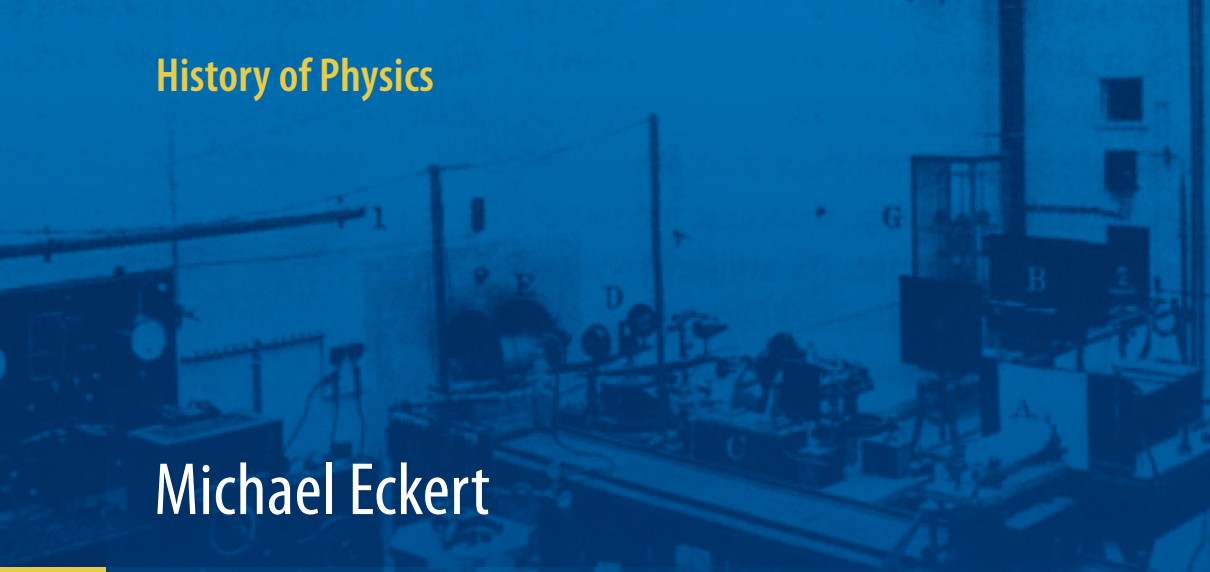
History of Physics



Michael Eckert

Turbulence—an Odyssey

Origins and Evolution of a Research Field at the Interface of Science and Engineering

Springer

History of Physics

The Springer book series *History of Physics* publishes scholarly yet widely accessible books on all aspects of the history of physics. These cover the history and evolution of ideas and techniques, pioneers and their contributions, institutional history, as well as the interactions between physics research and society. Also included in the scope of the series are key historical works that are published or translated for the first time, or republished with annotation and analysis.

As a whole, the series helps to demonstrate the key role of physics in shaping the modern world, as well as revealing the often meandering path that led to our current understanding of physics and the cosmos. It upholds the notion expressed by Gerald Holton that "science should treasure its history, that historical scholarship should treasure science, and that the full understanding of each is deficient without the other." The series welcomes equally works by historians of science and contributions from practicing physicists.

These books are aimed primarily at researchers and students in the sciences, history of science, and science studies; but they also provide stimulating reading for philosophers, sociologists and a broader public eager to discover how physics research – and the laws of physics themselves – came to be what they are today.

All publications in the series are peer reviewed. Titles are published as both print- and eBooks. Proposals for publication should be submitted to Dr. Angela Lahee (angela.lahee@springer.com) or one of the series editors.

More information about this series at https://link.springer.com/bookseries/16664

Michael Eckert

Turbulence—an Odyssey

Origins and Evolution of a Research Field at the Interface of Science and Engineering

Springer

Michael Eckert
Forschungsinstitut
Deutsches Museum
Munich, Germany

ISSN 2730-7549 ISSN 2730-7557 (electronic)
History of Physics
ISBN 978-3-030-91458-5 ISBN 978-3-030-91459-2 (eBook)
https://doi.org/10.1007/978-3-030-91459-2

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Odyssey: a long trip or period involving a lot of different and exciting activities, esp. while searching for something.

