

$$\nu_H \approx 2R_H cZ^2 \frac{\Delta n}{n^3} \left[1 - \frac{3}{2} \left(\frac{\Delta n}{n} \right) + 2 \left(\frac{\Delta n}{n} \right)^2 - \frac{5}{2} \left(\frac{\Delta n}{n} \right)^3 + \cdots \right], \qquad \Delta n \ll n.$$
(1.21)

 $^{^2}$ A property of the Bohr line series expressed by (1.17) and (1.19) is that the entire line series can be shifted in frequency by changing the Rydberg constant *R*. Radial velocities will cause similar shifts. This means that identification of the atomic species emitting these lines in a cosmic environment, in principle, cannot be made simply on the basis of the observed frequencies – as can be done for molecular emission lines with their less regularly spaced frequencies. In practice, radial velocities for observed optical and molecular lines along the same sight lines help identification of the atomic species of Bohr lines from cosmic gas.

³ The approximation comes from the first term of the binomial expansion of (1.17) when $n_2 \equiv n_1 + \Delta n$:

line $\lambda_{7\to 6} = 12.3 \,\mu\text{m}$, by Humphreys (1953) 40 years later as a result of a very fine measurements of spectra in a gas discharge.

With these studies, classical laboratory spectroscopy ran out of ability. Only new techniques could find new series and answer the question of how far the theory could go. In this quest, the frequencies of the lines moved from the optical through the infrared into the radio regime. Here, astronomy and, more exactly, its rapidly developing branch of radio astronomy came to the aid of physics.

1.4 Spectral Lines in Radio Astronomy

1.4.1 Theoretical Studies

The Dutch astronomer, van de Hulst (1945), was the first to consider the possibility of radio line radiation from transitions between highly excited levels of atoms in the ISM. In the same classical paper that predicted the $\lambda = 21 \text{ cm}$ line, van de Hulst also considered radiation from ionized hydrogen for both free-free and bound-bound transitions.

While calculation of the total emission in these lines is straightforward, the detectability of such RRLs would depend upon the distribution of this emission above the underlying continuum emission or, in other words, upon the shape of the emission lines. Although thermal conditions determine the amount of emission in the RRLs relative to the continuum emission, other effects like Stark broadening can widen the lines, spreading out the line emission in frequency, thereby reducing their peak intensities and, in turn, their detectability. van de Hulst derived an expression for the ratio of the peak intensity of the line I_L to the continuum intensity I_C :

$$\frac{I_L}{I_C} = 0.1 \frac{\nu}{\Delta \nu} \frac{h\nu}{kT},\tag{1.23}$$

where $\Delta \nu$ is the full frequency width of the line, T is the temperature of medium, and k is a Boltzmann's constant. Conceptually, (1.23) describes the total emission in the line to be the product $I_L \Delta \nu$.

To estimate the Stark broadening, van de Hulst drew from an analysis by Inglis and Teller (1939) of optical and infrared line series in stellar spectra. Within the hydrogenic line series of stellar spectra (see Sect. 1.3), there is a wavelength, short of the series limit, at which distinct lines can no longer be seen. In frequency units, (1.17) models these series (for any given n_1) as $n_2 \to \infty$. At a critical value of n_2 , the lines of that series merge into a continuum that continues until the series limit is reached.

The explanation for this line merging is simple. At this critical frequency (or wavelength), Stark broadening within the stellar atmosphere broadens the line to match the gap between it and the adjacent line at $n_2 + 1$. By counting lines, an astronomer determines this critical value of n_2 and calculates the wavelength (or frequency) separation to the next line at $n_2 + 1$. This separation must equal the amount of the line broadening and, consequently, is a measure of the electron density necessary to produce it. In this way, Inglis and Teller provided a method of determining gas densities in stellar atmospheres.

