Emergence, Complexity and Computation ECC

Andrew Adamatzky *Editor*

Automata and Complexity

Essays Presented to Eric Goles on the Occasion of His 70th Birthday

Springer

Emergence, Complexity and Computation

Volume 42

The Emergence, Complexity and Computation (ECC) series publishes new developments, advancements and selected topics in the fields of complexity, computation and emergence. The series focuses on all aspects of reality-based computation approaches from an interdisciplinary point of view especially from applied sciences, biology, physics, or chemistry. It presents new ideas and interdisciplinary insight on the mutual intersection of subareas of computation, complexity and emergence and its impact and limits to any computing based on physical limits (thermodynamic and quantum limits, Bremermann's limit, Seth Lloyd limits…) as well as algorithmic limits (Gödel's proof and its impact on calculation, algorithmic complexity, the Chaitin's Omega number and Kolmogorov complexity, non-traditional calculations like Turing machine process and its consequences,…) and limitations arising in artificial intelligence. The topics are (but not limited to) membrane computing, DNA computing, immune computing, quantum computing, swarm computing, analogic computing, chaos computing and computing on the edge of chaos, computational aspects of dynamics of complex systems (systems with self-organization, multiagent systems, cellular automata, artificial life,…), emergence of complex systems and its computational aspects, and agent based computation. The main aim of this series is to discuss the above mentioned topics from an interdisciplinary point of view and present new ideas coming from mutual intersection of classical as well as modern methods of computation. Within the scope of the series are monographs, lecture notes, selected contributions from specialized conferences and workshops, special contribution from international experts.

Indexed by zbMATH.

More information about this series at https://link.springer.com/bookseries/10624

Andrew Adamatzky
Editor

Automata and Complexity

Essays Presented to Eric Goles on the Occasion of His 70th Birthday

Springer

Editor
Andrew Adamatzky
Unconventional Computing Centre
University of the West of England
Bristol, UK

ISSN 2194-7287 ISSN 2194-7295 (electronic)
Emergence, Complexity and Computation
ISBN 978-3-030-92550-5 ISBN 978-3-030-92551-2 (eBook)
https://doi.org/10.1007/978-3-030-92551-2

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

Eric Goles is one of the world leaders in the field of automata and complexity. His made groundbreaking discovering theory and analysis of complex systems, particularly in the field of discrete systems dynamics such as neural networks, automata networks, majority networks, bootstrap percolation models, cellular automata, computational complexity theory, discrete mathematics and theoretical computer science. This book commemorates Eric Goles's achievements in science and engineering. The chapters are authored by world leaders in computer science, physics, mathematics and engineering.

The book will be a pleasure to explore for readers from all walks of life, from undergraduate students to university professors, from mathematicians, computers scientists and engineers to chemists and biologists.

Bristol, UK
July 2021

Andrew Adamatzky

Contents

Eric Goles

Andrew Adamatzky

Eric Goles, son of a theatre actress and a musician, was born in Antofagasta, in northern Chile, between the Pacific Ocean and the desert of Atacama. In 1970, he joined the University of Chile and graduated with a degree in mathematical engineering in 1975. He later went to the University of Grenoble, France, to carry out doctoral studies which culminated in 1980 with his thesis "Comportement oscillatoire d'une famille d'automates cellulaires non uniformes" where he proved the nowadays famous theorem that symmetric threshold automaton oscillates only with period one (fixed points) or two [12, 30, 57, 58, 62]. A nice review of this theorem can be found in [16]. In 1982, Eric joined the prestigious CNRS (French National Centre for Scientific Research) and moved to France. First to the Institute of Applied Mathematics, IMAG, at the University of Grenoble and later to the Laboratory on Network dynamics and Epistemology at the Polytechnic Institute in Paris. During this period, he carried out a state thesis in Mathematics, also in the field of automata networks [31]. After moving back to Chile to join the Engineering School of the Universidad de Chile, where he worked as a Professor till 2006. He then moved to the Faculty of Engineering and Sciences at the University Adolfo Ibáñez where he is still working. In 2004 he also founded the Institute of Complex Systems at Valparaiso, the first-ever Chilean research establishment devoted to complex systems. In 1993 he was honoured with the main scientific award of his country, the National Science Price. He has written more than two hundred articles and ten books. Further, he has trained a huge number of young scientists both in Chile and abroad. Eric excels in numerous fields like theoretical computer science, discrete mathematics, neural and Boolean networks, cellular automata and mathematical modelling in physics,

A. Adamatzky (✉)
Unconventional Computing Laboratory, University of the West of England,
Bristol BS16 1QY, UK
e-mail: andrew.adamatzky@uwe.ac.uk

A. Adamatzky (ed.), *Automata and Complexity*, Emergence, Complexity
and Computation 42, https://doi.org/10.1007/978-3-030-92551-2_1

biology, and social sciences. Here we provide just a few examples of his and his colleagues' outstanding research results.

Sand Piles and Chip Firing Game

The sand piles model, closely related to Spencer's chip firing game [75], was introduced as a tool to study self-organised criticality [2]. Eric analysed this model from complexity and computational universality points of view [38, 55, 56]. Results of the analysis were impressive. In 1992, Eric determines the lattice structure of the sand pile automata [33] and in Goles and Kiwi provided bounds for the transient time length of the sand pile modules, characterised the fixed points to which they converge and gave closed formulas for the sequential transient time [36, 37]. In the same year Bitar and Goles determined the two-cycle behaviour of the parallel chip firing game on trees and demonstrated that the chip firing game belongs to Wolfram class 4 automata [3]. In series of influential papers Eric and colleagues demonstrated that sand pile and chip firing game are universal computers. That is by representing logical truth by presence of a sand grain or a chip and logical false by absence of the grain/chip one route information as avalanches and implement logical gates via interaction of avalanches in an appropriate geometrical structure [23, 42, 43]. Other impressive results related to sand pile model, developed by Eric and colleagues, include estimation of computational complexity of sandpile avalanches [21], analysis of sandpile dynamical responses to reversal of the avalanche's source position [59], determination of non-polynomial (almost exponential) periods for the parallel chip firing game [65].

Sakoda and Schelling Models

In 1943 Sakoda developed a model of attitude-based social interaction of discrete agents, which was only published thirty years later in 1971 [73] at the same time as the similar model proposed by Schelling [74]. The models laid a foundation for studies of spatial mechanisms of social dynamics and economy. When Schelling was awarded a Nobel Prize in Economy "his models of segregation" was cited in an official documents as one reasons for the award [63]. In 2011 Goles et al. [17] studied exhaustively the behaviour of a generalized Schelling model in a two and three-dimensional grid with several neighbourhoods and they establish an energy operator associated with the model's dynamics. Other developments concerning the Schelling model can be seen in a combinatorial game developed by Eric and colleagues: a line (or cycle) graph with white and black tokens and an empty site where two players move alternately one of its colour's token to the hole trying to reach a connected configuration of its tokens [35]. Eric's interest in social models continued to develop precisely in the almost forgotten model of Sakoda through the characterization of the dynamics of all aptitude rules in one and two-dimensional grids [69]. More recently a generalisation of Schelling's model to other local functions in a two-dimensional grid has been done in [78].

Communication Complexity

This topic of computer science research attracted Eric's interest for a number of years. In 2008 Eric and colleagues proposed to define cellular automata via their communication protocols: "if we are able to give a protocol describing a cellular automaton, then we can understand its behaviour" [46]. In the same paper they proposed a hierarchy of complexity classes in cellular automata, based on their communication complexity of the automata. In [64] the same authors determine complexity equivalences between one round communication protocols and intrinsic cellular automata universality. Analysis of communicating protocols emerging from cellular automaton dynamics has been continued in [39] where authors developed a non-trivial communication protocol describing dynamics of elementary cellular automaton rule 218. In [28] Eric and colleagues have given protocols, and lower bounds of the same order, that together solve the one-round communication complexity of the prediction problem for nearly one-third of all elementary cellular automata, corresponding to the family of monotone rules. Most recent results of Eric's team have been about communication complexity of number-conserving cellular automata [28, 54].