https://dictionary.cambridge.org/dictionary/english/odyssey

Preface

Turbulence is a research field where high expectations are met with recurrent frustration. "Turbulence is the most important unsolved problem of classical physics," Nobel Laureate Richard Feynman (1918–1988) is quoted as one among other renowned scientists who voiced the riddle of turbulence. The famous author of a classic textbook on *Hydrodynamics*, Horace Lamb (1849–1934), is alleged to have said "When I meet my Creator, one of the first things I shall ask of Him is to reveal to me the solution to the problem of turbulence" (Peter A. Davidson, 2011, p. xiii). Theodore von Kármán (1881–1963), another pioneer of this science, put a similar quote in the mouth of Arnold Sommerfeld (1868–1951), the famous atomic physicist. Sommerfeld "once told me," Kármán recalled, "that before he died he would like to understand two phenomena–quantum mechanics and turbulence. Sommerfeld died in 1924. I believe he was somewhat nearer to an understanding of the quantum, the discovery that led to modern physics, but no closer to the meaning of turbulence." (von Kármán, 1967, p. 134). Recollections of this sort lack historical scrutiny and authenticity (Sommerfeld died in 1951, not in 1924), but they highlight a common perception among physicists, mathematicians and engineers: that there is a big mystery behind the phenomenon of turbulence.

There are different approaches towards a history of turbulence. From the vantage point of "the turbulence problem," I have traced the changing perception of what was regarded as the outstanding riddle in response to the research agendas in different environments and at different times (Eckert, 2019b). Another approach followed the outstanding pioneers of this research field (Peter A. Davidson, 2011). In this book, I sketch the history of turbulence in a different manner—as an odyssey through uncharted territories from the seventeenth century until a crucial stage in the mid of the twentieth century, a conference at Marseille in the year 1961. This event marked the inauguration of turbulence as "a new science" (Favre, 1962, p. 5), a perception that was confirmed by the subsequent developments and celebrated half a century later with another memorable conference at Marseille (Farge, 2013).

The portrayal of turbulence as an odyssey yields a less coherent view of its origins. There is room for contingency and haphazard developments within and beyond various scientific and engineering disciplines that were themselves in the making

when they are observed over longer periods of time. Yet the narrative of such a history must not result in a chaotic sequence of events: I structure this odyssey in three parts that explore the gateways (Part I) towards turbulence prior to the early twentieth century, the formation of mayor concepts (Part II) until World War II and the staging of the "new science" of turbulence prior to Marseille '61 (Part III). Besides some observations in the epilogue, I do not venture to review turbulence beyond 1961. Such a history would have to include in particular the rise of computational fluid dynamics, turbulence modelling (the "closure problem"), nonlinear dynamical systems ("strange attractors") and unresolved mathematics of the Navier–Stokes equation ("blow-up")—plenty of challenges for a future study of what appears to be another odyssey.

Munich, Germany Michael Eckert
September 2021

Acknowledgements

I am grateful to the staff of the archives and libraries cited in the footnotes for their help to find the pertinent source material and to my colleagues at the Deutsches Museum where I found the appropriate environment for work of this nature. This book benefited from previous projects at the Deutsches Museum on "Physik zwischen naturwissenschaftlich und technologisch orientierter Forschung: Das Beispiel der Strömungsforschung" and "Ludwig Prandtl—Wissenschaftler, Forschungsmanager und Politikberater," supported by the Deutsche Forschungsgemeinschaft (FOR 393/2-1, FOR 393/2-2 and TR 364/18-1). My thanks go also to the organizers of conferences that provided me opportunities to present one or another part of this work, in particular the International Colloquium 2011 in Marseilles on "Fundamental Problems of Turbulence: 50 years after the Turbulence Colloquium Marseilles 1961."

