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Algebra I

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Mary Jane Sterling
Algebra Queen



Algebra I

ALL-IN-ONE

by Mary Jane Sterling

for
dummies[®]
A Wiley Brand

Algebra I All-in-One For Dummies®

Published by: **John Wiley & Sons, Inc.**, 111 River Street, Hoboken, NJ 07030-5774, www.wiley.com

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Published simultaneously in Canada

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Library of Congress Control Number: 2021947882

ISBN 978-1-119-84304-7 (pbk); ISBN 978-1-119-84305-4 (ebk); ISBN 978-1-119-84306-1 (ebk)

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Introduction

“**W**hat is algebra?” “Is it really that important in the study of other math courses?” “Where did it come from?” And my favorite question from students: “What do I need this for?”

Algebra is really the basis of most courses that you take in high school and college. You can’t do anything in calculus without a good algebra background. And there’s lots of algebra in geometry. You even need algebra in computer science! Algebra was created, modified, and continues to be tweaked so that ideas and procedures can be shared by everyone. With all people speaking the same “language,” there are fewer misinterpretations.

Algebra, or *al-jabr* in Arabic, means “a reunion of broken pieces.” How appropriate!

About This Book

This book covers just about everything you’d ever want to know about basic algebra. And it provides opportunities for further discoveries. You’ll find explanations, examples, practice opportunities, and problems to test your comprehension. This book starts with basic operations and terminology, gives you information on simplifying and organizing expressions, runs through equation-solving, introduces applications, and goes visual with the graphing. When finished, you should:

- » Be familiar with notation and terminology.
- » Have confidence in finding the correct answer.
- » Look forward to more challenges with Algebra II and other courses.

Each new topic provides:

- » Example problems with answers and solutions.
- » Practice problems with answers and solutions.

Each chapter provides:

- » A test with problems representing the topics covered.
- » Solutions to the test problems.

Online quizzes are also available for even more practice and confidence-building.

This book also has a few conventions to keep in mind:

- » New terms introduced in a chapter, as well as variables, are in *italics*.
- » Keywords in lists and numbered steps are in **boldface**.
- » Any websites appear in monofont.
- » The final answers to problems appear in **bold**. Then the explanation follows.

Foolish Assumptions

You are reading this book to learn more about algebra, so I'm assuming that you have some of the other basic math skills coming in: familiarity with fractions and their operations, comfort with handling decimals and the operations involved, some experience with integers (signed whole numbers) and how they operate, and some graphing knowledge — how to place points on a graphing plane. If you don't have as much knowledge as you'd like related to some items mentioned, you might want to refer to some resources such as *Basic Math & Pre-Algebra For Dummies* or *Pre-Algebra Essentials For Dummies*.

I am also assuming that you're as excited about mathematics as I am. Oh, okay, you don't have to be that excited. But you're interested and eager and anxious to increase your mathematical abilities. That's the main thing you need.

Icons Used in This Book

You'll see the following five icons throughout the book:



EXAMPLE

Each example is an algebra question based on the discussion and explanation, followed by a solution. Work through these examples, and then refer to them to help you solve the practice problems that follow them as well as the quiz questions at the end of the chapter.



REMEMBER

This icon points out important information that you need to focus on. Make sure you understand this information fully before moving on. You can skim through these icons when reading a chapter to make sure you remember the highlights.



TIP

Tips are hints that can help speed you along when answering a question. See whether you find them useful when working on practice problems.



WARNING

This icon flags common mistakes that students make if they're not careful. Take note and proceed with caution!



When you see this icon, it's time to put on your thinking cap and work out a few practice problems on your own. The answers and detailed solutions are available so you can feel confident about your progress.

Beyond the Book

In addition to what you're reading right now, this book comes with a Cheat Sheet that provides quick access to some formulas and rules and processes that are frequently used. To get this Cheat Sheet, simply go to www.dummies.com and type **Algebra I All In One For Dummies Cheat Sheet** in the Search box.

You'll also have access to online quizzes related to each chapter. These quizzes provide a whole new set of problems for practice and confidence-building. To access the quizzes, follow these simple steps:

1. **Register your book or ebook at Dummies.com to get your PIN.** Go to www.dummies.com/go/getaccess.
2. **Select your product from the drop-down list on that page.**
3. **Follow the prompts to validate your product, and then check your email for a confirmation message that includes your PIN and instructions for logging in.**

If you do not receive this email within two hours, please check your spam folder before contacting us through our Technical Support website at <http://support.wiley.com> or by phone at 877-762-2974.