Computational Complexity and Universality
These two topics usually go a pair in Eric and colleagues' works because the computational complexity of a system is typically estimated by embedding a relevant Boolean circuit in the system. In 1993 Eric and colleagues constructed a simulation of a Turing machine by cellular automata based on the equivalence between programmable machines and Turing machines [41]. They proved that for this class of cellular automata the associated limit language is regular. In paper [22] Gajardo and Goles present designs of Boolean circuits embedded in the space-time evolution of the two-dimensional three-state automaton. To enhance their proof of the universality of the automaton they also simulate a Turing machine in the reaction-diffusion automaton. Later, Goles and Montealegre analysed computational complexity of majority automata networks (the state of a vertex being the most represented in its neighbour). One of the first to study the relationship between the dynamics of the majority automata and its computational complexity was C. Moore proving in [6] that the majority automaton is P-Complete in three or more dimensions, leaving open the complexity characterization in a two-dimensional grid. Later, in [53] Eric and his colleagues characterize the complexity of the frozen majority (i.e., state 1 remains invariant): if the maximum degree of the network is 5 or more then the problem is P-complete, otherwise (maximum degree less than 5) the problem is in the class NC. Further, the same authors proved in [49] that the majority automaton in planar graphs is P-complete. To achieve that they used the two periodic behaviour of the majority automaton [12, 57, 62] as traffic lights to cross information. Further, by considering other iteration schemes they proved that the majority automaton iterated under a block sequential updating scheme is NP-Hard [48]. Result which is improved in [52] by proving that the problem is PSPACE-Complete. Previous results are obtained for the usual majority automata, i.e., the networks admit weights 0 or 1, say to sites are friends if they are connected (i.e., the weight in the incidence matrix is 1). In [50] they studied the more general case where the weights in the matrix may be $-1, 0, 1$. Other problems related to dynamics and computational complexity are related to the diffusion limited segregation studied recently in [4]. Their most recent results deal with the complexity boundaries of the graphs with polynomially growing treewidth [51].

Neural and Boolean Automata Networks
First Eric's works on these subjects are dated back to his thesis in 1980, where he characterizes the dynamical behaviour of disjunctive networks and also proved his very well know theorem about the periodicity of symmetric neural networks [12, 57, 58, 62] . But that was only the beginning, in [32], where he obtained bounds on the cycle and transient length for parallel iterations of antisymmetric sign functions. Later in Cosnard et al. [15] studied and derived formalism related to the threshold neural automata with memory: bounds on transient periods, and characterised reversibility versus the coupling coefficients. From 1985 Goles and colleagues introduced decreasing energy functions as a tool for analyses a huge class of neural automata networks [8, 29, 45, 62]. This tool has been successfully applied in studying various types of continuous state and discrete state neural networks [13, 14, 20, 61]. Other notable results in Boolean and neural networks include but not limited to the following: (1) necessary and sufficient conditions for the existence of fixed points in discrete neural networks and an upper bound for the number of fixed points [1], (2) exponential transient classes of symmetric neural networks for synchronous and sequential updating [44] (3) complexity of high-order neural networks [9], (4) characterisation of attractor space of neural networks over undirected graphs [11] (5) universality, via embedding Boolean circuits and simulating Turing machine, of discrete neural networks [10, 27]. One of Eric's recent works [11] characterizes completely the existence of cycles for symmetric network iterated both in parallel or block-sequentially. The authors define an index to the graphs of the automaton such that there exist cycles other than fixed points (period two or more) if and only if such index is non-negative.

Discrete Ants
Advances of Langton's ant model [66] have been done in several directions. Eric and collaborators developed a generation of Langton's ant model and analysed the complexity of the ants' behaviour on the graphs. They shown a high degree of unpredictability in general case, especially in the families of finite graphs where the period of the system growth exponentially with the size of the graph. They also shown that a prediction of the dynamics of the generalised ants on finite graphs is P-hard [25]. In paper [26] they constructed Boolean circuits with the trajectory of a single ant. They prove P-hardness of the ants system. Further studies dealt with detailed characterisaton of space-time dynamics of 64 ant's rules [24] and analyses of complex pattern formation and sensitivity of traces of two-dimensional ants [67].

Tiling
Eric and colleagues contribution to the tiling theory has been manifested in three key results. First, they demonstrated the existence of coding that allows for an efficient transformation of an arbitrary degrees of freedom tiling problem into a restricted four degrees of freedom problem (the tiling with rotation) [60]. Second, they provided solution to the following problem: to tile a rectangle or a torus with only vertical and horizontal bars of a given length such that the number of bars in every column and row equals to some given numbers [18]. Third, they established a bridge between tiling and folding. Namely, given a finite word coding vertical and horizontal folds

they provided a necessary and sufficient condition in order to tile the plane with a set of tiles constructed with copies of the unfold surface [34].

Analysis of Natural Systems
From the beginning, during his doctoral studies, Eric was interested in Boolean networks and its applications. Actually, he told me that one of the first articles that he read seriously was Stuart Kauffman's paper, Metabolic stability and epigenesis in randomly constructed genetic nets [64]. From that Eric and colleagues continue to apply Boolean networks in modelling and analysis of natural systems. The results included characterisation of two-dimensional Boolean dynamics in a grid such that each Boolean rule has only two inputs [19], representation of microbial interactions in a human microbiome as threshold Boolean networks [76], modelling the immune control of macrophages and the genetic control of the floral morphogenesis [40, 70], the analysis of cell cycle models [47] bacterium quorum sensing [72], a novel framework to study the influence of minimal cognitive mechanisms on the formation and evolution of languages [77], plants response to salt stress [71] and contagion phenomena in a two-dimensional grid [6].

Decision Making and Social Science
One of Eric's first works in this subject was to characterize the behaviour of a population to choose an opinion among several [61]. In paper [7] Eric and colleagues analyzed two simple dynamical models of decision that represent the school choice problem under two views: (1) individual expectations when deciding for a school, without major consideration of the social environment, and (2) focused on social expectations modelled by the neighbourhood preferences when deciding for a school. The computational experiments demonstrated that the social expectations model represents a more socially efficient situation that may help families to stay informed about accessibility, information, social capital, and improved school performances. Recently in [5] Eric and colleagues present a model of competing activist and political polarization and in [68] a model related to social crisis. Other key topics of Eric Goles and colleagues research include block invariance and reversibility of one-dimensional linear cellular automata, Lyapunov operators to study the convergence of extreme automata, properties of positive functions and the dynamics of associated automata networks, effects of firing memory in the dynamics of conjunctive networks, the complexity of asynchronous freezing cellular automata, on the robustness of update schedules in Boolean networks, naming game automata networks, complexity of the majority rule on planar graphs, learning gene regulatory networks using the bees algorithm, a sequential operator for filtering cycles in Boolean networks, prime number selection of cycles in a predator-prey model.