Contents

Acronyms

APS	American Physical Society
AVA	Aerodynamische Versuchsanstalt, Göttingen
DFD	Division of Fluid Dynamics
DFDA	Division of Fluid Dynamics, Archives
DLR	Deutsches Zentrum für Luft- und Raumfahrt
DMA	Deutsches Museum, Archiv, Munich
DMV	Deutsche Mathematiker-Vereinigung
EUROMECH	European Mechanics Society
FIAT	Field Intelligence Agency, Technical
GALCIT	Guggenheim Aeronautical Laboratory at the California Institute of Technology, Pasadena
GAMM	Gesellschaft für Angewandte Mathematik und Mechanik
GOAR	Göttingen Archive of the DLR
IAS	Institute of the Aeronautical Sciences
IAU	International Astronomical Union
ICSU	International Council of Scientific Unions
IGY	International Geophysical Year
ISMT	Institute for the Statistical Mechanics of Turbulence
IUGG	International Union of Geodesy and Geophysics
IUTAM	International Union of Theoretical and Applied Mechanics
LFA	Luftfahrtforschungsanstalt Hermann Göring
MPGA	Max-Planck-Gesellschaft, Archiv, Berlin
NACA	National Advisory Committee for Aeronautics
NOL	Naval Ordnance Laboratory
NPL	National Physical Laboratory, Teddington
ONERA	Office National d'Études et de Recherches Aérospatiales
SUB	Staats- und Universitätsbibliothek, Göttingen
TKC	Theodore von Kármán Collection, California Institute of Technology, Pasadena
UAM	University Archive Munich
UNESCO	United Nations Educational, Scientific and Cultural Organization

ZAMG	Zentralanstalt für Meteorologie und Geodynamik, Vienna
ZAMM	Zeitschrift für Angewandte Mathematik und Mechanik
ZWB	Zentrale für Wissenschaftliches Berichtswesen über Luftfahrtforschung

Part I
Roots

Turbulence did not emerge as a specialty by branching off from physics or another scientific or engineering discipline. Its roots have grown since the early modern era in a host of fields, from hydraulics to meteorology. This part traces the gateways to turbulence until the early decades of the 20th century. Only then was turbulence discerned as a challenge that called for a concerted effort beyond disciplinary boundaries.

Chapter 1
Pipe Flow

Abstract Pipe flow has been a challenge that gave rise to investigations on turbulence—long before turbulence was discerned as a research problem in its own right. The discharge of water from elevated reservoirs through long conduits such as for the fountains at Versailles suggested investigations about the resistance in relation to the different diameters and lengths of the pipes as well as the speed of flow. Despite numerous measurements the data could not be reproduced by a commonly accepted formula, not to mention a theoretical derivation. Even more inaccessible to rational elaboration was the resistance of air flow in long pipes for the supply of blast furnaces or mine air. In the 19th century it became gradually clear that there were two modes of pipe flow, laminar and turbulent. While the former could be accommodated under the roof of hydrodynamic theory, the latter proved elusive.

1.1 The Conduits at Versailles

Conduits of pipes were central parts of water technology since antiquity. Roman pipes even seem to have been standardized. By the middle ages a variety of pipes was in use, made from wood, lead or ceramic. Long conduits were composed as a rule by pipes of ten to twelve feet length. Longer pipes would have been too cumbersome to handle, shorter pipes would have required more joints with the result of a higher friction. But this practice did not result from flow measurements. The design and construction of medieval pipes was entirely based on practical experience (Magnusson, 2001, Chap. 3).

The first published record of pipe flow measurements concerned the requirements for the water art at Versailles and other royal gardens of the "Sun King", Louis XIV., such as the height of fountain jets driven from the discharge of elevated water reservoirs. Under these premises, hydraulics was put on the agenda of the newly founded royal academy in Paris (Blay, 1986). Two years after its foundation, on 11 July 1668, the academicians Jean Picard (1620–1682) and Edme Mariotte (1620–1684) were charged with a study of "the force of running water for pressing and

M. Eckert, *Turbulence—an Odyssey*, History of Physics,
https://doi.org/10.1007/978-3-030-91459-2_1

moving",[1] a task that led to a number of experimental investigations on the relation of water discharge through pipes from reservoirs at different heights and through circular holes of various diameter. The results were published in academy memoirs and summarized in a *Traité du Mouvement des Eaux et des Autres Corps Fluides* (Mariotte, 1686). Not least because of this work, published posthumously by his academic colleague Philippe de La Hire (1640–1718), Mariotte is regarded as "the father of the experimental method in France" (Rouse and Ince, 1957, p. 63).

