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Preface

The International Conference on "Recent Advances in Mechanical Engineering Research and Development (ICRAMERD 2021)" focused on the latest research findings and advances in the mechanical and materials engineering areas to society, specifically centered on the development of new materials and their applications, thermal engineering, design and manufacturing system engineering, industrial and system engineering, automotive engineering energy research, etc.

The proceedings received full research papers, experimental reports and review articles from different areas of mechanical engineering. All of these submissions went through a rigorous peer review process by inviting reviewers from different themes of national and international repute.

This conference received 156 papers from across the globe, and after peer review, only 79 research papers were selected and registered for presentations in 9 different parallel sessions according to the themes. Nine session chairs (9 in numbers) from various reputed institutes of our country were invited to preside over the session. The presented papers were evaluated based on their quality of work and presentations. Finally, one paper from each session was awarded as the best paper.

ICRAMERD 2021 invited a number of high-profile keynote speakers from India and abroad. Professor A. Rolstadas, Norwegian University of Science and Technology, Norway, and Prof. Jim Browne, President of Irish Academy of Engineering, jointly expressed their views on the topic "New Challenges in Manufacturing Engineering Education." Professor David Scott Sink, Senior Advisor, Poirier Group, Adjunct Professor Virginia Tech., Canada, discussed on the topic "Large Scale Enterprise Transformation," while Prof. Bopaya Bidanda from Pittsburgh, USA, showed light on the topic "Thriving under Uncertain and Disruptive Conditions." Professor Md. Mamun Habib from Independent University, Bangladesh, explained about the disruption of supply chain due to COVID-19.

Professor Pinakeswar Mahanta, Director, NIT, Arunachal Pradesh and Professor, IIT, Guwahati discussed on the topic "Recent Development in Co-Gasification of Biomass with Coal".

Eight different invited speakers from academic, research institutes and industries delivered lectures on their field of research and expertise.

We would like to thank everyone including authors, session chairs, reviewers, invited speakers, keynote speakers, organizing committee members, volunteers, students, media persons, and advisors for their contribution to make the event a success. We extend our sincere gratitude to Siksha ‘O’ Anusandhan, Deemed to be University as the host organization for support provided by the Institute for successfully co the event.

Bhubaneswar, India Dr. Premananda Pradhan

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About the Editors

Dr. Premananda Pradhan is the convenor of the International Conference (ICRAMERD-21) and is currently working as an associate professor at the Department of Mechanical Engineering, Institute of Technical Education and Research, affiliated to Siksha 'O' Anusandhan (Deemed to be University), Bhubaneswar, Odisha. He obtained his degree in mechanical engineering from the Institution of Engineers (Kolkata) and M.Tech. in Thermal Engineering from Siksha 'O' Anusandhan (Deemed to be University) and Ph.D. from KIIT University, Bhubaneswar, Odisha. Dr. Pradhan has got 13 years of working experience in reputed industries and 8 years of teaching and research experience. His major areas of research include thermal system design and optimization, combustion and heat transfer in porous media, bio-energy conversion, and thermal performance analysis of IC Engines and porous media combustion using both fossil and biofuels. Dr. Pradhan has published more than a dozen of research papers in reputed international journals and five conference proceedings. In addition to the above, he is engaged in several agricultural and medical equipment design projects of S'O'A (Deemed to be University). Dr. Pradhan has guided five M.Tech. scholars and several B.Tech. projects and at presently supervising two research Scholars.

Dr. Binayak Pattanayak is currently working as assistant professor at the Department of Mechanical Engineering, Siksha 'O' Anusandhan (Deemed to be University), Bhubaneswar. He obtained his B.Tech. (Mechanical Engineering) from Biju Patnaik University of Technology, Rourkela; M.Tech. and Ph.D. from the Siksha 'O' Anusandhan (Deemed to be University), Bhubaneswar. His major research area includes drying behavior of agricultural products, gasification of coal and biomass, dynamic modeling of solar collectors for power plant applications. He has more than 8 years of experience in teaching and research and published more than 10 papers in reputed international journals and conferences. He has guided more than 20 B.Tech. projects, five M.Tech. projects and currently two Ph.D. research scholars are enrolled under his supervision. He has also actively participated as a reviewer in peer reviewed journals as well as international/national conference proceedings.

Dr. Pattanayak has also conducted/organized academic-industrial interaction programs among the research community in the institute level.

Prof. Harish Chandra Das is currently working as Professor at Department of Mechanical Engineering, NIT Meghalaya. He has obtained his Ph.D. from NIT Rourkela. The research interest of Prof. Das includes vibration analysis, composite materials and explosion modeling. He has more than 30 years of teaching and research experience with more than 60 national and international publications. He has guided 12 Ph.D. scholars, 25 M.Tech. projects, and numerous B.Tech. projects. He has successfully completed more than 10 industrial consultancy projects. Currently, he holds the position of the Dean (Faculty Welfare) and Chairman (IQAC) at NIT, Meghalaya.