Using results from Inglis and Teller and an estimate for the density of the ISM, van de Hulst estimated the magnitude of Stark broadening to be

$$\frac{\Delta\nu}{\nu} = \left(\frac{\lambda}{100\,\mathrm{m}}\right)^{3/5}.\tag{1.24}$$

Although the original paper gives few details regarding this formula, the quantity $\Delta\nu/\nu \approx 0.02$ if $\lambda = 20$ cm, a typical wavelength considered for radio astronomy in 1944, e.g., the $\lambda = 21$ cm line. At this wavelength, (1.24) shows the Stark broadening to be dramatically bigger than the thermal broadening that he correctly estimated as $\Delta\nu/\nu = 10^{-4}$. In the meter wavelength range where radio astronomers (Reber, 1944) were actually observing at that time, the Stark broadening would be even larger. Consequently, van de Hulst concluded that hydrogen lines caused by transitions between highly excited levels would be too broad and, therefore, too weak to be observed.⁴

Other astronomers were also pessimistic. Reber and Greenstein (1947) had considered hydrogen radio lines in their examination of the astronomical possibilities of radio wavelengths but had excluded them, "these [lines] have small intensity." Wild (1952) also considered RRLs but dismissed them because "these lines are so numerous that, without the presence of some selection mechanism they may be regarded merely as contributing toward a continuous spectrum."

Kardashev (1959) reached just the opposite conclusion. Although he was aware that Wild (1952) had dismissed the possibility of detecting lines, he was unaware of the very pessimistic van de Hulst (1945) study.⁵ Kardashev made

⁴ Many years later, Sullivan (1982) analyzed the working notes of van de Hulst while studying the history of radio astronomy. He found a place in these notes where van de Hulst appeared to have inverted the exponent in (1.24), i.e., the Stark broadening should vary as $(\lambda/100m)^{5/3}$. In fact, combining expressions from the two relevant papers (van de Hulst, 1945; Inglis and Teller, 1939) show that the exponent indeed should have been 5/3. The correct formula – not (1.24) – would predict smaller Stark broadening at radio wavelengths and, correspondingly, more intense line intensities. Sullivan did not comment on the choice of electron density used to derive (1.24). Probably, van de Hulst assumed N_e to be 1 cm⁻³. See Appendix C.1 for more details.

⁵ The van de Hulst paper was published in a very rare edition of "Nederlands Tijdschrift voor Natuurkunde" that Soviet libraries did not have. The disruption of scientific contact during the second world war and, later, during the cold war years also played a role. According to Shklovsky (1956b; 1960), Soviet scientists had learned about the $\lambda = 21$ cm line only from references and comments that appeared much later in journals that were

detailed calculations of the expected line widths and intensities of excited hydrogen RRLs in ionized nebulae (H II regions).

The earlier papers by highly respected astronomers created a difficult climate for optimism with respect to detections of RRLs. Parijskij (2002) recalls an ad hoc meeting at the IAU General Assembly in Moscow in August 1958, where well-known radio astronomers discussed with the then young Nicolay Kardashev the validity of his new, encouraging calculations (Kardashev, 1959). These probably included W.L. Erickson, G.B. Field, L. Goldberg, F.T. Haddock, J.P. Hagen, D.S. Heeschen, T.K. Menon, C.A. Muller, H.F. Weaver, and G.L. Westerhout (Kardashev, 2002). The discussion took place in a small room in a new building of Moscow University. Parijskij acted as interpreter. He recalls that the discussion was interesting but quite intense – one "of the deepest I have heard in my life" – with the experienced astronomers examining every calculation made by Kardashev. At the end of the 2-h meeting, they took some kind of a vote and decided that Kardashev might well be correct. This must have been a challenging experience for the young astronomer.

The principal difference between Kardashev and van de Hulst in these calculations lies in their approach to Stark broadening. Kardashev also used the Inglis–Teller relationship, but only for a rough estimate. Independently, he calculated Stark broadening from collisions of excited atoms with electrons as well as from quasistatic broadening. From this analysis, he concluded that, in H II regions with typical values of electron temperature $T_e = 10^4 K$ and density $N_e = 10^2 \text{ cm}^{-3}$, pressure broadening would have no significant influence on the line broadening at frequencies greater than 7,000 MHz. In other words, he concluded that line widths would be determined solely from thermal effects, i.e., from the frequency redistribution of emission from a Maxwellian gas according to Doppler effects giving rise to a Gaussian line shape.