Pipe flow measurements at Versailles were not a one-time task, nor were Mariotte and Picard the only academicians concerned with this investigation. In the1660s and1670s Versailles and its water supply system was a giant construction site under permanent academic surveillance with regard to the levelling of the canals, pipes and reservoirs for the fountains in the park (Descamps, 2003). It involved outstanding scholars such as Christiaan Huygens (1629–1695) and Ole Rømer (1644–1710) as well as less known academicians like Claude-Antoine Couplet (1642–1722), for whom Versailles was an opportunity to develop his hydraulic skills with pipe flow measurements until he rose to prominence as the academy's permanent treasurer (de Fontenelle, 1722). In 1696 Couplet's son, Pierre (1670–1743), also became a member of the academy—and a collaborator of his father on the pipe flow experiments at Versailles (Heyman, 1976). Many years later, on 22 March 1732, he presented to the academy a comprehensive report about these investigations (Couplet, 1732). The "rules" of pipe flow, as reported in Mariotte's treatise, were derived from measurements on very short conduits, Couplet junior criticized the earliest pipe flow measurements at Versailles. The crucial question concerned the resistance of the water in the conduits:

> This is a question which despite its importance has hardly been clarified; its solution seems to depend on the knowledge of the friction of the water in the pipes in relation to the different diameters, lengths and speed of the water.[2]

Couplet argued that only a great number of experiments could provide this knowledge, and that Versailles was the perfect site for these measurements. By the early 18th century water supply systems did not yet involve large pipes made from cast iron like those at Versailles, where only the best was good enough to achieve the King's demands. Couplet's experiments focused on the discharge of water through five conduites made of cast iron pipes that varied in length from 296 to 2340 toises (1 toise = 6 ft = 1.83 m) and in diameter from 4 to 18 pouces (1 pouce = 1 in. = 2.54 cm). Through these long pipes the discharge was quite different from what was to be expected according to Mariotte's rules. Couplet attributed this discrepancy to the friction of the water against the inner wall of the pipes:

---

[1] "de la force des eaux courantes à presser et à mouvoir." (Blay, 1986, p. 93).

[2] "C'est une question qui, toute importante qu'elle est, n'a point encore été éclaircie, et dont il paroît que la solution dépend de la connoissance du frottement des eaux dans leurs conduites par rapport à différents diamètres et longueurs différentes, et par rapport à leurs différentes vitesses." (Couplet, 1732, p. 114).

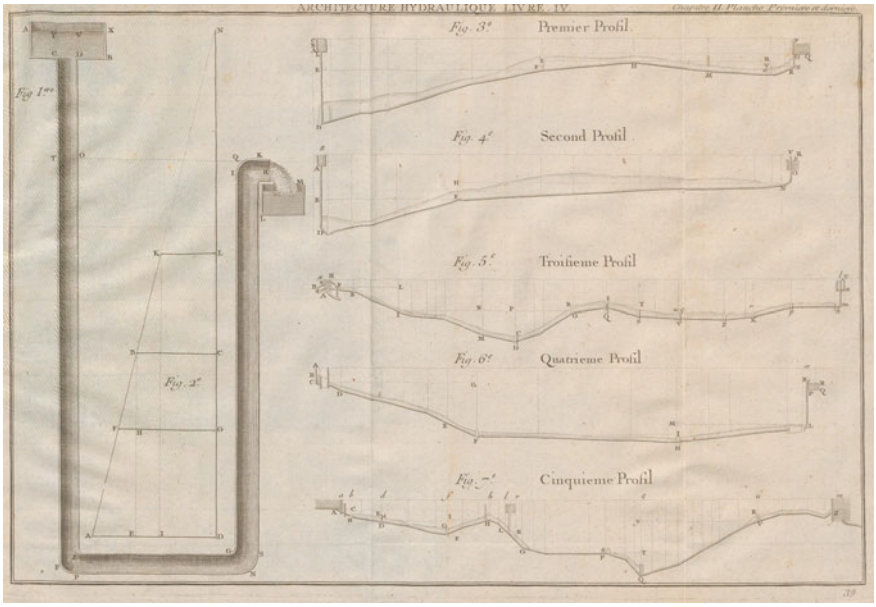


Fig. 1.1 The five conduits at Versailles used for Couplet's pipe flow experiments (Belidor, 1737, Livre IV, Chapitre II, plate I)

> ... and it is astonishing that this friction of the water against the walls of this pipe caused a decrease in the flow of water about 30 times greater than the quantity of water which came out through this pipe.[3]

The conduits included elbows where trapped air disrupted the water column in the pipes. In the case of the longest conduit it could take more than ten days until the water from the reservoir was discharged. Couplet did not arrive at a formula for pipe flow friction, but the conclusion from his measurements was obvious: Friction retarded the flow of water in the pipes considerably—the smaller the pipe diameter and the longer the conduit and the larger the flow speed, the more. Elbows and bends, too, added to the friction.