Now you're ready to go! You can come back to the practice material as often as you want — simply log on with the username and password you created during your initial login. No need to enter the access code a second time.

Your registration is good for one year from the day you activate your PIN.

Where to Go from Here

This book is organized so that you can safely move from whichever chapter you choose to start with and in whatever order you like. You can strengthen skills you feel less confident in or work on those that need some attention.

If you haven't worked on any algebra recently, I'd recommend that you start out with Chapter 1 and some other chapters in the first unit. It's important to know the vocabulary and basic notation so you understand what is being presented in later chapters.

If you're all set with the basic operations, then a good place to go is Chapter 11, where you find out about factoring different types of expressions. Factoring makes it possible to simplify expressions and solve equations.

When you're ready for the "What do I need this for?" question, go to Chapter 20 or Chapter 21, where you see some of the many types of applications that use algebraic expressions and solutions.

There are other resources, such as *Basic Math & Pre-Algebra For Dummies* and *Pre-Algebra Essentials For Dummies* (John Wiley & Sons, Inc.), if you think you need more background. And, of course, when you've finished here and are ready for the next challenge, be sure to check out *Algebra II For Dummies* and *Pre-Calculus For Dummies*. And that's just the beginning!

1

**Starting Out with
Numbers and
Properties**

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IN THIS CHAPTER

- » Identifying the different types of numbers
- » Placing numbers on a number line
- » Becoming familiar with the vocabulary
- » Recognizing the operations of algebra

Chapter 1

Assembling Your Tools: Number Systems

You've undoubtedly heard the word *algebra* on many occasions, and you know that it has something to do with mathematics. Perhaps you remember that algebra has enough stuff in it to require taking two separate high school algebra classes — Algebra I and Algebra II. But what exactly is algebra? What is it really used for?

This book answers these questions and more, providing the straight scoop on some of the contributions to algebra's development, what it's good for, how algebra is used, and what tools you need to make it happen. In this chapter, you find some of the basics necessary to make it easier to find your way through the different topics in this book.

In a nutshell, *algebra* is a way of generalizing arithmetic. Through the use of *variables* (letters representing numbers) and formulas or equations involving those variables, you solve problems. The problems may be in terms of practical applications, or they may be puzzles for the pure pleasure of the solving. Algebra uses positive and negative numbers, integers, fractions, operations, and symbols to analyze the relationships between values. It's a systematic study of numbers and their relationships, and it uses specific rules.

Identifying Numbers by Name

Where would mathematics and algebra be without numbers? Numbers aren't just a part of everyday life, they are the basic building blocks of algebra. Numbers give you a value to work with. Where would civilization be today if not for numbers? Without numbers to figure the distances, slants, heights, and directions, the pyramids would never have been built. Without numbers to figure out navigational points, the Vikings would never have left Scandinavia. Without numbers to examine distance in space, humankind could not have landed on the moon.

Even the simplest tasks and the most common of circumstances require a knowledge of numbers. Suppose that you wanted to figure the amount of gasoline it takes to get from home to work and back each day. You need a number for the total miles between your home and business, and another number for the total miles your car can run on a gallon of gasoline.

The different sets of numbers are important because what they look like and how they behave can set the scene for particular situations or help to solve particular problems. It's sometimes really convenient to declare, "I'm only going to look at whole-number answers," because whole numbers don't include fractions or negatives. You could easily end up with a fraction if you're working through a problem that involves a number of cars or people. Who wants half a car or, heaven forbid, a third of a person?

Algebra uses different sets of numbers, in different circumstances. I describe the different types of numbers here.

Realizing real numbers

Real numbers are just what the name implies. In contrast to imaginary numbers, they represent *real* values — no pretend or make-believe. Real numbers cover the gamut and can take on any form — fractions or whole numbers, decimal numbers that can go on forever and ever without end, positives and negatives. The variations on the theme are endless.

Counting on natural numbers

A *natural number* (also called a *counting number*) is a number that comes naturally. What numbers did you first use? Remember someone asking, "How old are you?" You proudly held up four fingers and said, "Four!" The natural numbers are the numbers starting with 1 and going up by ones: 1, 2, 3, 4, 5, 6, 7, and so on into infinity. You'll find lots of counting numbers in Chapter 8, where I discuss prime numbers and factorizations.