References

1. Aracena J, Demongeot J, Goles E (2004) Positive and negative circuits in discrete neural networks. IEEE Trans Neural Netw 15(1):77–83
2. Bak P, Tang C, Wiesenfeld K (1987) Self-organized criticality: an explanation of the 1/f noise. Phys Rev Lett 59(4):381
3. Bitar J, Goles E (1992) Parallel chip firing games on graphs. Theor Comput Sci 92(2):291–300
4. Bitar N, Goles E, Montealegre P (2019) Computational complexity of restricted diffusion limited aggregation. arXiv:1904.10011
5. Lucas B, Pedro M, Eric G, Hans G (2020) Competing activists-political polarization. Physica A: Stat Mech Appl 545:123713
6. Böttcher L, Woolley-Meza O, Goles E, Helbing D, Herrmann HJ (2016) Connectivity disruption sparks explosive epidemic spreading. Phys Rev E 93(4):042315
7. Canals C, Goles E, Mascareño A, Rica S, Ruz GA (2018) School choice in a market environment: individual versus social expectations. Complexity
8. Goles EC (1985) Dynamics of positive automata networks. Theor Comput Sci 41:19–32
9. Goles EC, Matamala M (1994) Dynamical and complexity results for high order neural networks. Int J Neural Syst 5(3):241–252
10. Goles EC, Matamala M (1996) Symmetric discrete universal neural networks. Theor Comput Sci 168(2):405–416
11. Goles EC, Ruz GA (2015) Dynamics of neural networks over undirected graphs. Neural Netw 63:156–169
12. Goles EC (1980) Comportement oscillatoire d'une famille d'automates cellulaires non uniformes. PhD thesis, Grenoble Institute of Technology, France
13. Cosnard M, Goles EC (1995) A characterization of the existence of energies for neural networks. In: Fülöp Z, Gécseg F (eds) Proceedings of the automata, languages and programming, 22nd international colloquium, ICALP95, Szeged, Hungary, 10–14 July 1995. Lecture notes in computer science, vol 944. Springer, pp 570–580
14. Michel Cosnard and Eric Goles Ch (1997) Discrete state neural networks and energies. Neural Netw 10(2):327–334
15. Cosnard M, Moumida D, Goles E, de St Pierre T (1988) Dynamical behavior of a neural automaton with memory. Compl Syst 2(2):161–176
16. Delahaye JP (2018) Faut-il adopter l'avis de ces voisins. Pour la Sci 494
17. Domic NG, Goles E, Rica S (2011) Dynamics and complexity of the schelling segregation model. Phys Rev E 83(5):056111
18. Dürr C, Goles E, Rapaport I, Rémila E (2003) Tiling with bars under tomographic constraints. Theor Comput Sci 290(3):1317–1329
19. Fogelman-Soulie F, Goles-Chacc E, Weisbuch G (1982) Specific roles of the different boolean mappings in random networks. Bull Math Biol 44(5):715–730
20. Fogelman-Soulié F, Mejía C, Goles E, Aguilera SM (1989) Energy functions in neural networks with continuous local functions. Compl Syst 3(3)
21. Formenti E, Goles E, Martin B (2012) Computational complexity of avalanches in the kadanoff sandpile model. Fundamenta Informaticae 115(1):107–124
22. Gajardo A, Goles E (2001) Universal cellular automaton over a hexagonal tiling with 3 states. Int J Algebr Comput 11(03):335–354
23. Gajardo A, Goles E (2006) Crossing information in two-dimensional sandpiles. Theor Comput Sci 369(1–3):463–469
24. Gajardo A, Goles E (2004) Dynamics of a class of ants on a one-dimensional lattice. Theor Comput Sci 322(2):267–283
25. Gajardo A, Goles E, Moreira A (2001) Generalized langton's ant: dynamical behavior and complexity. In: Annual symposium on theoretical aspects of computer science. Springer, pp 259–270
26. Gajardo A, Moreira A, Goles E (2002) Complexity of langton's ant. Disc Appl Math 117(1–3):41–50

27. Goles E, Matamala M (1997) Reaction-diffusion automata: three states implies universality. Theory Comput Syst 30(3):223–229
28. Goles E, Moreira A, Rapaport I (2011) Communication complexity in number-conserving and monotone cellular automata. Theor Comput Sci 412(29):3616–3628
29. Goles E, Vichniac G (1990) Energy and attractors in parallel potts dynamics. J Phys A: Math Gen 23(7):1329
30. Goles E (1982) Fixed point behavior of threshold functions on a finite set. SIAM J Algebr Disc Methods 3(4):529–531
31. Goles E (1985) Comportement dynamique de reseaux d'automates. PhD thesis, Université de Grenoble, France
32. Goles E (1986) Antisymmetrical neural networks. Disc Appl Math 13(1):97–100
33. Goles E (1992) Sand pile automata. Ann de l'IHP Physique théorique 56:75–90
34. Goles E (2004) Folding and tiling. Theor Comput Sci 322(2):285–296
35. Goles E, Gómez L (2018) Combinatorial game associated to the one dimensional schelling's model of social segregation. Nat Comput 17(2):427–436
36. Goles E, Kiwi MA (1992) Dynamics of sand-piles games on graphs. In: Latin American symposium on theoretical informatics. Springer, pp 219–230
37. Goles E, Kiwi MA (1993) Games on line graphs and sand piles. Theor Comput Sci 115(2):321–349
38. Goles E, Latapy M, Magnien C, Morvan M, Phan HD (2004) Sandpile models and lattices: a comprehensive survey. Theoret Comput Sci 322(2):383–407
39. Goles E, Little C, Rapaport I (2008) Understanding a non-trivial cellular automaton by finding its simplest underlying communication protocol. In: International symposium on algorithms and computation. Springer, pp 592–604
40. Goles E, Lobos F, Ruz GA, Sené S (2020) Attractor landscapes in boolean networks with firing memory: a theoretical study applied to genetic networks. Nat Comput 19(2):295–319
41. Goles E, Maass A, Martinez S (1993) On the limit set of some universal cellular automata. Theor Comput Sci 110(1):53–78
42. Goles E, Margenstern M (1996) Sand pile as a universal computer. Int J Mod Phys C 7(02):113–122
43. Goles E, Margenstern M (1997) Universality of the chip-firing game. Theor Comput Sci 172(1–2):121–134
44. Goles E, Martinez S (1989) Exponential transient classes of symmetric neural networks for synchronous and sequential updating. Complex Syst 3(6):589–597
45. Goles E, Martţnez S (1991) Lyapunov functionals for automata networks defined by cyclically monotone functions. SIAM J Disc Math 4(2):200–206
46. Goles E, Meunier P-E, Rapaport I, Theyssier G (2009) Communications in cellular automata. In: Neary T, Woods D, Seda T, Murphy N (eds) Proceedings international workshop on the complexity of simple programs, Cork, Ireland, 6–7th Dec 2008, volume 1 of electronic proceedings in theoretical computer science. Open Publishing Association, pp 81–92
47. Goles E, Montalva M, Ruz GA (2013) Deconstruction and dynamical robustness of regulatory networks: application to the yeast cell cycle networks. Bull Math Biol 75(6):939–966
48. Goles E, Montealegre P (2014) Computational complexity of threshold automata networks under different updating schemes. Theor Comput Sci 559:3–19
49. Goles E, Montealegre P (2015) The complexity of the majority rule on planar graphs. Adv Appl Math 64:111–123
50. Goles E, Montealegre P, Perrot K, Theyssier G (2018) On the complexity of two-dimensional signed majority cellular automata. J Comput Syst Sci 91:1–32
51. Goles E, Montealegre P, Ríos-Wilson M, Theyssier G (2020) On the impact of treewidth in the computational complexity of freezing dynamics. arXiv:2005.11758
52. Goles E, Montealegre P, Salo V, Törmä I (2016) Pspace-completeness of majority automata networks. Theor Comput Sci 609:118–128
53. Goles E, Montealegre-Barba P, Todinca I (2013) The complexity of the bootstraping percolation and other problems. Theor Comput Sci 504:73–82

