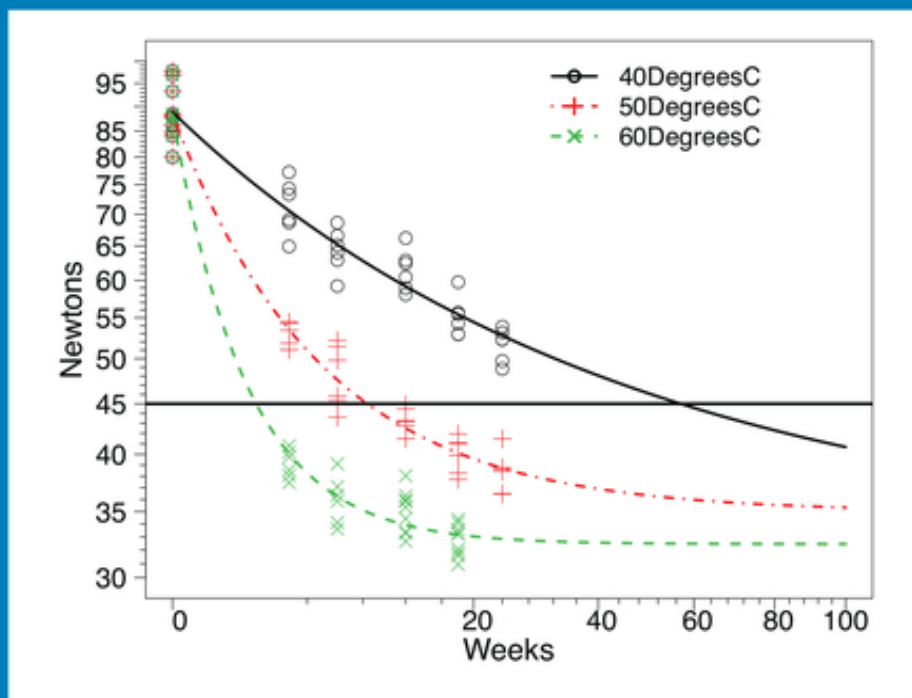


Wiley Series in Probability and Statistics

Second Edition

# STATISTICAL METHODS FOR RELIABILITY DATA

WILLIAM Q. MEEKER | LUIS A. ESCOBAR | FRANCIS G. PASCUAL



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# Statistical Methods for Reliability Data

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# Statistical Methods for Reliability Data

**Second Edition**

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To Karen, Katherine, Josh, Liam, and Ayla, and my parents.  
W. Q. M.

To my grandchildren: Olivia, Lillian, Nathaniel, Gabriel,  
Samuel, and Jackson.  
L. A. E.

To my parents and Brother Michael O'Keefe, F.M.S.  
F. G. P.



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# *Preface to the Second Edition*

The first edition of *Statistical Methods for Reliability Data* (SMRD1) was published 23 years ago. We believe SMRD1 successfully met its goal of providing a comprehensive overview of statistical methods for reliability data analysis and test planning for practitioners and statisticians and we have received much positive feedback. Despite, and perhaps because of this, there were compelling reasons for a second edition (SMRD2). We (both the profession and the authors) have learned much since 1998. In our experiences consulting on reliability applications and in presenting on the order of one hundred short courses to both statisticians and reliability engineers in industry and government, we have gained a solid appreciation for what are the most important topics in the field of statistical methods for reliability, suggesting a slight change in focus for SMRD2. In teaching our respective university courses, we have discovered some improved methods for presenting some topics. These experiences helped us to develop our plan for SMRD2. For the SMRD2 project, Bill and Luis have been most fortunate to have Jave Pascual join them as a co-author.

## **Goals for SMRD2**

Our goals for the SMRD2 were to:

- Update and improve or expand on various previously presented statistical methods for reliability data, using statistical and computational methods that have been developed or become readily available since SMRD1 was published.
- Improve the organization of the material to make it possible to cover more and the most important topics in a one-semester course.
- Completely rewrite chapters where there have been important developments or changes in what are considered to be best practices in the analysis of reliability data.
- Provide a more extensive treatment of the use and application of Bayesian methods in reliability data analysis.

- Provide, via technical appendices, additional justification and theory underlying the statistical methods presented in this book.
- Provide a webpage that gives up-to-date information about available software for doing reliability data analysis, supplementary information such as presentation slides, additional data sets, and exercises, as well as other important up-to-date information developed or coming to our attention after publication.

## What Has Not Changed

In SMRD2, we have paid special care to retain the appealing features of SMRD1. Specifically, the *special features of the book*, listed in the preface of SMRD1 are still intact.

## Important Changes in SMRD2

Important changes in SMRD2 include the following:

- Although SMRD1 has been popular with both engineers and statisticians, in the preparation of SMRD2 there has been a concerted effort to look for ways to improve the presentation and usability of the book. Means for doing this have included additional words of explanation, additional examples using simulation to provide insights, moving some technical details to appendices, and omitting topics that, while of some technical interest, have had little or no value in practical applications.
- We have added a section on the important topic of the distribution of remaining life to the chapter *Models, Censoring, and Likelihood for Failure-Time Data* (Chapter 2).
- We now include a section describing the important Fréchet distribution in the chapter *Some Parametric Distributions Used in Reliability Applications* (Chapter 4).
- SMRD1 Chapter 5 (*Other Parametric Distributions*) has been eliminated, with the important and useful material being moved to either Chapter 4 or the chapter *Special Parametric Models* (Chapter 11).
- The chapter *Parametric Bootstrap and Other Simulation-Based Confidence Interval Methods* (Chapter 9), has been completely rewritten to reflect many new developments since SMRD1 was published, including the use of the fractional-random-weight bootstrap and generalized pivotal quantities.
- The chapter *An Introduction to Bayesian Statistical Methods for Reliability* (Chapter 10) replaces, updates, and expands SMRD1 Chapter 14. The chapter has been completely rewritten with a more modern slant on prior distribution specification and computational methods, with several illustrations of Bayesian applications. Then, in most subsequent chapters, Bayesian methods are integrated into the development and presentation of many statistical analyses where some prior information is available (and ignoring it would be wrong) or where there is other strong motivation to use Bayesian methods.
- SMRD1 Chapter 10 has been completely rewritten to focus on *Planning Life Tests for Estimation* (now Chapter 13) and to improve clarity and usability of the material.
- The material in the chapter *Planning Reliability Demonstration Tests* (Chapter 14) is mostly new, where the SMRD1 material (previously in a small part of SMRD1 Chapter 10) has been completely rewritten and generalized to allow planning demonstration tests for any log-location-scale distribution and tests that allow failures to occur (and still