Prof. Pinakeswar Mahanta is currently the Director of National Institute of Technology Arunachal Pradesh, Yupia, Arunachal Pradesh, India. Ha has served as professor of mechanical engineering at IIT Guwahati. He did his Bachelors in Mechanical Engineering from Regional Engineering College, Rourkela in 1985. His areas of research include thermodynamics, heat transfer and renewable energy. He has established an international platform for research in bio-energy with the University of Nottingham, University of Birmingham, and Loughborough University, UK. He also has established collaboration in the field of clean coal technology with the University of Cranfield, UK and UCL, Belguim. He has published more than 120 papers in peer reviewed international journals, and edited four books. He has guided/mentored more than 30 B.Tech. projects, 35 M.Tech. projects, 18 PhD students and one PDF. Professor Mahanta's latest achievements include JSPS fellowship by invitation to Gifu University, Japan, Guest Lecturer at Hof University, Germany, and Indian Distinguishing visiting fellowship by the University of Nottingham, UK.

Chaotic Oscillations in Axially Travelling String with Time Varying Velocity

Shashendra Kumar Sahoo, Lokanath Panda, and Harish Chandra Das

Abstract The chaotic oscillations of an axially travelling string with harmonic fluctuation in velocity are investigated. The partial differential equation of motion is discretized via Galerkin approach. Both single-mode and two-mode expansions are considered. Numerical simulations are performed to study the system behaviour with variation in control parameters. The numerical simulation indicates the existence of complex dynamical behaviours including chaos and period-doubling bifurcations for the two-mode expansion. The single-mode expansion exhibits periodic and quasiperiodic oscillations.

Keywords Axially travelling string · Galerkin approach · Periodic · Quasiperiodic · Period-doubling · Chaotic

1 Introduction

The linear and nonlinear vibration of travelling strings have been investigated for last sixty years. The harmonic fluctuation in velocity is one of the main causes of nonlinear oscillations of travelling string. The chaotic oscillations are dangerous among all the nonlinear oscillations of the string because the amplitude of vibration is larger than that of periodic and quasiperiodic vibration. Using modal analysis, Swope and Ames [1] examined the vibration behaviour of axially travelling string. Applying Floquet theory, Pakdemirli et al. [3, 4] explored the vibration behaviour of travelling string via Galerkin method. Chen et al. [5, 6] examined the bifurcation

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and chaotic behaviour for a travelling string with viscoelastic damping via Galerkin approach. Ghayesh et al. [7] studied the vibration characteristics of translating string having an elastic foundation applying multi-timescales analysis. Kesimili et al. [8] analysed the dynamic behaviour of axially moving string with number of supports applying multi-timescale analysis. Yang et al. [9] examined the nonlinear vibration behaviour of axially translating string applying gyroscopic modes of decoupling.

The paper investigates the chaotic oscillations of axially travelling string having harmonic fluctuation in velocity. The governing partial differential equation is discretized via Galerkin approach. Both single-mode and two-mode expansions are considered. The numerical simulation is performed to study the system behaviour with variation in control parameters. The numerical simulation shows a cascade of period-doubling bifurcations culminating to chaos for two-mode expansion. However, single-mode expansion exhibits periodic and quasiperiodic oscillations.

2 The Analytical Model

We consider a string having length *L*, cross-sectional area *A*, linear density ρ and axial tension *P* travelling over two pulleys as shown in Fig. 1.

The equation governing the transverse deflection of string is written as [3]

$$\rho A(\ddot{w} + \dot{v}w' + 2v\dot{w}') + (\kappa \rho A v^2 - P)w'' = 0 \tag{1}$$

with boundary conditions

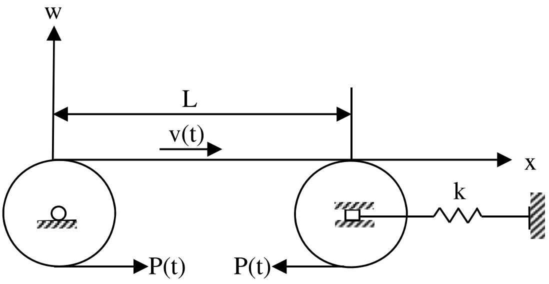
$$w(0, t) = w(L, t) = 0 \tag{2}$$

where $w(x, t)$ is the transverse deflection of the string. The dot and prime are used to denote partial derivatives with respect to t and x, respectively.