54. Goles E, Moreira A (2012) Number-conserving cellular automata and communication complexity: a numerical exploration beyond elementary CAS. J Cell Automata 7(3)
55. Goles E, Morvan M, Phan HD (2002) Lattice structure and convergence of a game of cards. Ann Comb 6(3–4):327–335
56. Goles E, Morvan M, Phan HD (2002) Sandpiles and order structure of integer partitions. Disc Appl Math 117(1-3):51–64
57. Goles E, Olivos J (1980) Periodic behaviour of generalized threshold functions. Disc Math 30(2):187–189
58. Goles E, Olivos J (1981) Comportement périodique des fonctions à seuil binaires et applications. Disc Appl Math 3(2):93–105
59. Goles E, Prisner E (2000) Source reversal and chip firing on graphs. Theor Comput Sci 233(1–2):287–295
60. Goles E, Rapaport I (1999) Tiling allowing rotations only. Theor Comput Sci 218(2):285–295
61. Goles E, Tchuenté M (1983) Iterative behaviour of generalized majority functions. Math Soc Sci 4(3):197–204
62. Goles-Chacc E, Fogelman-Soulié F, Pellegrin D (1985) Decreasing energy functions as a tool for studying threshold networks. Disc Appl Math 12(3):261–277
63. Hegselmann R, Schelling TC, Sakoda JM (2017) The intellectual, technical, and social history of a model. J Artif Soc Soc Simul 20(3)
64. Kauffman SA (1969) Metabolic stability and epigenesis in randomly constructed genetic nets. J Theor Biol 22(3):437–467
65. Kiwi MA, Ndoundam R, Tchuente M, Goles E (1994) No polynomial bound for the period of the parallel chip firing game on graphs. Theor Comput Sci 136(2):527–532
66. Langton CG (1986) Studying artificial life with cellular automata. Physica D: Nonlinear Phenom 22(1-3):120–149
67. Markus M, Schmick M, Goles E (2006) Tracks emerging by forcing langton's ant with binary sequences. Complexity 11(3):27–32
68. Mascareño A, Goles E, Ruz GA (2016) Crisis in complex social systems: a social theory view illustrated with the chilean case. Complexity 21(S2):13–23
69. Medina P, Goles E, Zarama R, Rica S (2017) Self-organized societies: on the sakoda model of social interactions. Complexity
70. Ruz GA, Goles E, Sené S (2018) Reconstruction of boolean regulatory models of flower development exploiting an evolution strategy. In: 2018 IEEE congress on evolutionary computation (CEC). IEEE, pp 1–8
71. Ruz GA, Timmermann T, Goles E (2016) Neutral space analysis of gene regulatory network models of salt stress response in arabidopsis using evolutionary computation. In: 2016 IEEE congress on evolutionary computation (CEC). IEEE, pp 4281–4288
72. Ruz GA, Zúñiga A, Goles E (2018) A boolean network model of bacterial quorum-sensing systems. Int J Data Mining Bioinf 21(2):123–144
73. Sakoda JM (1971) The checkerboard model of social interaction. J Math Sociol 1(1):119–132
74. Schelling TC (1971) Dynamic models of segregation. J Math Sociol 1(2):143–186 (1971)
75. Spencer J (1986) Balancing vectors in the max norm. Combinatorica 6(1):55–65
76. Travisany D, Goles E, Latorre M, Cortés M-P, Maass A (2020) Generation and robustness of boolean networks to model clostridium difficile infection. Nat Comput 19(1):111–134
77. Vera J, Goles E (2016) Automata networks for memory loss effects in the formation of linguistic conventions. Cogn Comput 8(3):462–466
78. Vieira AP, Goles E, Herrmann HJ (2020) Dynamics of extended schelling models. J Stat Mech: Theory Exp 2020(1):013212

Seven Things I Know About Them

Jacques Demongeot

Abstract In this paper, we intend to present a series of 7 application examples inspired by the work of Eric Goles with his numerous collaborators, while remaining focused on the field of Boolean automata. This constitutes a sort of anthology showing the extent of the mathematical domain defined by the study of Boolean automata dynamics, showing the relevance of open paths and results obtained by Eric Goles and the high explanatory power of the models which arise from his work during 45 years.

Keywords Boolean automata · Automata gradient dynamics · Automata Hamiltonian dynamics · Updating schedule

1 Introduction

A discrete dynamical system has the same definition that the continuous ones. It involves a flow function *f* defined on ExT, where T is a discrete time space (in general $\mathbb{N}$) and E a discrete state space ($\{0, 1\}^n$ in the Boolean case and more generally a finite subset of $\mathbb{R}_+{}^n$), $f(x, t)$ representing for each state x and time t, the state reached after time t by the trajectory starting in state x at time 0. We denote in general $f(x(0), t)$ by $x(t) = (x_i(t))_{i=1,\ldots,n}$, which permits to have a coherent notation for all the states of a trajectory. The set of such states is called the orbit of $x(0)$. Following [1], we can now define the discrete time derivative for the state vector $(x_i(t))_{i=1,\ldots,n}$ by:

$$\Delta x_i/\Delta t = (x_i(t + \Delta t) - x_i(t))/\Delta t,$$

which reduces to $x_i(t + 1) - x_i(t)$, if $\Delta t = 1$. By using the same formula, we can also define:

J. Demongeot (✉)
Faculty of Medicine, Laboratory AGEIS EA 7407, Team Tools for E-Gnosis Medical, University Grenoble Alpes, 38700 La Tronche, France
e-mail: Jacques.Demongeot@univ-grenoble-alpes.fr

A. Adamatzky (ed.), *Automata and Complexity*, Emergence, Complexity and Computation 42, https://doi.org/10.1007/978-3-030-92551-2_2

- the space derivative: $\Delta g(x)/\Delta i = (g(x_{i+\Delta i}) - g(x_i))/\Delta i = g(x_{i+1}) - g(x_i)$, if i is 1-dimensional and $\Delta i = 1$. If i is n-dimensional, it is possible to partially derive in each dimension.
- the partial state derivative: $\Delta g(x)/\Delta x_i = [g(x_1,\ldots, x_i + \Delta x_i,\ldots, x_n) - g(x_1,\ldots, x_i,\ldots, x_n)]/\Delta x_i$.

A discrete automaton is defined by a transition function F:

$$\Delta x_i/\Delta t = (x_i(t+\Delta t) - x_i(t))/\Delta t = F_i(x(t)),$$

where F_i depends only on coordinates $(x_j(t))_{j\in V(i)}$, $V(i)$ being a neighbourhood of i in the space set (in general the Manhattan—or L_1—unit ball of $E \cap \mathbb{R}_+^n$ centred on i and having a radius equal to 1), with conditions defining V on the boundary of E (e.g. periodic) and with constraints on the discrete velocity ensuring that flow remains in E.

We will first define a discrete analogous of a potential (or gradient) continuous dynamical system (called here potential automaton). A continuous potential differential equation on $\mathbb{R}^n$ is defined by: $\forall i = 1,\ldots,n$, $dx_i/dt = -\partial P/\partial x_i$, where P is a real continuously differentiable function (e.g., a polynomial with real coefficients) on $\mathbb{R}^n$. In the same way, a potential automaton on the discrete state space E is defined by:

$$x_i(t+1) = h(-\Delta P/\Delta x_i + x_i(t)), \tag{1}$$

where P is a real function (e.g., a polynomial with real coefficients) on E and h a function from $\mathbb{R}$ to E, with boundary conditions ensuring that the flow remains in E. For example, in the Boolean case, we will choose for h the Heaviside function H: $H(s) = 1$, if $s > 0$, and $H(s) = 0$, if $s \leq 0$. In the integer case (E subset of $\mathbb{N}^n$), h can be the identity, if P has integer coefficients and if $\forall i = 1,\ldots,n$, $\Delta x_i \in \{-1, 0, 1\}$.

Provided by the above definitions, we will give now some examples of application of Boolean networks inspired by the work of Eric Goles and his numerous co-workers.