Couplet's measurements were regarded important enough to be included five years later in Bernard Forest de Bélidor's (1697–1761) famous *Architecture hydraulique* (Belidor, 1737, Livre IV, Chapitre II). They deserved particular interest also because of their location at Versailles, "perhaps the only place of the world where one finds everything one could wish in order to perform experiments of that sort we are dis-

[3] "... il est étonnant que ce frottement de l'eau contre les parois de ce tuyau ait causé une diminution d'écoulement d'eau environ 30 fois plus grande que la quantité d'eau qui est sortie par cette conduite." (Couplet, 1732, p. 148).

cussing here."[4] He paid tribute to Couplet by extensive quotes from his academy memoir, including a copper engraving that showed the profile of the five conduits at which the measurements had been performed (Fig. 1.1).

1.2 Pipe Flow Accounts in the Late 18th Century

As soon as metallic pipes became used more broadly for the water supply of cities, pipe flow measurements on long conduits were no longer confined to royal gardens. In 1771 Charles Bossut (1730–1814), professor of mathematics at the École du Génie in Mézières, published a treatise on hydrodynamics where he included the results of his measurements from conduits used for the water supply of Mézières (Bossut, 1771, Chap. VI). By and large he agreed with Couplet's findings, so that he allowed himself to proceed one step further to "form a general and sufficient practical idea of the loss of speed that water makes in pipes, either straight or curvilinear."[5] Instead of a formula for the friction he provided tables for the presentation of his results from which a user might choose the case closest to the one he wants to treat.

As a member of the Paris academy, Bossut's expert knowledge was requested also in other hydraulic projects. In 1776, for example, he witnessed water raising trials with the famous machine de Marly. This giant machine pumped water via intermediate reservoirs up to an aquaeduct 162 m above the level of the Seine from where it was conducted to Versailles. The plan was to dispose of the intermediate reservoirs and pump the water in a single step up to the aquaeduct. However, the pipes used for this purpose proved inadequate. These are matters, Bossut excused the impotence of his expertise in this affair, "whose knowledge is based on practical trial and error and which can only be imperfectly appreciated by theory and reasoning."[6]

Bossut's pipe flow measurements called for more extensive investigations about the friction in long conduits. "I devoured the part of Abbé Bossut's Hydrodynamics, which deals with the movement of water," Pierre du Buat (1734–1809) introduced his *Principes d'Hydraulique, Vérifiés par un grand nombre d'Expériences faites par ordre du Gouvernement*. He regarded the pipe flow data as "the key to hydraulics." Du Buat reasoned that the flow on an inclined river bed reached a uniform speed only because the accelerating force of gravity was balanced by friction, and

> that the movement of water in a pipe had a great analogy with the uniform course of a river bed, since in both cases gravity was the motor, and the resistance of the bed the moderator. I

[4] "... sur la dépense des tuyaux de conduite qui amènent l'eau dans les réservoirs de Versailles, qui est peutêtre le seul endroit du monde où l'on trouve tout ce que l'on peut désirer, pour faire des expériences de la nature de celles dont nous parlons." (Belidor, 1737, p. 273).

[5] "En combinant nos expériences avec celles de M. Couplet, on se formera une idée générale et suffisante dans la pratique, de la perte de vitesse que l'eau fait dans les tuyaux de conduite, soit rectilignes, soit curvilignes." (Bossut, 1771, p. 159).

[6] "... dont la connaissance tient à un tâtonnement pratique et qui ne peuvent être qu'imparfaitement appréciés par la théorie et le raisonnement". Quoted in Brandstetter (2006, p. 186). See also (Barbet, 1907, pp. 141–142).