The axial tension of the string is given as [2]

$$P = P_0 + \eta \rho A v^2 \tag{3}$$

Fig. 1 Physical model of travelling string



where $0 \leq \eta \leq 1$ and the axial support rigidity parameter η is given by

$$\eta = 1 - \kappa \tag{4}$$

Inserting Eqs. (3) and (4) in Eq. (1), we get

$$\begin{aligned} &\rho A(\ddot{w} + \dot{v}w' + 2v\dot{w}') + (k\rho A v^2 - P_0)w'' = 0 \\ &w(0, t) = w(L, t) = 0 \end{aligned} \tag{5}$$

3 Discretization

According to Galerkin approach, the solution to Eq. (5) may be assumed to have the form

$$w(x, t) = \sum_{n=1}^{N} q_n(t)\varphi_n(x) \tag{6}$$

where $q_n(t)$ and $\varphi_n(x)$ are the generalized coordinates and mode shapes of the stationary string, respectively.

The mode shapes of the stationary string satisfying the geometric boundary conditions is given by

$$\varphi_n(x) = \sin(n\pi x/L), \quad n = 1, 2, 3, \ldots N. \tag{7}$$

Inserting Eq. (6) in Eq. (5) and using Galerkin approach and letting $N = 1$, we get for single-mode expansion the linear ordinary differential equation of motion as

$$\ddot{q}_1 + \left(\frac{p_0}{\rho A} - \kappa v^2\right)\frac{\pi^2}{L^2}q_1 = 0 \tag{8}$$

Letting $N = 2$, we get for two-mode expansion linear ordinary differential equations which are gyroscopically coupled as

$$\ddot{q}_1 - \frac{16v}{3L}\dot{q}_2 + \left(\frac{p_0}{\rho A} - \kappa v^2\right)\frac{\pi^2}{L^2}q_1 - \frac{8\dot{v}}{3L}q_2 = 0 \tag{9}$$

$$\ddot{q}_1 + \frac{16v}{3L}\dot{q}_1 + \frac{8\dot{v}}{3L}q_1 + \left(\frac{P_0}{\rho A} - \kappa v^2\right)\frac{4\pi^2}{L^2}q_2 = 0 \tag{10}$$

Assuming the axial velocity of the string to vary harmonically with time

$$v(t) = V \sin \Omega t \tag{11}$$

where V and Ω are velocity amplitude and fluctuation frequency, respectively.

4 Numerical Simulation

Numerical simulation is carried out to examine the system behaviour with variation in control parameters, viz. velocity amplitude and fluctuation frequency. The time trace, phase plane and frequency spectra are plotted. The physical parameters values are given in Table 1. The initial conditions chosen are $q_1(0) = q_2(0) = 0$ and $\dot{q}_1 = \dot{q}_2(0) = 0.001$.

4.1 Single-Mode Expansion

Figure 2 shows periodic oscillations for single-mode expansion at $V = 10$ m/s and $\Omega = 20$ rad/s. The phase portrait is a closed trajectory, and frequency spectrum shows a sharp peak at a single frequency. Figure 3 shows quasiperiodic oscillations for V

Table 1 Physical parameters for a band-saw [2]

Physical parameter	Parameter value	Unit
P_0	76.22	N
ρ	7754.0	Kg/m^3
A	0.5201×10^{-5}	m^2
k	0.22	–
L	0.3681	m

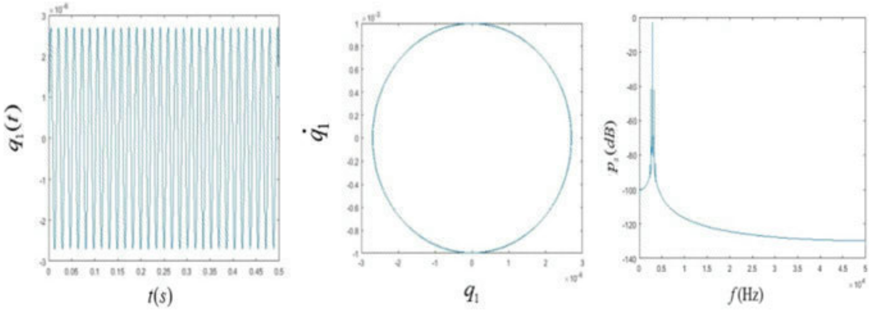


Fig. 2 Periodic oscillations at $V = 10$ m/s and $\Omega = 20$ rad/s (Time trace, phase plane and frequency spectrum)