2 Seven Remarks About Boolean Automata Theory

2.1 Potential Boolean Automata (Inspired by Cosnard and Goles [2])

Proposition 1 In the Boolean case, let suppose that $A = 0$, $P(x) = {}^t xAx + Bx$, with $a_{ii} = 0$ and each sub-matrix on any subset J of indices in $\{1, \ldots,n\}$ of A is non positive and less than the linear operator -B restrained on J. Then P decreases on the trajectories of the potential automata defined by $x_i(t+1) = H(-\Delta P/\Delta x_i + x_i(t))$

for any mode of implementation of the dynamics (sequential, block sequential and parallel). These Boolean automata constitute a Hopfield-like network whose weights are $w_{ii} = 1$ and $w_{ij} = -a_{ij} - a_{ji}$, $\forall j \neq i$, thresholds are the b_i's, and stable fixed configurations correspond to the minima of P.

Proof It is easy to check that: $\Delta P/\Delta x_i = \Sigma_{j \neq i} (a_{ij} + a_{ji}) x_j + b_i$ and: $x_i(t+1) = H(-P/x_i + x_i(t)) = H(-[j \neq i\ (a_{ij} + a_{ji})\, x_j(t) + b_i] + x_i(t)) = H(j\ w_{ij} x_j(t) - b_i)$

We can calculate for the block sequential iteration at any step of block J:

$$x_i(t+1) = H(-\Delta P/\Delta x_i + x_i(t)) = H(-[\sum\nolimits_{j \neq i} (a_{ij} + a_{ji}) x_j(t) + b_i] + x_i(t))$$
$$= \mathrm{H}(\sum\nolimits_j w_{ij} x_j(t) - b_i)$$

$$P(x(t+1)) - P(x(t)) = \sum\nolimits_{(i,j) \in J \times J} a_{ij} \Delta x_i \Delta x_j + \sum\nolimits_{i \in J} b_i \Delta x_i \leq 0,$$

the result coming from the hypothesis on the sub-matrices J of W or from [2] ■

The interest of the Proposition 1 is to show that the Hopfield-like network defined by:

$$\forall i = 1, \ldots, n,\ x_i(t+1) = H\left(\sum\nolimits_j w_{ij} x_j(t) - b_i\right), \text{ with}$$
$$w_{ii} = 1,\ w_{ij} < 0,\ b_i \geq \mathrm{e} > 0,$$

has not only P as Lyapunov function as proved in [2], but more it can be considered as a potential automaton with a potential equal to P, because the opposite of the gradient of P is related to the velocity of the automaton, what is quite different in general for a system with simply a Lyapunov function (Fig. 1) [3].

Another example of potential Boolean automata is given by the n-switch often used in morphogenesis modelling [4, 5], for example in dorsal somites (Fig. 2) [6] or skin appendages [7, 8] models. It is easy to show that its Hopfield-like dynamics with all weights w_{ij} equal to -1, except $w_{ii} = 1$ and all $b_i \geq \mathrm{e} > 0$, is gradient for any updating mode and its attractors are the 6 fixed points (10,000), (01,000), (00,100),

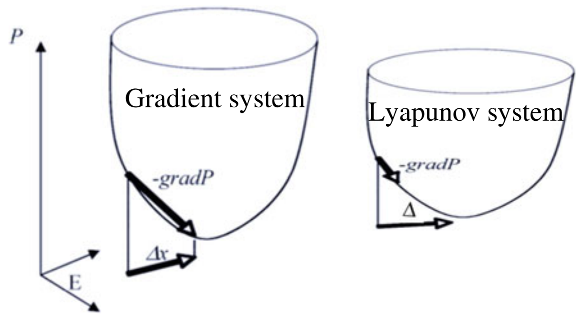

Fig. 1 Potential automaton with $\Delta x = -gradP$ (on the left) and an automaton with a Lyapunov function decreasing on its trajectories (on the right) (after [3])

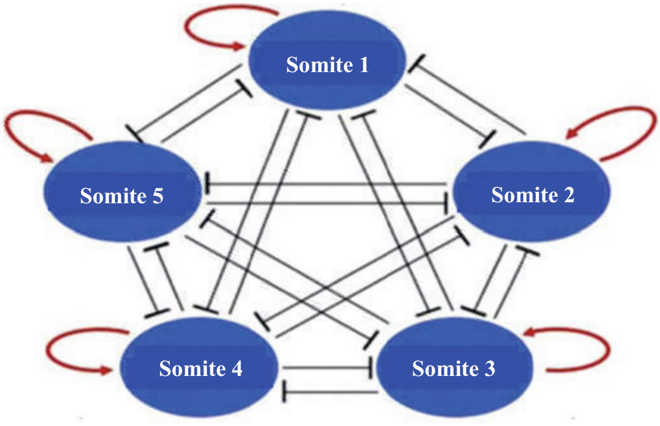

Fig. 2 "Metatron" interaction graph of a 5-switch (after [4, 5])

(00,010) and (00,001) and (00,000). The equivalent continuous model [5] has the same property.

2.2 Hamiltonian Boolean Automata

Proposition 2 Let us consider a deterministic Hopfield-like network of size n, which is a circuit sequentially or synchronously updated with constant absolute value w for its non-zero interaction weights. Then, its dynamics is conservative, keeping constant on the trajectories the Hamiltonian function L defined by:

$$L(x(t)) = \sum\nolimits_{i=1,n} \frac{(x_i(t) - x_i(t-1))^2}{2}$$
$$= \sum\nolimits_{i=1,n} \frac{H\left(w_{i(i-1)modnxi-1}(t-1) - x_i(t-1)\right)^2}{2}$$

where H denotes the classical Heaviside function. L($x(t)$) is the total discrete kinetic energy of the network, equal to the half of the global dynamic frustration:

$$F(x(t)) = \sum\nolimits_{i=1-n} F_{i,(i-1)modn}(x(t)),$$

with $F_{i,\,(i-1)\,\mathrm{mod}n}$ is the local dynamic frustration defined between nodes $(i-1)$ and i by:$F_{i,\,(i-1)}\,(x(t)) = 1$, if $\{\mathrm{sign}(w_{i(i-1)}) = 1, x_i(t) \neq x_{i-1}(t-1)\}$ $\{\mathrm{sign}(w_{i(i-1)}) = -1, x_i(t) = x_{i-1}(t-1)\}$, $= 0$, if not.

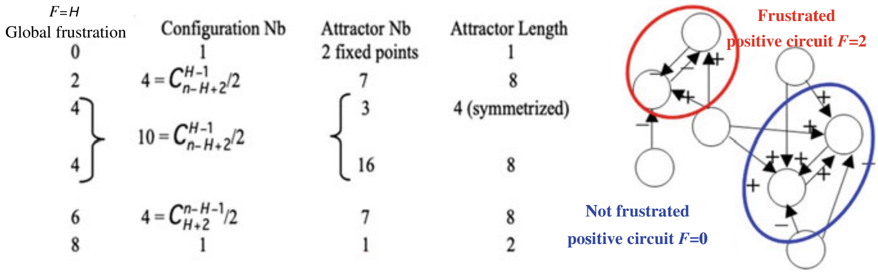

Fig. 3 On the left: calculation of the global frustration F for circuits of length 8 with only identities and negations as local transitions. On the right: "Arabidopsis" interaction graph with eight nodes and two circuits of length 2 (after [4, 5]), one frustrated (in red) and the other not frustrated (in blue)

Proposition 2 still holds if the network is a circuit whose transition functions are Boolean identity or negation [10], on which it is easy to calculate the global frustration F and show that it characterizes attractors by remaining constant along them (Fig. 3).

A general program of characterizing potential and Hamiltonian Boolean automata could extend the energetic notion of dissipative (potential) and conservative (Hamiltonian) energy, from the Hopfield-like Boolean networks to the most general Boolean automata. This work would be in the direct continuation of the pioneering work of the two PhD students of François Robert, Eric Goles and Françoise Fogelman [11].