> therefore made use, in order to make a formula of uniform movement, of the experiments of Abbé Bossut on the pipes, and even of those he had made on dummy canals, although one datum was missing, which was the depth of the current. This is how I composed the work that I gave to the public, under the title of Principles of Hydraulics, in the year 1779.[7]

As indicated by the title of du Buat's treatise, his investigations were commissioned by the government, more precisely by the war ministry which also placed at his disposal two officers from the Royal Engineering Corps for performing the experiments. Unlike Bossut's pipes of conduit which had only a "mediocre" slope, du Buat's experiments comprised very small and very large inclinations. The slope was calculated by dividing the length of the pipe by the height of the reservoir from which the water was discharged through the pipes. Furthermore, du Buat and his officers experimented with pipes bent in different ways, comparing the results with those of straight pipes of the same diameter and length. In his treatise, du Buat displayed the results of 56 experiments in tables with five entries for each measurement: the length of the pipe, the height of the reservoir, the slope, the experimentally measured velocity (derived from the discharge divided by the cross section of the pipe), and the velocity calculated from a formula that combined theoretical speculation with empirically adjusted coefficients (Buat, 1786, I, p. xxv and pp. 71–74).

1.3 Early Pipe Flow Formulae

Bossut's and Du Buat's accounts on pipe flow marked the beginning of an era in the history of fluid mechanics in which experiments challenged pre-conceived theoretical concepts about the nature of fluid resistance (Darrigol, 2005, pp. 221–222). Pioneers of ideal flow theory in the 18th century, such as Johann (1667–1748) and Daniel Bernoulli (1700–1782) (father and son) or Leonhard Euler (1707–1783), contributed little to elucidate pipe flow friction. Euler's effort in this regard was largely ignored—and would have been doomed to failure if it had been scrutinized by hydraulic engineers (Bistafa, 2015). Both Bossut's *Traité élémentaire d'hydrodynamique* and du Buat's *Principes d'Hydraulique*, however, were widely appreciated as valuable contributions. They encouraged further investigations on pipe flow and were translated into German. "This book certainly is one of the most useful and best ones that has appeared since some time in hydraulics," Reinhard Woltmann (1757–1837), a German pioneer in hydraulic engineering, praised du Buat's treatise. "Theory and experience go together step by step, and it testifies on almost every page to the skill, diligence and love of truth of the author." The "almost" alluded to du Buat's specu-

[7] "que le mouvement de l'eau dans un tuyau de conduite avait une grande analogie avec le cours uniforme d'un lit de riviere, puisque de part et d'autre la pesanteur était le moteur, et la résistance du lit le modérateur. Je me servis donc, pour faire une formule du mouvement uniforme, des expériences de M. l'abbé Bossut sur les tuyaux de conduite, et même de celles qu'il avait faites sur des canaux factices, quoiqu'il y manquât une donnée, qui était la profondeur du courant. C'est ainsi que je composai l'ouvrage que j'ai donné au public, sous le titre de Principes d'Hydraulique, en l'année 1779." (Buat, 1786, I, pp. xvi–xx).

lations on the nature of friction. "I confess that I don't like the author's imagination on the resistance due to friction," Woltmann qualified his praise. He missed a more detailed consideration of the movement of the water particles close to the wall where, in his view, "they become hindered by collisions, deviated into other directions and into an eddying motion whereby the entire flow is decelerated." Furthermore, Woltmann was unimpressed by du Buat's formula with which the experimental data were fitted. By means of arbitrary quantities "Mr. Buat has modified his formula until it fitted precisely all trials."[8]

Another German expert on hydraulics, Johann Albert Eytelwein (1764–1848), who had started his career in the Prussian administration of constructions and co-founded in 1799 the Berlin Building Academy (Bauakademie), also paid tribute to du Buat in a study "On the motion of water in conduit pipes".[9] He praised Couplet's, Bossut's and du Buat's pipe flow measurements and the effort to arrive at a "general expression" for these results, but he, too, did not like du Buat's formula—even though for other reasons than Woltmann. He regarded the formula as largely useless "because of its complicated form".[10] Eytelwein derived from the same experiments the following formula:

$$u = 6,42\sqrt{\frac{50dh}{l+50d}},$$

where u is the mean flow velocity in the conduit of pipes, d and l the diameter and length of the conduit, and h the pressure head, i.e. the height between the reservoir and the lower end of the conduit of pipes; all length units in "rheinisch" feet. The discharge M is given by the product of the cross section of the pipe times the velocity

$$M = \frac{1}{4}\pi d^2 u = 5.04d^2\sqrt{\frac{50dh}{l+50d}}.$$

Eytelwein's pipe flow formula did not fit the data as well as du Buat's formula—but had the virtue of being useful for determining the dimensions of pipes for given purposes. It could be used to solve engineering problems such as: "How large has to