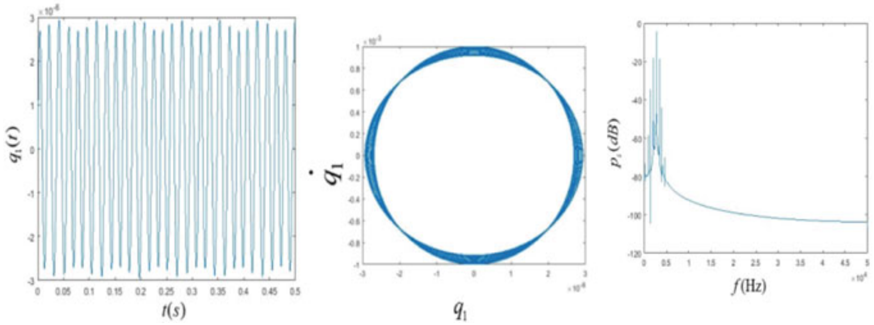


Fig. 3 Quasiperiodic oscillations at V = 30 m/s and Ω = 20 rad/s (Time trace, phase plane and frequency spectrum)

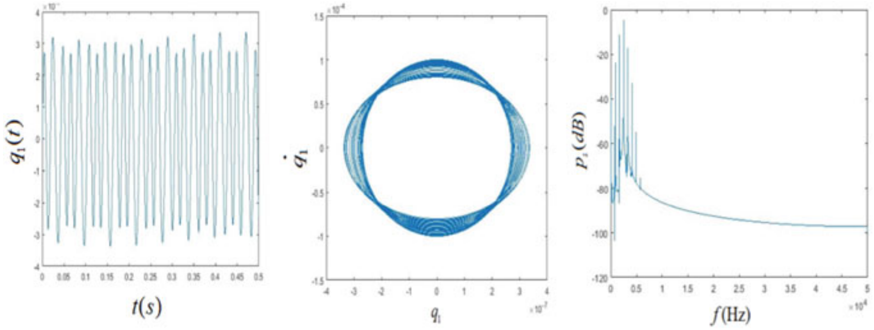


Fig. 4 Quasiperiodic oscillations at V = 50 m/s and Ω = 20 rad/s (Time trace, phase plane and frequency spectrum)

= 30 m/s and Ω = 20 rad/s. Figure 4 also shows quasiperiodic oscillations at V = 50 m/s and Ω = 20 rad/s.

4.2 Two-Mode Expansion

Figures 5 and 6 show that both the first and second modes undergo cascade of period-doubling bifurcations at V = 10 m/s and Ω = 20 rad/s. Figures 7 and 8 show chaotic oscillations for both first and second modes at V = 30 m/s and Ω = 20 rad/s. The first mode shows ‘wide-band’ chaotic oscillation, whereas second mode shows ‘narrow-band’ chaotic oscillation. Figures 9 and 10 show strong chaotic oscillations for both first and second modes at V = 50 m/s and Ω = 20 rad/. Again the first mode shows ‘wide-band’ chaotic oscillation, whereas the second mode shows ‘narrow-band’ chaotic oscillation.

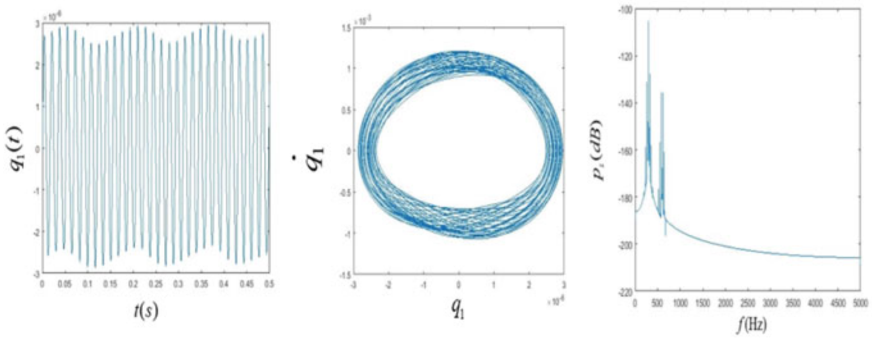


Fig. 5 Cascade of period-doubling bifurcations at $V = 10$ m/s and $\Omega = 20$ rad/s for the first mode (Time trace, phase plane and frequency spectrum)

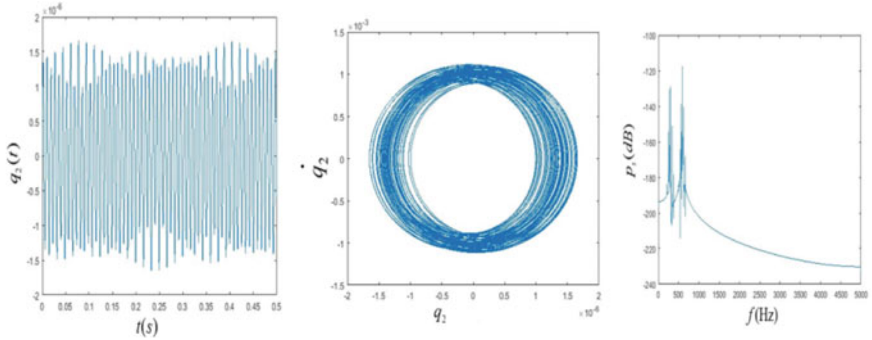


Fig. 6 Cascade of period-doubling bifurcations at $V = 10$ m/s and $\Omega = 20$ rad/s for the second mode (Time trace, phase plane and frequency spectrum)