After calculating an oscillator strength to determine the line intensities, Kardashev predicted that excited hydrogen radio lines would be observable by radio astronomical techniques in the range from the FIR to decimeter waves. He also showed that the $n \rightarrow n-1$ transition would have highest intensity and, in addition to the hydrogen lines, the radio lines of helium would be detectable. The frequencies of the helium lines would be shifted relative to hydrogen because of the difference in the Rydberg constant (see Table A.2) due to its greater mass.

Subsequent calculations made it possible to define the intensities of expected radio lines more accurately and, thereby, to plan a search optimized in both frequency and in target sources (Sorochenko, 1965). To re-estimate the line intensities, the attention was again focused on Stark broadening. This time, the calculations used the theory of line broadening in a plasma as developed in early 1960s (Griem, 1960).

available in the USSR. In fact, the van de Hulst paper only appeared in the USSR after its translation (Sullivan, 1982) into English.



Fig. 1.1 Predicted line-to-continuum ratios for radio recombination lines as a function of frequency. VdH van de Hulst (1945) from (1.23) and (1.24), K Kardashev (1959) who calculated that Stark broadening may be neglected for $\nu > 7,000$ MHz, S Sorochenko (1965) who considered both thermal and Stark broadening for the two values of electron density $N_e = 100 \text{ cm}^{-3}$ (1) and $N_e = 1,000 \text{ cm}^{-3}$ (2). All calculations assume an electron temperature of $T_e = 10^4 K$

Figure 1.1 summarizes the line-to-continuum ratios (I_L/I_C) from the papers mentioned. All calculations refer to the $n \to n-1$ transitions. One can see that van de Hulst (1945) strongly underestimated I_L/I_C , especially taking into account the probable adopted density $N_e = 1 \text{ cm}^{-3}$. For Doppler broadening alone as calculated by Kardashev (1959) for the centimeter wavelength range, the L/C ratio is a few percent, at values of $N_e = 10^2 \text{ cm}^{-3}$ appropriate for H II regions. If Stark broadening is taken into account, at $N_e \geq 10^2 \text{ cm}^{-3}$ the L/C ratio decreases noticeably at frequencies $\nu < 10 \text{ GHz} (\lambda = 3 \text{ cm})$.

To estimate realistic circumstances for the detection of the radio lines, Sorochenko (1965) calculated their intensities in the units of brightness temperature T_b customary in radio astronomy. Figure 1.2 shows these expected values at line center $T_{b,l,c}$ as a function of wavelength λ and of N_e . These data are normalized to the value of the emission measure (EM) of the H II region. EM is a physical parameter calculated from observations of the continuum emission of an H II region and defined as $N_e^2 L \,\mathrm{cm}^{-6}$ pc, where L is the depth of an H II region in parsecs⁶ (pc) along the line of sight through the H II region.

⁶ Abbreviation for "parallactic arcsecond," the distance at which the average Earth–Sun distance subtends an angle of 1". One $pc = 3.0856 \times 10^{18}$ cm.



Fig. 1.2 Brightness temperature at the line center as a function of wavelength: (a) $N_e = 100 \text{ cm}^{-3}$, (b) $N_e = 200 \text{ cm}^{-3}$, (c) $N_e = 500 \text{ cm}^{-3}$, and (d) $N_e = 1,000 \text{ cm}^{-3}$

In the millimeter range, the effective size of the atoms is small, there is less collisional interaction with the ambient H II gas, and Stark (pressure) broadening is insignificant as a result. Here, only Doppler broadening determines the line widths and Fig. 1.2 shows the brightness temperature of the lines to increase with wavelength. For an electron density of $N_e > 100 \,\mathrm{cm}^{-3}$, Stark broadening of the lines begins to manifest itself at centimeter wavelengths, spreading the line emission over a broader wavelength (frequency) range and reducing the peak intensity of the lines. As the wavelength increases further, the line intensities decline sharply. There is a peak or "turnover" in each of the curves, with the maximum of the brightness temperature shifting toward shorter wavelengths with larger densities.

Simple Bohr atom physics easily explains this effect. Because the longer wavelength lines are generated by atoms whose electrons are in larger orbits, the effective size of these atoms is larger, and their larger sizes render them more likely to interact or collide with the charged particles of the ambient H II gas. These collisions strip the atoms of the outer electrons, thereby removing their ability to radiate and, correspondingly, reducing the aggregate line intensity emitted by the H II region. The wavelength of the turnover is directly related to the probability of these collisions and, therefore, decreases as the gas density increases.