2.3 *Social Choice and Majority Rule*

In the spirit of the games theory of the seventies [12], Eric Goles and Maurice Tchuente considered in [13] a society of n persons $\{P_1,\ldots,P_n\}$ having at time t opinions $\{x_1(t),\ldots,x_n(t)\}$ with interaction coefficient $a_{ij} = a_{ji}$ between P_i and P_j. Let $\{\theta_1,\ldots,\theta_p\}$ be the set of possible opinions which may be assumed by any person, with a local hierarchy h_i adopted by each person P_i (a reordering of opinion indices without ex-æquo, that is a permutation of $\{1,\ldots, p\}$). The dynamical behaviour of such a society depends on local majority rules, where, if $a(k)$ denotes the global weight of the opinion θ_k, the change of opinion is made as follows:

$$x_\mathrm{i}(t+1) = k, \text{ with}$$

$$k = \sup\left\{ i/\forall r = 1, \ldots, p, a(i) \sum\nolimits_{j/x_j(t)=k} a_{ij} \geq a(r) \sum\nolimits_{j/x_j(t)=r} a_{ij} \right\}$$

Then, the main result is the following.

Proposition 3 In such a society the opinion of any member P_i, after a certain number of steps, either remains constant or oscillates between two values.

This work inspired general studies on social choices as those described in [14, 15].

2.4 Eberhard-Robert Scholia

Recently, François Robert has established in collaboration with André Eberhard a very interesting result we can call a "scholia", because it is an original explanatory comment opening a new domain of research concerning interactions between automata networks. The example treated in [16] concerns cross interactions between two Boolean automata of size 2, one with transition F_0 and the other with transition G_0, evolving in time with the following rules, where symbol ¬ denotes Boolean complementary (negation):

- Sequential dependence: $G_1(x_1, x_2) = G_0(F_0(x_1, x_2), x_2)$, $F_1(x_1, x_2) = F_0(x_1, G_1(x_1, x_2))$,..., $G_i(x_1, x_2) = G_{i-1}(F_{i-1}(x_1, x_2), x_2)$, $F_i(x_1, x_2) = F_{i-1}(x_1, G_i(x_1, x_2))$,...
- Parallel dependence: $G_1(x_1, x_2) = G_0(F_0(x_1, x_2), x_2)$, $F_1(x_1, x_2) = F_0(x_1, G_0(x_1, x_2))$,...,
  $G_i(x_1, x_2) = G_{i-1}(F_{i-1}(x_1, x_2), x_2)$, $F_i(x_1, x_2) = F_{i-1}(x_1, G_{i-1}(x_1, x_2))$,...
- Sequential opposition: $G_1(x_1, x_2) = G_0(\neg F_0(x_1, x_2), x_2)$, $F_1(x_1, x_2) = F_0(x_1, \neg G_1(x_1, x_2))$,..., $G_i(x_1, x_2) = G_{i-1}(\neg F_{i-1}(x_1, x_2), x_2)$, $F_i(x_1, x_2) = F_{i-1}(x_1, \neg G_i(x_1, x_2))$,...
- Parallel opposition: $G_1(x_1, x_2) = G_0(\neg F_0(x_1, x_2), x_2)$, $F_1(x_1, x_2) = F_0(x_1, \neg G_0(x_1, \ldots\ldots x_2))$,...,
  $G_i(x_1, x_2) = G_{i-1}(\neg F_{i-1}(x_1, x_2), x_2)$, $F_i(x_1, x_2) = F_{i-1}(x_1, \neg G_{i-1}(x_1, x_2))$,...

Then, the Eberhard-Robert scholia says:

Proposition 4

(i) Sequential and parallel dependence (resp. opposition) rules give same fixed points.
(ii) The transform by sequential (resp. parallel) opposition of (¬*F*, ¬*G*) is the complementary of the transform by sequential (resp. parallel) dependence of (*F*, *G*).

Such a result can be applied to situations in which two groups of actors (political, ethnic, social, neural, genetic, etc.) or two age classes evolve by taking social choices with cross interactions. It could serve namely to revisit and interpret the asymptotic properties of complex Boolean biological networks as those studied in [17, 18].

2.5 *Arabesques*

A close friend of Eric Goles, René Thomas, invented the notion of arabesques systems, and studied their dynamics both in discrete and continuous framework [17, 19, 20]. The Boolean equations of such a system of size n is given by:

$$\forall i = 1, \ldots, n, x_i = \mathrm{H}\big(w_{ii} X_i - w X_{j+1(mod n)} + w X_{i-1(mod n)}\big)$$

The weights verify: $0 \leq w_{ii} < w$. If $n = 2$ and $w_{22} = 0$, the arabesque is called regulon [17]. It is the smallest network containing one positive and one negative circuit (Fig. 4). The dynamics of the arabesque of size 2, with $w_{11} = w_{22} > 0$, has been used for representing for example the functioning of the hippocampus [19, 20]. The asymptotic behaviour of the arabesque of size 3 in sequential updating (Fig. 4) is represented by two unstable fixed points (000), (111), and a stable cycle of order 6 (010*, 0*11, 01*1, 001*, 0*01, 10*1, 101*, 1*00, 10*0, 110*, 1*10, 01*0), * denoting the node to update, with (011*, 0*10, 01*0), (00*1, 001*), (1*01, 0*01), (100*, 1*00) and (11*0, 110*) as attraction basin trajectories.

In parallel updating mode, (000) and (111) are unstable fixed point and there is a cycle of order 6 (010, 011, 001, 101, 100, 110). This cycle exists in the continuous analog (useful to interpret the mechanism of memory evocation [19]) and both in discrete parallel and in continuous case with $w_{ii} = 0$, the system is Hamiltonian, with conservation of the kinetic energy, independently of the number of tangent circuits in the arabesque. A deeper study of such intersecting circuits can be found in [20].

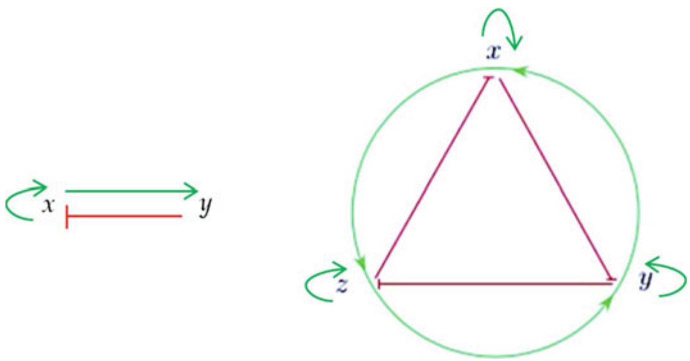

Fig. 4 On the left, regulon of size 3. On the right, arabesque of size 3. The positive interactions (activations) are represented by green arrows, the negative (inhibitions) by red ones

E. Goles	
F. Robert	
N. Gastinel	
J. Kuntzmann	
G. Valiron (also advisor of L. Schwartz, himself advisor of J.L. Lions)	
E. Borel	
G. Darboux (also advisor of H. Lebesgue)	
E. Borel	
M. Chasles	
S.D. Poisson	
J.L. Lagrange (also advisor of J. Fourier)	P.S. Laplace
L. Euler	J. Le Rond d'Alembert
Johann Bernoulli (also advisor of D. Bernoulli)	
Jacob Bernoulli	N. Eglinger
N. Malebranche	
G.W. Leibniz	
C. Huyghens	E. Weigel
F. van Schooten	J.J. Stampioen
J. Golius	M. Mersenne (also advisor of B. Pascal)
W. Snellius	T. Erpenius
R. Snellius	J.J. Scaliger
V. Naibod	A. Turnèbe
E. Reinhold	J. Toussain
J. Milich	G. Budé
D. Erasmus	

Fig. 5 Mathematical genealogy of Eric Goles. A mathematician under another is his PhD advisor

2.6 *Block Parallel Updating*

The problem of the influence of the updating rule has been emphasized since the start of the team “Iteration behaviours”, whose members were brought together by François Robert and Roger Maynard at University Joseph Fourier of Grenoble (IMAG) in 1983 [22]. A great progress has been done by our PhD students Julio Aracena, Adrien Elena, Lilian Salinas and Andrès Moreira, when they searched to escape the classical parallel and sequential updating modes for finding more realistic schedules [23, 24].