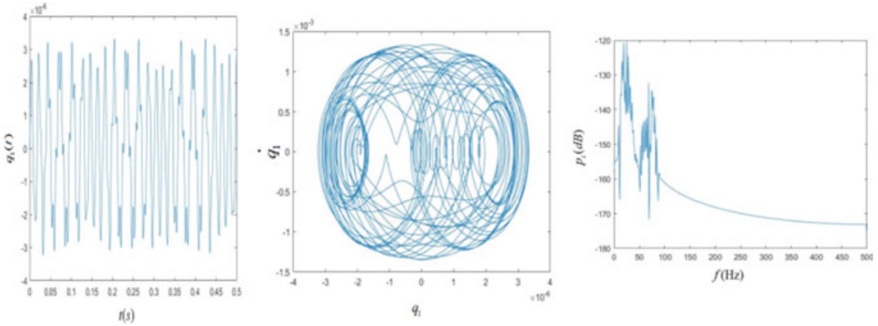


Fig. 7 Wide-band chaotic oscillations at $V = 30$ m/s and $\Omega = 20$ rad/s for first mode (Time trace, phase plane and frequency spectrum)

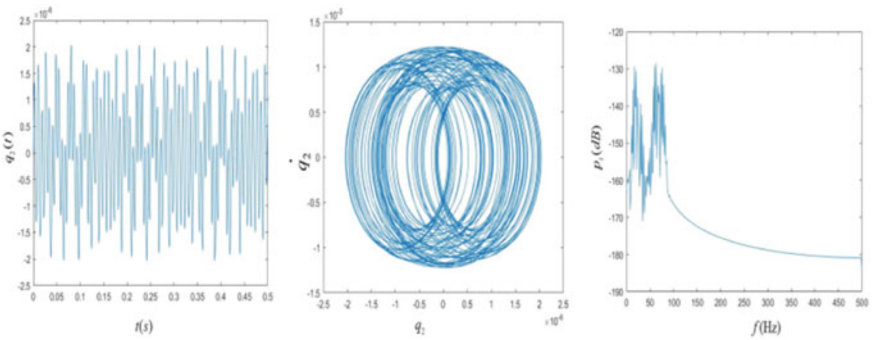


Fig. 8 Narrow-band chaotic oscillations at $V = 30$ m/s and $\Omega = 20$ rad/s for the second mode (Time trace, phase plane and frequency spectrum)

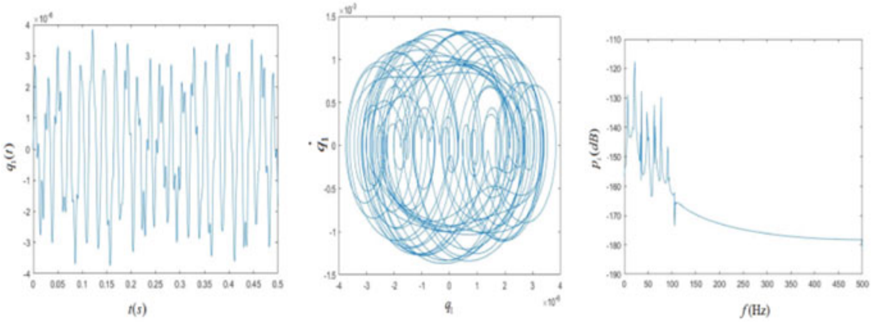


Fig. 9 Wide-band chaotic oscillations at $V = 50$ m/s and $\Omega = 20$ rad/s for the first mode (Time trace, phase plane and frequency spectrum)

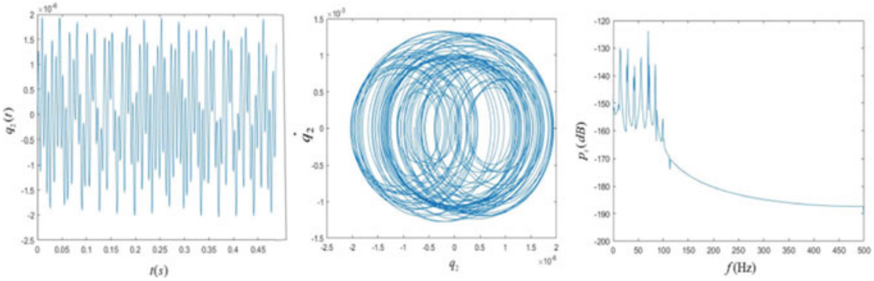


Fig. 10 Narrow-band chaotic oscillations at $V = 50$ m/s and $\Omega = 20$ rad/s for the second mode (Time trace, phase plane and frequency spectrum)

