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Optimization Aided Design

Reinforced Concrete





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Foreword by Manfred Curbach

Intensifying Creativity in Construction

There is hardly a topic among building professionals that is discussed more intensively than sustainable construction. In view of the emphasis on this topic, it appears that intensive work is being done on the implementation of this challenge, both in research and in realization. After all, it is about nothing less than building in a way that enables all people of the generations to come to live a decent life on this earth. Because we have only this one. In 1994, the astronomer and astrophysicist Carl Sagan had the idea of taking a photo of the Earth with the help of the Voyager 1 space probe after it left the solar system. In a lecture on 13 October 1994 at the Cornell University, he said the following about this:

Our planet is a lonely speck in the great enveloping cosmic dark. In our obscurity, in all this vastness, there is no hint that help will come from elsewhere to save us from ourselves. There is perhaps no better demonstration of the folly human conceits than this distant image of our tiny world. To me, it underscores our responsibility to deal more kindly with one another, and to preserve and cherish the pale blue dot, the only home we've ever known.¹

In fact, we are overexploiting and consuming the resources of our earth and changing them massively. The consequences are climate change, scarcity of resources, natural disasters, hunger, flight, and misery. And the construction industry is massively involved in these developments.

The building industry in Germany accounted for 5.3% of nominal gross value added in 2018 (€179.6 billion GDP of €3388.2 billion GDP)² but causes around 25% of CO₂ emissions and uses around 40% of the energy generated.³

This discrepancy alone should lead to enormous productive activities. But what is the reality in terms of efficiency and research?

- 1 Sagan, C. (1997). Pale Blue Dot. United States: Random House USA Inc.
- $2\,$ Statistisches Bundesamt (2019). Bruttoinlandsprodukt 2018 für Deutschland. Wiesbaden: Statistisches Bundesamt.
- 3 Hong, J., Shen, G.Q., Feng, Y., et al. (2015). Greenhouse gas emissions during the construction phase of a building: a case study in China. *Journal of Cleaner Production* 103: 249–259.

In sectors such as manufacturing (excluding construction), productivity increased by around 70% from 1995 to 2016, whereas in construction it only increased by around 5%.4

In terms of industry investment in research and development, out of a total of 436 571 people (in full-time equivalents) in 2017, only 1147 people came from the construction industry, i.e. 0.26%.5

The Federal Government of the Federal Republic of Germany spent a total of €17 250 million on research and development in 2018. Of this, €118.1 million was allocated to the area of "Regional planning and urban development; construction research," i.e. 0.69%,6

Considering only the Federal Ministry of Education and Research, a total of €10 486.7 million was invested in 2018. The area of "Regional planning and urban development; construction research" accounted for a share of only €27.5 million, i.e. 0.26%.

In 2019, the annual grant total from the German Research Foundation amounted to around €3285.3 million. The field of Civil Engineering and Architecture received grants totaling €51.5 million, i.e. 1.57%.⁷

The result of this small survey illustrates that in one of the most important industries in Germany, which contributes disproportionately to climate change, efficiency is stagnating and, at the same time, research is receiving severely below-average

Every 12 years, the population of the earth grows by 1 billion people⁸ who need a decent home, infrastructure, and energy supply. In view of the continuing increase in the world's population, we will not build less, but more. Contrary to this, we need to radically limit resource consumption and CO₂ emissions. It is obvious that in the future, building will have to be completely different, not just marginally, but fundamentally.

It is thus clear that we must significantly intensify research in the construction industry. Because of its enormous leverage effect, this is therefore one of the most important tasks for the future, both nationally and internationally, with extremely great significance for society as a whole. At all levels, from basic research to realization, for all available and newly to be developed building materials and combinations of building materials, in all areas of our social life up to politics, we have to become much more creative. Only through our inventiveness, our power of imagination for realization, our abilities to mentally penetrate complex processes

⁴ Statistisches Bundesamt (2020). Inlandsproduktberechnung - Lange Reihen ab 1970, Fachserie 18 Reihe 1.5. https://www.destatis.de/DE/Themen/Wirtschaft/Volkswirtschaftliche-

Gesamtrechnungen-Inlandsprodukt/Publikationen/Downloads-Inlandsprodukt/inlandsproduktlange-reihen-pdf-2180150.html (accessed 27 August 2021).