Whittling out whole numbers

Whole numbers aren't a whole lot different from natural numbers. Whole numbers are just all the natural numbers plus a 0: 0, 1, 2, 3, 4, 5, and so on into infinity.

Whole numbers act like natural numbers and are used when whole amounts (no fractions) are required. Zero can also indicate none. Algebraic problems often require you to round the answer to the nearest whole number. This makes perfect sense when the problem involves people, cars, animals, houses, or anything that shouldn't be cut into pieces.

Integrating integers

Integers allow you to broaden your horizons a bit. Integers incorporate all the qualities of whole numbers and their opposites (called their *additive inverses*). *Integers* can be described as being positive and negative whole numbers and zero: $-3, -2, -1, 0, 1, 2, 3$.

Integers are popular in algebra. When you solve a long, complicated problem and come up with an integer, you can be joyous because your answer is probably right. After all, it's not a fraction! This doesn't mean that answers in algebra can't be fractions or decimals. It's just that most textbooks and reference books try to stick with nice answers to increase the comfort level and avoid confusion. This is my plan in this book, too. After all, who wants a messy answer — even though, in real life, that's more often the case. I use integers in Chapter 14 and those later on, where you find out how to solve equations.

Being reasonable: Rational numbers

Rational numbers act rationally! What does that mean? In this case, acting rationally means that the decimal equivalent of the rational number behaves. The decimal eventually ends somewhere, or it has a repeating pattern to it. That's what constitutes "behaving."

Some rational numbers have decimals that end such as: $3.4, 5.77623, -4.5$. Other rational numbers have decimals that repeat the same pattern, such as 3.164164164 , or $0.6666666\bar{6}$. The horizontal bar over the 64 and the 6 lets you know that these numbers repeat forever.

In *all* cases, rational numbers can be written as fractions. Each rational number has a fraction that it's equal to. So one definition of a *rational number* is any number that can be written as a fraction, $\frac{p}{q}$, where p and q are integers (except q can't be 0). If a number can't be written as a fraction, then it isn't a rational number. Rational numbers appear in Chapter 16, where you see quadratic equations, and later, when the applications are presented.

Restraining irrational numbers

Irrational numbers are just what you may expect from their name: the opposite of rational numbers. An *irrational number* cannot be written as a fraction, and decimal values for irrationals never end and never have a nice pattern to them. Whew! Talk about irrational! For example, π , with its never-ending decimal places, is irrational. Irrational numbers are often created when using the quadratic formula, as you see in Chapter 16, because you find the square roots of numbers that are not perfect squares, such as: $\sqrt{6}$ and $\sqrt{85}$.

Picking out primes and composites

A number is considered to be *prime* if it can be divided evenly only by 1 and by itself. The prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and so on. The only prime number that's even is 2, the first prime number. Mathematicians have been studying prime numbers for centuries, and prime numbers have them stumped. No one has ever found a formula for producing all the primes. Mathematicians just assume that prime numbers go on forever.

A number is *composite* if it isn't prime — if it can be divided by at least one number other than 1 and itself. So the number 12 is composite because it's divisible by 1, 2, 3, 4, 6, and 12. Chapter 8 deals with primes, but you also see them throughout the chapters, where I show you how to factor primes out of expressions.

Numbers can be classified in more than one way, the same way that a person can be classified as male or female, tall or short, blonde or brunette, and so on. The number -3 is negative, it's an integer, it's an odd number, it's rational, and it's real. The number -3 is also a negative prime number. You should be familiar with all these classifications so that you can read mathematics correctly.

Zero: It's Complicated

Zero is a very special number. It wasn't really used in any of the earliest counting systems. In fact, there is no symbol for zero in the Roman numerals!

Zero is a very useful number, but it also comes with its challenges. You can't divide by zero, but you can add zero to a number and multiply a number by 0. You'll find zero popping up in the most interesting places!

Imagining imaginary numbers

Yes, there are imaginary numbers in mathematics. These numbers were actually created by mathematicians who didn't like not finishing a problem! They would be trying to solve a quadratic equation and be stumped by the situation where they needed the square root of a negative number. There was no way to deal with this.