5 Conclusion

The chaotic oscillations of travelling string having harmonic velocity fluctuations are explored. The governing equation of motion is discretized applying Galerkin approach. Numerical analysis is performed to study the system behaviour with variation in control parameters. The numerical analysis yields the following conclusions:

1. The nonlinear vibration response of the system depends on initial motion conditions.
2. The single-mode expansion exhibits periodic and quasiperiodic oscillations.
3. The two-mode expansion undergo a cascade of period-doubling bifurcations leading to chaotic oscillations. The first mode shows ‘wide-band’ chaotic oscillation, whereas the second mode shows ‘narrow-band’ chaotic oscillation.

References

1. Swope RD, Ames WF (1963) Vibrations of a moving thread line. J Franklin Inst 275:36–55
2. Mote CDJ (1965) A study of band-saw vibrations. J Franklin Inst 279(6):431–444
3. Pakdemirli M, Ulsoy AG, Ceranoglu A (1994) Transverse vibration of an axially accelerating string. J Sound Vib 169(2):179–196
4. Pakdemirli M, Ulsoy AG (1997) Stability analysis of an axially accelerating string. J Sound Vib 203:815–832
5. Chen LQ, Zhang NH, Zu JW (2002) The regular and chaotic vibrations of an axially moving viscoelastic string based on 4-order Galerkin Truncation. J Sound Vib 261(4):764–773
6. Chen LQ, Zhang NH, Zu JW (2003) Bifurcation and chaos of an axially moving viscoelastic string. Mech Res Commun 29(2/3):81–90
7. Ghayesh MH (2009) Stability characteristics of an axially accelerating string supported by an elastic foundation. Mechasism Mach Theory 44:1964–1979
8. Kesimili A, Ozkaya E, Bagdatli SM (2015) Nonlinear vibrations of spring supported axially moving string. Nonlinear Dyn 81:1523–1534
9. Yang XD, Wu H, Qian YJ, Zhang W, Lim CW (2017) Nonlinear vibration analysis of axially moving strings based on gyroscopic modes of decoupling. J Sound Vib 393:308–320

Nonlinear Dynamics of Translating String with Geometric Nonlinearity

Shashendra Kumar Sahoo, Lokanath Panda, and Harish Chandra Das

Abstract The dynamic response of translating string considering geometric nonlinearity is investigated. The nonlinear partial differential equation is nondimensionalized first and then discretized via Galerkin approach. Taking two mode truncation gives gyroscopically coupled nonlinear ordinary differential equations. Numerical simulation is performed applying Runge-kutta technique. Time trace, phase plane and frequency spectra are plotted to examine the influence of non-dimensional velocity on dynamic response of axially translating string. The numerical simulations indicate that the system exhibits both periodic and quasiperiodic oscillations.

Keywords Translating string · Geometric nonlinearity · Galerkin approach · Quasi-periodic · Periodic

1 Introduction

The nonlinear dynamics of axially translating string has been studied since last sixty years. The axially translating string forms the classical model for many mechanical devices such as aerial cables, power transmission belt, textile fiber, magnetic tapes, paper sheets, serpentine belts. Applying the theory of characteristics, Mote [1, 2] studied the dynamics of axially traveling string and verified that the fundamental period of vibrations is dependent on initial tension and the axial velocity. Applying the method of multi-timescales, Chen et al. [3, 4] studied the nonlinear dynamics and chaos of axially accelerating viscoelastic string via Galerkin method. Applying

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method of multi-time scales, Ghayesh [5] investigated the parametric combination resonance of an axially translating string having an elastic foundation. Applying multi-timescale analysis, Kesimili et al. [6] investigated the vibration behavior of axially translating string consisting of number of supports. Basing on gyroscopic modes of decoupling Yang et al. [7] analyzed the vibration behavior of axially traveling string. Applying assumed mode and linear superposition method, Lu et al. [8] investigated the dynamic behavior of an axially traveling string having time-varying supports.

The present work focuses on the nonlinear dynamic response of the axially translating string considering geometric nonlinearity. The nonlinear partial differential equation is nondimensionalized and then discretized via Galerkin approach. The two mode truncation results in gyroscopically coupled nonlinear ordinary differential equations. The vibration behavior is highlighted through time trace, phase plane diagram and frequency spectra for different values of nondimensional velocity.

2 The Analytical Model

The schematic model consists of a string of length L, linear density ρ, Young's modulus E, axial tension P traveling between two fixed eyelets with an axial velocity V as shown in Fig. 1.