From an experimental viewpoint, this analysis indicated that the search for RRLs would be more effective at low centimeter wavelengths where Stark broadening would be weakest and the line intensities would be the strongest. Specifically, it suggested that the search should take place at $\lambda = (2-5)$ cm in the brightest, extended H II regions, the Omega and Orion nebulae. Furthermore, at these wavelengths, the angular sizes of these bright H II regions would be well matched to the beam of typical radio telescopes available at that time, thereby ensuring maximum sensitivity for the search. The stage had now been prepared for the main act: the actual detection of RRLs. In actuality, of course, the stories of theoretical refinements and the searches were complex and intertwined.

1.4.2 Detection of Radio Recombination Lines

The first attempt to detect radio lines emitted by highly excited atoms was undertaken at the end of 1958 in Pulkovo by Egorova and Ryzkov (1960) just after they learned about Kardashev's calculations. Utilizing the receiver developed to search for the deuterium lines ($\lambda = 91.6$ cm), and the unmovable parabolic antenna 20 × 15 m, they searched for hydrogen radio line corresponding to the $n_{272} \rightarrow n_{271}$, or H271 α , transition in the Galactic plane over the longitude range $l = 60^{\circ}-115^{\circ}$, but without success.

Five years later, Pulkovo radio astronomers repeated their attempt. At this time, the search was done in 1963 by Z.V. Dravskikh and A.F. Dravskikh (1964) during the testing of the new 32-m paraboloid antenna of the Space Research Center in the Crimea. A simple $\lambda = 5 \text{ cm}$ mixer receiver with filter width of 2 MHz and a tuning accuracy of about 1–3 MHz scanned over a 20-MHz band to search for the $n_{105} \rightarrow n_{104}$ hydrogen line at 5.76 GHz in the Omega and Orion nebulae.

According to Dravskikh (1994; 1996), a strong wind arose during their scheduled time, making it difficult to point the telescope and resulting in only eight spectrograms for the Omega nebula and five for Orion. The quality of these spectra were accordingly poor, and the Dravskikhs were reluctant to consider them further. However, a young colleague, Yuri Parijskij, insisted that spectra should be processed further, believing that the wind effects could be removed. After this processing, the lines appeared to be present in the spectra of each nebula – although too weak to convince everyone of the reality of their detection, shown in Fig. 1.3. The authors themselves estimated the detection probability to be 0.9, corresponding to a signal-to-noise (S/N) ratio of 2.

At that time, the situation was very competitive. Two Soviet groups had been preparing to search for RRLs. Besides A.F. Dravskikh and Z.V. Dravskikh in Pulkovo, the other group for detecting lines was located at the Lebedev Physical Institute in Moscow, where they had been preparing since 1963. The competition involved the quality as well as the timing of the searches. The Pulkovo group had been able to begin their observations earlier but the detections were marginal, having been achieved by necessarily salvaging the unfortunate wind-damaged spectra. On the other hand, the Lebedev group wanted to make detections that would be convincing to everyone and were willing to delay their observing until their specially designed equipment was ready.



Fig. 1.3 Figures 1–5 of Z.V. Dravskikh and A.F. Dravskikh (1964): (1a) average of the eight spectra of the Omega nebula at 5.7 GHz, (1b) previous spectrum convolved with a 1-MHz filter, (1c) a different smoothing scheme; (2) same as (1), but only four spectra of the Omega nebula; (3) average of all five spectra of Orion; (4) average of only three spectrograms from Orion; and (5) the average of the left and right sides of the spectra (centered about the line proposed position) in the Omega (a) and Orion (b) nebulae. The vertical line marks the radial velocity position of 0 km s^{-1} with respect to the Local Standard of Rest (LSR) for the $n_{105} \rightarrow n_{104}$ line

Based upon a closer analysis of the lines' expected properties and intensities with regard to their 22-m radio telescope (Sorochenko, 1965), the Lebedev group came to the conclusion that a new receiver would be needed – one with a sensitivity at least an order of magnitude greater than the existing spectrometers being used for observations of the $\lambda = 21 \text{ cm}$ line. The recombination lines were not only expected to be very weak in themselves but also expected to be weak with respect to the stronger background continuum emission emitted by the H II regions (see Fig. 1.1). In other words, the very objects in which the weak lines should appear would also be emitting strong background emission that would make detection more difficult.