So some clever mathematicians came up with a solution. They declared that $\sqrt{-1}$ must be equal to i . Yes, the i stands for "imaginary." You'll see how this works in Chapter 16.

Coping with complex numbers

A complex number isn't really all that mysterious. This is just a designation that allows for you to deal with both real and imaginary parts of a number. A complex number has some of each! Complex numbers have the general format of $a + bi$, where a and b are real numbers, and the i is that imaginary number, $\sqrt{-1}$.



EXAMPLE

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number 8?

A. **Natural, whole, integer, rational.** The number 8 fits all of these descriptions. It is rational, because you can write it as a fraction such as $\frac{8}{1}$ or $\frac{24}{3}$.

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number $-\frac{2}{3}$?

A. **Rational.** This is written as a fraction but cannot be reduced to create an integer.

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number $\sqrt{17}$?

A. **Irrational.** The number 17 isn't a perfect square, so the decimal equivalence of $\sqrt{17}$ is a decimal that goes on forever without repeating or terminating.

Q. Using the choices: natural, whole, integer, rational, irrational, prime, and imaginary, which of these can be used to describe the number $\sqrt{-9}$?

A. **Imaginary.** Even though 9 is a perfect square, so you can write the number as $\sqrt{-1} \cdot \sqrt{9}$ and then simplify it to read $i \cdot 3$ or $3i$, this number stays imaginary.



YOUR TURN

1 Identify which of the following numbers are natural numbers:

$$-41, 15, -5.2, 11, 3.2121\dots, -\frac{12}{3}, \frac{14}{11}, \sqrt{-5}, \sqrt{10}, \sqrt{9}$$

2 Identify which of the following numbers are integers:

$$-41, 15, -5.2, 11, 3.2121\dots, -\frac{12}{3}, \frac{14}{11}, \sqrt{-5}, \sqrt{10}, \sqrt{9}$$

3 Identify which of the following numbers are rational numbers:

$$-41, 15, -5.2, 11, 3.2121\dots, -\frac{12}{3}, \frac{14}{11}, \sqrt{-5}, \sqrt{10}, \sqrt{9}$$

4 Identify which of the following numbers are irrational numbers:

$$-41, 15, -5.2, 11, 3.2121\dots, -\frac{12}{3}, \frac{14}{11}, \sqrt{-5}, \sqrt{10}, \sqrt{9}$$

5 Identify which of the following numbers are prime numbers:

$$-41, 15, -5.2, 11, 3.2121\dots, -\frac{12}{3}, \frac{14}{11}, \sqrt{-5}, \sqrt{10}, \sqrt{9}$$

6 Identify which of the following numbers are imaginary numbers:

$$-41, 15, -5.2, 11, 3.2121\dots, -\frac{12}{3}, \frac{14}{11}, \sqrt{-5}, \sqrt{10}, \sqrt{9}$$

Placing Numbers on the Number Line

A number line is labeled with numbers that increase as you move from left to right. And numbers are listed with an equal amount or value between any two consecutive numbers.

Numbers are placed on a number line to give you a visual picture of how they compare, how far apart they are, and what is missing between them. The two number lines shown here are examples of some versions that are possible. In Figure 1-1, you see the half-way mark indicated between units. And in Figure 1-2, the negative and positive integers are shown, with 0 in the middle.

FIGURE 1-1:

A number line from 0 to 5 with half-unit increments.

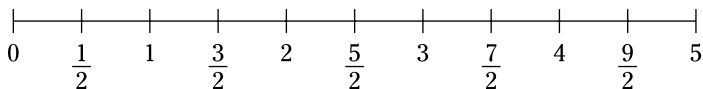
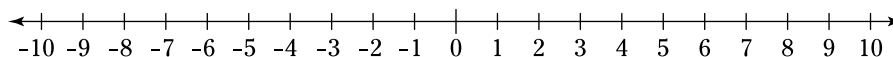


FIGURE 1-2:

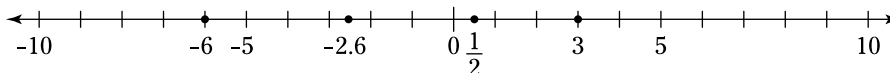
A number line from -10 to 10 with one-unit increments.



EXAMPLE

Q. Place the numbers 3, -6, $\frac{1}{2}$, -2.6 on a number line.

A.