A good example of application for a new updating mode is the nuclear and mitochondrial genetic expression in which the combination of cellular repressors (microRNAs and circular RNAS), mitochondrial genes (relative to the cell respiration) and nuclear genes under the control of the chromatin clock, present a dynamical schedule close to a new updating mode called block-parallel in [22], for which the updating is sequential inside blocks, these blocks being updated parallelly with not necessary the same internal clock. The problems linked to the search for asymptotic behavior of such updated Boolean automata are very hard, but will be surely extensively studied in the future by the Goles’ school.

2.7 Discrete Convolution (to Regularize, We Convolve, L. Schwartz)

Yann Le Cun (PhD student of a friend of Eris Goles, Maurice Milgram) has defined in 1983 at a Colloquium held at Les Houches (University J. Fourier of Grenoble) and co-organized by J. Demongeot and B. Lacolle from the IMAG team "Iteration behaviours", the notion of learning process [25] and then, he used for the deep learning [26, 27] several discrete convolution operators, useful in writing or image processing [29]. The Boolean automata $z(t)$ resulting from the discrete convolution between two Boolean automata $x(t)$ and $y(t)$ (called the kernel filter) can be defined as follows:

$$z_i(t) = \sum\nolimits_{i-k \in v(i)} x_{i-k}(t) y_k(t)$$

where the size of the neighbourhood $V(i)$ equals p and the positive kernel filter of size p, $y(t)$, verifies: for any t, $k = 1,\ldots, p$ $y_k(t) = 1$. If $x_i(t)$ has a regular derivative (in the sense of F. Robert [1]), that is, for any spatial derivative and any t, $\Delta x_i(t)/\Delta i \leq 1$, then $z_i(t)$ is also regular, because we have: $\Delta z_i(t)/\Delta i \leq {}_k\sum{}_{V(i)} \Delta x_{i-k}(t)/\Delta k \; y_k(t) \leq 1$.

3 Mathematical Genealogy of Eric Goles as a Conclusion

Eric Goles has a prestigious mathematical genealogy starting with François Robert and ending with the two most renowned philomaths (like him) of the Renaissance, Guillaume Budé and Desiderius Erasmus both born in 1467, 584 years before him. This genealogy is described in Fig. 5 (after [29]).

The first ancestor of Eric Goles intends to end this genealogy with the following tribute:

«Mon billet pour Éric», by François Robert.

«Éric, la scène se passe à Grenoble quelques années après ta soutenance de thèse. Retour d'Israël via Paris, et avant de rentrer à Santiago, tu débarques impromptu ce matin-là dans mon bureau de la Tour IRMA au Campus. Nous sommes contents de nous revoir, et tu commences à m'expliquer au tableau ce qui préoccupe ton esprit en ce moment. S'agissait-il du tas de sable qui s'effondre sur lui-même ? Je crois que oui. Et tu me montres la relation mathématique que tu as écrite au tableau: « Tu vois, cette expression, eh bien elle ne me plaît pas, elle n'est pas casher !» Devant mon air d'incompréhension, tu corriges immédiatement: « Disons qu'elle n'est pas très catholique !», ce qui n'a pas amélioré ma compréhension pour autant, mais j'ai alors saisi en un éclair le concept de mathématicien international que tu commençais à incarner alors.

J’ai par ailleurs été très sensible au témoignage chaleureux que tu as rendu récemment à Jacques Demongeot [30], dans lequel tu insistes beaucoup pour rendre effectivement sensible cette part d’éternité et de globalité de vos échanges dînatoires, mathématiques ou non, que ce soit au Bar national de Santiago ou ailleurs: une caractéristique importante qui vous situe bien. Et de plus, ce témoignage m’a redonné comme instantanément aussi la vive perception du compagnonnage artisanal qui animait les membres de notre groupe « Comportement d’Itérations». Quelle équipe, en effet, d’excellents chercheurs ! Je les cite par ordre d’entrée en scène: Michel Cosnard, Maurice Tchuente, toi Eric, Françoise Fogelman, Houcine Snoussi, Yves Robert, les premiers couteaux en quelque sorte, tous chercheurs C.N.R.S. (sauf Françoise, universitaire, et Houcine, boursier marocain) et une huitaine de jeunes en troisième cycle. Jacques Demongeot entretenait des relations suivies avec tout ce petit monde. Nous partagions chaque semaine notre conviction dans la dimension intemporelle de notre réalité mathématique en cours d’élaboration, assidûment méditée et travaillée, et c’était là que résidait le ciment du groupe.

En ce qui te concerne, ta façon très spécifique de « faire des maths» est maintenant largement connue: fougueux, jovial, parfois brouillon et fâché de l’être, tu déploies au tableau une puissante séduction mathématique qui jaillit de ta très forte conviction personnelle, manipulant un maelström de notions entrelacées… Tu convaincs, car tu es habité, et spécifiquement toi !

Il n’empêche: les deux théorèmes de Golès-Martinez sur les cycles de longueur deux dans les réseaux d’automates à seuils symétriques (il y a trente-cinq ans !) resteront pour moi le signal fort d’un accomplissement à venir, qui s’est grandement réalisé depuis. J’ai même vu récemment que tu avais contribué à faire, de la fourmi de Langton, la brique de base d’un calculateur universel, dans la ligne de ce que vous élaboriez à l’époque avec Maurice Tchuente et Yves Robert sur d’autres modèles.

Cher Éric, bon et heureux 70, et longue vie ! Avec toute mon amitié, François Robert.»

References

1. Robert F (1986) Discrete iterations: a metric study. Springer, Berlin
2. Cosnard M, Goles E (1997) Discrete states neural networks and energies. Neural Netw 10:327–334
3. Demongeot J, Elena A, Weil G (2006) Potential-Hamiltonian decomposition of cellular automata. Application to degeneracy of genetic code and cyclic codes III. Comptes Rendus Biol 329, 953–962
4. Cinquin O, Demongeot J (2002) Positive and negative feedback : striking a balance between necessary antagonists. J Theor Biol 216:229–241
5. Cortez MJV, Rabajante JF, Tubay JM, Babierra AL (2017) From epigenetic landscape to phenotypic fitness landscape: evolutionary effect of pathogens on host traits. Infect Genet Evol 51:245–254
6. Goldbeter A, Pourquié O (2008) Modeling the segmentation clock as a network of coupled oscillations in the Notch, Wnt and FGF signaling pathways. J Theor Biol 252:574–585
7. Michon F, Forest L, Collomb E, Demongeot J, Dhouailly D (2008) BMP-2 and BMP-7 play antagonistic roles in feather induction. Development 135:2797–2805