To overcome these difficulties, the Lebedev staff developed a nulling-type spectral radiometer at a wavelength of $\lambda = 3.4$ cm using low noise parametric amplifiers. With great accuracy, this radiometer ensured that the noise in the 20-MHz band was the same for the source (antenna) and the reference load. In this way, it was insensitive to fluctuations of the background continuum such as pointing errors, changes in atmospheric emission, etc. At the same time, it was capable of detecting weak, narrow spectral lines superimposed upon the strong, background continuum emission.

On 27 April 1964, using this radiometer and the 22-m radio telescope of the Physical Institute in Pushchino shown in Fig. 1.4, Sorochenko and Borodzich (1965) detected the hydrogen radio line $n_{91} \rightarrow n_{90}$ (H90 α) at 8,872.5 MHz in the spectrum of the Omega nebula on their first attempt. Figure 1.5 shows these spectra. Unlike the earlier observations of the Pulkovo group 4 months earlier in December 1963, this line was clearly present even in the individual spectrograms. The specially designed receiver and the better observing conditions had made a definitive difference. Observations carried out over the next 3 months showed shifts in the line frequency corresponding to the Doppler shifts expected from the Earth's orbital rotation. These frequency shifts dispelled any doubts about the cosmic origin of the line.

Nearly simultaneously with the Lebedev group, the group (Dravskikh, Dravskikh, Kolbasov, Misezhnikov, Nikulin and Shteinshleiger, 1965) at Pulkovo observatory also convincingly detected an excited hydrogen line. Only a month separated these two detections. After improving their radiometer by installing a maser amplifier for the receiver, they were able to detect the hydrogen radio line $n_{105} \rightarrow n_{104}$ at 5,762.9 MHz with the 32-m radio telescope. This time, in May and July of 1964, there was no doubt. The H104 α line had definitely been detected in the Omega nebula. Figure 1.5 shows their spectra as well. The Doppler shift of line frequency due to orbital motion of Earth was also found, confirming this detection as well.

On 31 August 1964, the results of both groups were communicated to astronomers attending the XII General Assembly of the International Astronomical Union in Hamburg, Germany. In a joint session of Commissions 33, 34, and 40 organized by Westerhout, Yuri Parijskij presented a paper on behalf of Dravskikh et al. (1966), and Vitkevitch did the same for Sorochenko and Borodzich (1966). Figure 1.5 shows the first spectrograms of the excited hydrogen lines $n_{91} \rightarrow n_{90}$ and $n_{105} \rightarrow n_{104}$ with good S/R ratios that were presented to the IAU General Assembly.

At that presentation, there were a number of questions from the audience (Dravskikh, 1996). Accustomed to the much higher S/N ratios of

1.4 Spectral Lines in Radio Astronomy



Fig. 1.4 The 22-m radio telescope of the P.N. Lebedev Physical Institute in Pushchino, 100 km south of Moscow. With this instrument, the excited hydrogen line $n_{91} \rightarrow n_{90}$ was clearly detected on 27 April 1964

the $\lambda = 21 \,\mathrm{cm}$ radio spectra of atomic hydrogen and having experienced the technical difficulties of observing weak spectral lines at that time, many astronomers were skeptical of these clearly noisy results (Price, 2002). In addition, the visual material at that time was less than ideal, and the presentations came near the end of the day, being the 23rd and 24th papers of the 26 presented. Alan H. Barrett, codiscoverer of the second known radio astronomical line (OH) during the previous October (Weinreb, Barrett, Meeks and Henry, 1963), asked Parijskij, "Are you saying that you detected the excited hydrogen line $n = 105 \rightarrow 104$?" Parijskij replied, "Yes." Barrett repeated his question, "Are you saying that these lines can exist?" Parijskij again replied, "Yes." Despite their reservations, this exchange (Parijskij, 2002) also shows