The equation of motion for transverse vibration of axially translating string considering geometric nonlinearity can be expressed as [1].

$$\rho W,_{TT} + 2\rho V W,_{XT} + (\rho V^2 - P) W,_{XX} = \frac{3}{2} E (W,_X)^2 W,_{XX} \tag{1}$$

with boundary conditions

$$W(0, T) = W(L, T) = 0 \tag{2}$$

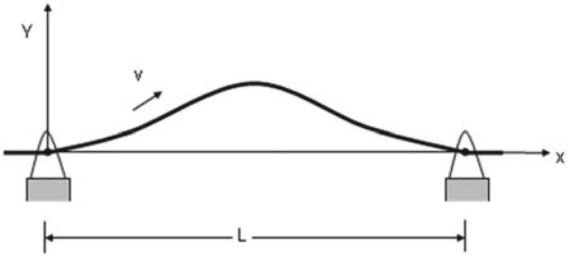

Fig. 1 Physical model of an axially translating string

$W(X, T)$ being the transverse deflection of the axially translating string and comma-subscript notation represents partial differentiation with respect to coordinate X and time T. The different terms ρW_{TT},$2\rho V W_{XT}$ and $\rho V^2 W_{XX}$ represent inertia force, coriolis force and centrifugal force respectively.

Defining nondimensional variables

$$w = \frac{W}{L}, x = \frac{X}{L}, t = T\sqrt{P/\rho L^2}, \gamma = V\sqrt{\rho/P}, \kappa = E/P \tag{3}$$

Inserting Eq. (3) into Eq. (1) yields the dimensionless governing equation

$$w_{,tt} + 2\gamma w_{,xt} + (\gamma^2 - 1)w_{,xx} = \frac{3}{2}\kappa w_{,x}^2 w_{,xx} \tag{4}$$

with dimensionless boundary conditions

$$w(0, t) = w(1, t) = 0 \tag{5}$$

3 Discretization

According to Galerkin's approach, the solution to partial differential equation of motion (4) may be assumed to be

$$w(x, t) = \sum_{r=1}^{N} q_r(t) \sin(r\pi x) \tag{6}$$

where $q_r(t)$ and $\sin(r\pi x)$ represent modal coordinate and the mode shapes of the stationary string respectively.

Substituting Eq. (6) into Eq. (4), and using Galerkin approach, yields a series of nonlinear ordinary differential equations

$$\begin{aligned} &\ddot{q}_r + 8\gamma \sum_{r+j_{\text{odd}}}^{N} \frac{nr}{r^2 - j^2}\dot{q}_j + (1 - \gamma^2)(r\pi)^2 q_r \\ &= 3\kappa \sum_{j=1}^{N}\sum_{s=1}^{N}\sum_{m=1}^{N} d_{jsmr} q_j q_s q_m (r = 1, 2, \ldots N) \end{aligned} \tag{7}$$

where

$$d_{jsmr} = \begin{cases} \dfrac{jsmr\pi^4}{8}, j+s = \pm(m+r), \pm(m-r) \text{ or} \\ j-s = \pm(m+r), \pm(m-r) \\ 0, \text{otherwise} \end{cases} \tag{8}$$

Taking two mode truncations ($N = 2$), yields the nonlinear ordinary differential equations that are gyroscopically coupled

$$\ddot{q}_1 - \frac{16}{3}\gamma\,\dot{q}_2 + (1-\gamma^2)\pi^2 q_1 + \frac{3}{8}\kappa\pi^4 q_1^3 + 3\kappa\pi^4 q_1 q_2^2 = 0 \tag{9}$$

$$\ddot{q}_2 + \frac{16}{3}\gamma\,\dot{q}_1 + 4(1-\gamma^2)\pi^2 q_2 + 3\kappa\pi^4 q_1^2 q_2 + 6\kappa\pi^4 q_2^3 = 0 \tag{10}$$

4 Numerical Simulation

The numerical simulation is carried out using Runge-Kutta technique. The initial conditions chosen are $q_1(0) = q_2(0) = 0$ and $\dot{q}_1(0) = \dot{q}_2(0) = 0.01$.The effect of dimensionless velocity (γ) on the nonlinear dynamic response is investigated through time trace, phase plane trajectory and frequency spectra. The Figs. 2 and 3 show periodic oscillations at nondimensional velocity, $\gamma = 0$ for both first and second modes respectively. The phase plane is a closed trajectory and a dominant peak at single basic frequency is observed in the power spectrum. The Figs. 4 and 5 shows quasiperiodic oscillations at nondimensional velocity $\gamma = 0.2$ for both first and second modes respectively. The time trace shows beating effect for both the modes. The phase plane trajectories for both the modes represents a torus. The power spectra show two sharp peaks at two different basic frequencies. The Figs. 6 and 7 show quasiperiodic oscillations at nondimensional velocity $\gamma = 0.5$ for both the