[8] "Dieses Buch ist gewiss eines der nützlichsten und besten, die seit einiger Zeit in der Hydraulik erschienen sind. Theorie und Erfahrung gehen in demselben Schritt für Schritt beisammen; und es zeugt von der Geschicklichkeit, dem Fleiße und der Wahrheitsliebe des H. Verf. fast auf allen Seiten. [...] Ich gestehe, dass mir des H. Verf. Vorstellungsart über den Widerstand wegen des Reibens nicht gefällt [...] Dagegen stoßen die Teilchen an, werden aufgehalten, in andere Richtungen und mancherlei wirbelnde Bewegung gebracht, wodurch der ganze Strom verzögert wird [...] hat Hr. Buat seine Formel so lange abgeändert bis sie auf alle Versuche genau passte." (Woltmann, 1791, pp. 142, 152, 162). On Woltmann's career see (Ruehlmann, 1885, pp. 280–283).

[9] "Von der Bewegung des Wassers in Röhrenleitungen". (Eytelwein, 1801, 9. Kapitel). On Eytelwein's career see (Ruehlmann, 1885, pp. 284–289).

[10] "Herr du Buat hatte das groß e Verdienst, zuerst einen allgemeinen Ausdruck mitgetheilt zu haben, welcher mit den bekannten Erfahrungen übereinstimmt, und der bloß den Fehler hat, dass er wegen seiner verwickelten Form, nur mit vieler Weitläufigkeit Anwendung findet." (Eytelwein, 1801, p. 216). Du Buat's formula is discussed in (Rouse and Ince, 1957, pp. 131–132).

be the diameter of a straight conduit pipe with a length of 100 ft at a pressure head of 5 ft in order to discharge a half cubic feet per second?"[11]

In 1804 Gaspard de Prony (1755–1839), director of the École des Ponts et Chaussées and perhaps the most prominent representative of hydraulic engineering in France, made a major step forward by suggesting a formula which combined pipe flow and open channel flow. He shared the view—introduced by Charles Coulomb (1736–1806) a few years earlier in the wake of his famous torsion balance experiments— that there are two forces involved in fluid friction: one due to the mutual interaction of the molecules of the fluid, and one due to the collision of the fluid molecules with surface irregularities at the bounding wall. The former resulted in a friction proportional to the flow velocity, the latter implied an inertial retardation proportional to the square of the velocity (Darrigol, 2005, pp. 104–106). Prony, therefore, attempted to account for the experimental data by making the ansatz

$$\frac{gdh}{4l} = au + bu^2$$

where g is the gravitational acceleration and a, b are constants to be determined from measurements. Like Eytelwein, Prony relied on the pipe flow experiments of Couplet, Bossut and Du Buat. Furthermore, he employed the newly introduced metric system. Altogether, the set of data comprised the results of 51 experiments with conduits of tin and cast iron pipes that differed considerably in diameter (from 2.7 to 49 cm) and length (from 9.7 to 2280 m). For the determination of a and b Prony adopted a method developed by Pierre-Simon Laplace (1749–1827) in celestial mechanics for matching observational data to mathematical expressions. As a result, Prony's *Recherches Physico-Mathematiques sur la Théorie des Eaux Courantes* represented empirical hydraulic knowledge combined with sophisticated mathematical elaboration like no other contemporary account (de Prony, 1804).

Prony's formula dominated hydraulic engineering for several decades. The two terms proportional to u and u^2 seem to foreshadow the distinction of laminar and turbulent flow, but this is a retrospective interpretation. The notion of viscosity as introduced by Claude-Louis Navier (1785–1836) in the 1820s in the fundamental equations of flow did not enter pipe flow formulae before the second half of the 19th century (Darrigol, 2005, Chap. 3).

1.4 Air Flow in Long Conduits of Pipes

While Prony's formula made the pipe flow of water accessible to hydraulic engineering, the flow of air in conduits of pipes remained mysterious—although the

[11] "Wie groß wird man den Durchmesser einer graden 100 Fuß langen Röhrenleitung bei einer Druckhöhe von 5 Fuß annehmen müssen, damit solche in jeder Sekunde einen halben Kubikfuß Wasser liefert?" Inserting $M = \frac{1}{2}, h = 5, l = 100$ yields an equation for d with the result $d = 0,341$ ft (Eytelwein, 1801, p. 223).