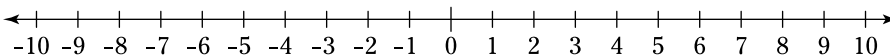


Note that the marks representing these numbers on the number line are marked with dots or points. The points for the fraction and decimal numbers are approximated, because the tickmarks for these numbers aren't on the number line to make them exact.

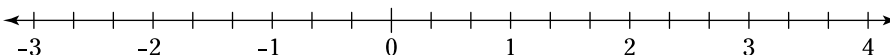


YOUR TURN

7 Place the following numbers on the number line: -6, -1, 0.5, 2, 3.2



8 Place the following numbers on the number line: $-2\frac{2}{3}$, $\frac{1}{3}$, $2\frac{1}{3}$, $3\frac{2}{3}$



Speaking in Algebra

Algebra and symbols in algebra are like a foreign language. They all mean something and can be translated back and forth as needed. It's important to know the vocabulary in a foreign language; it's just as important in algebra.

Being precise with words

The words used in algebra are very informative. You need to know their exact meaning, because they convey what is happening.

- » An *expression* is any combination of values and operations that can be used to show how things belong together and compare to one another. $2x^2 + 4x$ is an example of an expression. Think of an expression as being the equivalent of a phrase or part of a sentence; you have some subjects and conjugates, but no verbs. You see how items are distributed over expressions in Chapter 9.
- » A *term*, such as $4xy$, is a grouping together of one or more *factors* (variables and/or numbers) all connected by multiplication or division. In this case, multiplication is the only thing connecting the number with the variables. Addition and subtraction, on the other hand, separate terms from one another. For example, the expression $3xy + 5x - 6$ has three *terms*.
- » An *equation* uses a sign to show a relationship — that two things are equal. By using an equation, tough problems can be reduced to easier problems and simpler answers. An example of an equation is $2x^2 + 4x = 7$. See Chapters 14 through 18 for more information on equations.
- » An *operation* is an action performed upon one or two numbers to produce a resulting number. Operations include addition, subtraction, multiplication, division, square roots, and so on. See Chapter 7 for more on operations.
- » A *variable* is a letter representing some unknown; a variable always represents a number, but it *varies* until it's written in an equation or inequality. (An *inequality* is a comparison of two values. For more on inequalities, turn to Chapter 19.) Then the fate of the variable is set — it can be solved for, and its value becomes the solution of the equation. By convention, mathematicians usually assign letters at the end of the alphabet to be variables to be solved for in a problem (such as x , y , and z).
- » A *constant* is a value or number that never changes in an equation — it's constantly the same. The number 5 is a constant because it is what it is. A letter can represent a constant if it is assigned a definite value. Usually, a letter representing a constant is one of the first letters in the alphabet. In the equation $ax^2 + bx + c = 0$, c is a constant and x is the variable.
- » A *coefficient* is another type of constant. It is a multiplier of a variable. In the equation $ax^2 + bx + c = 0$, a and b are coefficients. They have constant, assigned values and are factors, but they have the special role of multiplying variables.
- » An *exponent* is a small number written slightly above and to the right of a variable or number, such as the 2 in the expression 3^2 . It's used to show repeated multiplication. An exponent is also called the *power* of the value. For more on exponents, see Chapter 5.



EXAMPLE

Q. Identify the terms, coefficients, factors, exponents, and constants in the expression $4x^2 - 3x + 2$.

A. There are three terms, separated by the subtraction and addition symbols. In the first term, the 4 is the coefficient, and the 4 and x^2 are factors. The 2 is the exponent. In the second term, the 3 and the x are factors. The exponent 1 isn't shown on the x ; it's just assumed. And the final term, the 2, is a constant.

Q. Identify the terms, coefficients, factors, exponents, and constants in the expression $P\left(1 + \frac{r}{n}\right)^{nt}$.

A. This expression has just one term. The P is a factor, and the parentheses form the other factor. There are two terms in the parentheses, and the exponent on the parentheses is nt .



YOUR
TURN

9 How many terms are there in the expression: $4x - 3x^3 + 11$?

10 How many factors are found in the expression: $3xy + 2z$?

11 Which are the variables and which are the constants in the expression:

$$\frac{(x-h)^2}{a} + \frac{(y-k)^2}{b} = 1?$$

12 Which are the exponents in the expression: $z^2 + z^{1/2} - z$?