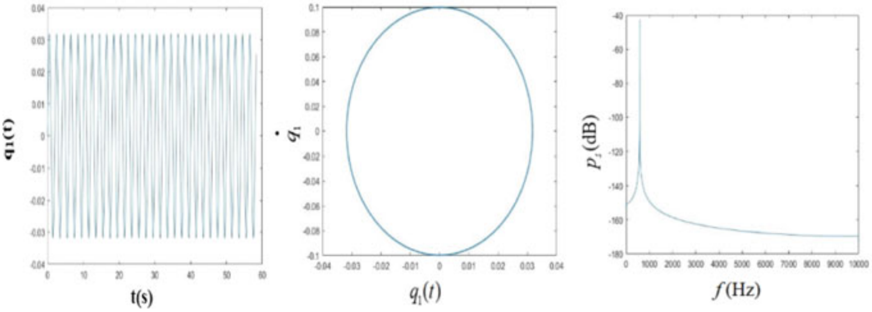

Fig. 2 Periodic oscillations at non-dimensional velocity $\gamma = 0$ (Time trace, phase plane and frequency spectrum of first mode)

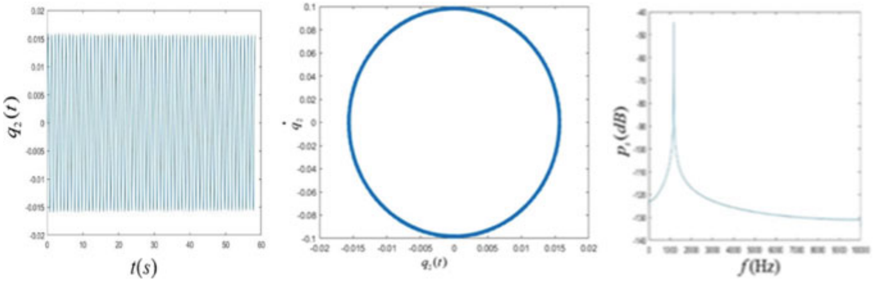

Fig. 3 Periodic oscillations at non-dimensional velocity $\gamma = 0$ (Time trace, phase plane and frequency spectrum of second mode)

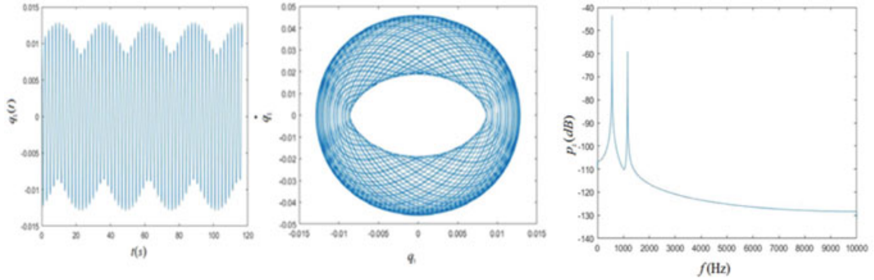

Fig. 4 Quasiperiodic oscillations at non-dimensional velocity $\gamma = 0.2$ (Time trace, phase plane and frequency spectrum of first mode)

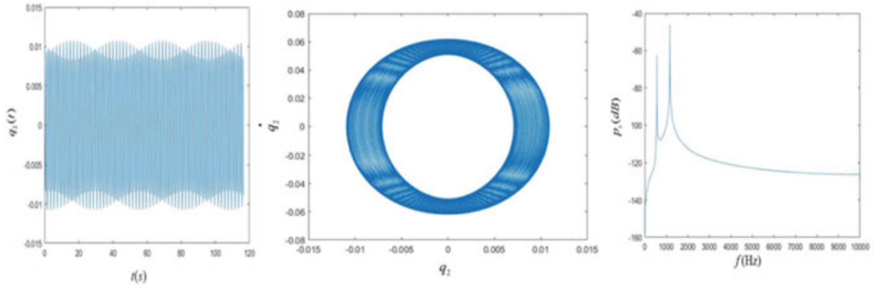

Fig. 5 Quasiperiodic oscillations at non-dimensional velocity $\gamma = 0.2$ (Time trace, phase plane and frequency spectrum of second mode)

modes. The phase plane trajectories of both the modes represent a torus. The power spectra show sharp peaks at two basic frequencies. The Figs. 8 and 9 show again quasiperiodic oscillations at nondimensional velocity $\gamma = 0.7$ for both the modes. The Figs. 10 and 11 show periodic oscillations at nondimensional velocity $\gamma = 1.0$ for both the modes. The phase plane diagram represent a closed trajectory for both the modes. The power spectra show dominant peak at single basic frequency.