8. Abbas L, Demongeot J, Glade N (2009) Synchrony in Reaction-diffusion models of morphogenesis: applications to curvature-dependent proliferation and zero-diffusion front waves. Phil Trans R Soc A 367:4829–4862
9. Demongeot J, Ben Amor H, Hazgui H, Waku J (2014) Stability, complexity and robustness in population dynamics. Acta Biotheor 62:243–284
10. Rachdi M, Waku J, Hazgui H, Demongeot J (2020) Entropy as robustness marker in genetic regulatory networks. Entropy 22:260
11. Goles E, Fogelman-Soulié F, Pellegrin D (1985) Decreasing energy functions as a tool for studying threshold networks. Disc Appl Math 12:261–277
12. Moulin H (1976) Extensions of two person zero sum games. J Math Anal Appl 55:490–508
13. Goles E, Tchuente M (1983) Iterative behaviour of generalized majority functions. Math Soc Sci 4:197–204
14. Demongeot J, Volpert V (2015) Dynamical system model of decision making and propagation. J Biol Syst 23:339–353
15. Banerjee M, Demongeot J, Volpert V (2016) Global regulation of individual decision making. Math Methods Appl Sci 39:4428–4436
16. Robert F (2020) Une aventure mathématique. Personal Commun
17. Demongeot J, Aracena J, Ben Lamine S, Meignen S, Tonnelier A, Thomas R (2001) Dynamical systems and biological regulations. In: Goles E, Martinez S (eds) COMPLEX SYSTEMS. Kluwer, Amsterdam, pp 105–151
18. Aracena J, Demongeot J, Goles E (2004) Mathematical modelling in genetic networks. IEEE Trans Neural Netw 15:77–83
19. Antonopoulos C, Basios V, Demongeot J, Nardone P, Thomas R (2013) Linear and nonlinear arabesques: a study of closed chains of negative 2-element circuits. Int J Bifurc Chaos 23:30033
20. Demongeot J, Elena A, Noual M, Sené S, Random boolean networks and attractors of their intersecting circuits. In: Barolli L, Xhafa F, Jeong HY (eds) IEEE AINA' 11. IEEE Proceedings, Piscataway, pp 483–487
21. Demongeot J, Khlaifi H, Istrate D, Mégret L, Taramasco C, Thomas R (2020) From conservative to dissipative non-linear differential systems. an application to the cardio-respiratory regulation. Disc Contin Dyn Syst 13:2121–2134
22. Robert F (1969) Blocs-H-matrices et convergence des méthodes itératives classiques par blocs. Linear Algebra Appl 2:223–265
23. Demongeot J, Elena A, Sené S (2008) Robustness in neural and genetic networks. Acta Biotheor 56:27–49
24. Aracena J, Goles E, Moreira A, Salinas L (2009) On the robustness of update schedules in Boolean networks. Biosystems 97:1–8
25. le Cun Y (1986) Learning processes in an asymmetric threshold network. In: Fogelman-Soulié F, Bienenstock E, Weisbuch G (eds) Disordered systems and biological organization, *Les Houches*, France, 1985. North Holland, Amsterdam, pp 233–240
26. le Cun Y, Bottou L, Bengio Y, Haffner P (1998) Gradient-based learning applied to document recognition. Proc IEEE Soc 86:2278–2324
27. Kavukcuoglu K, Sermanet P, Boureau Y, Gregor K, Mathieu M, Le Cun Y (2010) Learning convolutional feature hierarchies for visual recognition. In: Lafferty JD, Williams CKI, Shawe-Taylor J, Zemel RS, Culotta A (eds) Advances in neural information processing systems (NIPS), vol 23. Curran Associates, Red Hook, pp 1090—1098
28. Demongeot J, Bezy-Wendling J, Mattes J, Haigron P, Glade N, Coatrieux JL (2003) Multiscale modeling and imaging: the challenges of biocomplexity. Proc IEEE Soc 91:1723–1737
29. Genealogy (2020). https://www.genealogy.math.ndsu.nodak.edu/id.php?id=160281. Accessed 20 May 20
30. Goles E (2020) Tribute to Prof. Jacques Demongeot: conversations at the *National Bar*. Nat Comput 19**:**3–4

Distortion in Automorphisms of Expansive Systems

Nicolas Bitar, Sebastian Donoso, and Alejandro Maass

Abstract In this work we study the role distortion plays on automorphisms groups of expansive dynamical systems. We begin by generalizing results from subshifts, linking distortion and non-expansivity, to arbitrary expansive systems, and explore the subset of symmetrically distorted automorphisms. Due to the generalization, we are able to determine that expansive automorphisms can never be distorted.

1 Introduction

In symbolic dynamics and group theory, distortion generally refers to an object that grows or moves sub-linearly. In particular, we say that a cellular automaton or an endomorphism acting on a symbolic space is *range distorted* if the local radius of the iterated applications grows at the aforementioned rate. Examples of this behavior can be constructed from Turing machines when viewed as endomorphisms or automorphisms of a symbolic space. Indeed, Guillon and Salo showed in [7] that any aperiodic Turing machine is range distorted. Also, using the so-called conveyor belt technique one can show that on any sofic shift one can define a non-trivial range distorted automorphism.

Analogously, an element of a finitely generated group is said to be *distorted* if its minimal expression on the generating set grows sub-linearly with successive iterations of the element. The two mentioned concepts are related: a group distorted

All authors thank ANID Grant BASAL FB210005 and ACE210010.

N. Bitar · S. Donoso · A. Maass (✉)
Department of Mathematical Engineering and Center for Mathematical Modeling, Universidad de Chile, IRL-CNRS 2807 Santiago, Chile
e-mail: amaass@dim.uchile.cl

N. Bitar
e-mail: nbitar@dim.uchile.cl

S. Donoso
e-mail: sdonoso@dim.uchile.cl

A. Adamatzky (ed.), *Automata and Complexity*, Emergence, Complexity and Computation 42, https://doi.org/10.1007/978-3-030-92551-2_3

automorphism is always range distorted (here we are considering the group of automorphisms). This prompts the fundamental question: does the converse hold?

To address this question, several notions of discrete Lyapunov exponents have been introduced. These objects are closely related to range distortion due to the fact that they quantify the average rate at which any kind of endomorphism on a symbolic space moves information. One recent example are the exponents introduced by Cyr, Franks and Kra in [6], that quantify the rate at which information is moved asymptotically. The novelty of these exponents is that they relate distortion to geometrical properties of the space-time defined by endomorphisms.

In this article, we will show that the phenomenon of distortion is not exclusive to the realm of symbolic systems. This is achieved through the use of M. Boyle and D. Lind's work on expansive dynamical systems [2]. We generalize both the notions of local radius and asymptotic Lyapunov exponents to endomorphisms of arbitrary expansive dynamical systems. Furthermore, the connections between distortion and geometry are preserved. We exemplify this generalization by considering automorphisms of the n-torus.

There is a second question we want to address in this article. Since Boyle and Lind introduced the notions of expansive and non-expansive directions for the study of directional dynamics of an action, there has been one persistent question: which sets can occur as sets of non-expansive directions?

In [2] they showed that this set is closed and, if the domain is infinite, non-empty. Furthermore, they showed that any closed set of directions, with two or more elements is the set of non-expansive directions for some action. Later, Hochman showed in [9] that for every direction, there exists an automorphism of a subshift such that its unique non-expansive direction is the selected one, effectively solving the realization problem. Nevertheless, the subshift built to achieve this result lacks of many natural dynamical properties one would like to get, as transitivity or minimality. This motives the author to ask the following, still open, question: Does any closed non-empty set of directions arise as the set of non-expansive directions of a $\mathbb{Z}^2$-action that is transitive or minimal?

We begin by introducing the necessary concepts from the field of symbolic dynamics from the theory of expansive dynamical systems. We then proceed to generalize the concept of radius to the context of expansive systems, introducing the concept of distorted automorphism. Next, we introduce alternative notions of distortion through the generalization of discrete Lyapunov exponent to the realm of expansive systems. This allows us to establish a connection between non-expansive directions and the asymptotic behavior of an automorphism. In addition, we establish that no expansive automorphism can be distorted. We continue by addressing the question of group distortion in relation to range distortion by studying the set of distorted automorphisms with distorted inverse. Finally, we look at examples of distorted automorphisms, first in the context of subshifts through the use of Turing Machines and then in a non-symbolic example through the study of automorphisms of the torus.