⁵ Stifterverband für die Deutsche Wissenschaft e.V. (2019). Forschung und Entwicklung in der Wirtschaft 2017. Essen: SV Wissenschaftsstatistik GmbH.

⁶ Bundesministerium für Bildung und Forschung (2020) Bildung und Forschung in Zahlen 2020 (It should be noted that the figures therein for 2019 and 2020 are target figures).

⁷ Köster, T.; Lüers, K.; Schneeweiß, U.; Hohlfeld, C.; Kaufmann-Mainz, N. (2020). Deutsche Forschungsgemeinschaft - Jahresbericht 2019 - Aufgaben und Ergebnisse.

⁸ United Nations (2019). World Population Prospects: The 2019 Revision. https://www.dsw.org/ infografiken/ (03 April 2021).

will we change the entire building process from design, planning, calculation, structure, material extraction, production, transport, on-site construction, operation, maintenance, data storage, strengthening up to further use, reuse, and recycling in such a way that we achieve climate- and resource-neutral building.

The methods, procedures, and calculations described in this book represent an important step toward a kind of building that has little to do with the way we know it today. And this is a good thing.

At the same time, may this book also promote the idea that it is worthwhile for everyone to think about change in the building industry to contribute ideas, to conduct research, and to work on realization. May the amount of research increase to a degree that is both appropriate and necessary to the challenge we all face.

Dresden, June 2021

Manfred Curbach

Prof. Dr.-Ing. Dr.-Ing. E.h. Manfred Curbach. Since 1994 professor for concrete structures at the Technische Universität Dresden, from 1999 to 2011 speaker of the SFB 528 "Textile Reinforcement for Structural Strengthening and Repair," since 2013 speaker of the BMBF consortium C^3 – Carbon Concrete Composite, since 2020 speaker of the SFB/TRR 280 "Design Strategies for Material-Minimised Carbon Reinforced Concrete Structures," 2014 foundation of the company CarboCon for the practical implementation of carbon concrete, 2016 winner of the German Future Prize, Award of the Federal President for Technology and Innovation.

Foreword by Werner Sobek

Building Emission-Free for More People with Less Material

Concrete is the only building material that can be cast into almost any shape on the construction site. It is available worldwide, is cheap, is easy to use, has a comparatively high strength, and is resistant to most environmental conditions. Concrete is the building material for everyone and for everything; it is the most widely used building material in the world. On the other hand, concrete is more unpopular for most people than almost any other material. In the past decades, this dislike was mainly based on its color, the quality of its surface, and its "coldness" (i.e. its low heat radiation). Today, it is the massive criticism of the CO_2 emissions caused by the production of cement, which, at around 8% of global CO_2 emissions, make a significant contribution to global warming and which, in terms of volume, even exceed the emissions of the entire global air traffic that labels concrete as an unloved, even demonized building material.

The currently widespread message that it will be possible to replace concrete as a building material with timber in the short to medium term is mostly based on an ignorance of the interrelationships. Mankind currently needs approximately 60–100 Gt of building materials per year in order to create a home for the new inhabitants on earth and to expand the existing built environment. This number does not include the so-called pent-up demand of the Third World, which with 6.3 billion people represents approx. 80% of the world's population and which, with a volume of approx. 60 t per capita, has a significantly lower building standard than the citizens of the industrialized nations, who account for approx. 335 t of building materials per capita.

If all the forests in the world were managed according to the principles of sustainable forestry, as is the case in many countries in Central Europe, for example, then a maximum of 10 Gt of construction timber could be obtained per year. An increased supply of construction timber through a higher logging rate in the forests would mean a reduction of the urgently needed ${\rm CO_2}$ sink potential of these forests and must therefore be rejected. A redistribution of the available timber toward the construction industry would mean a reduction in the availability of wood to produce

cellulose, paper or, for example, the abandonment of the cooking of daily food, as is still the case for many hundreds of millions of people every day.

To the scenario described above, the already mentioned pent-up demand for the inhabitants of the Third World. If the frequently voiced demand for an increase in prosperity and thus also a reduction in the birth rate in these countries were actually to be met through easier access to health care and education, especially for the female part of the population, the corresponding construction activities would have to be carried out, for example the building of schools and universities, medical practices and hospitals, including the associated infrastructure. Raising the level of construction in the Third World countries to that of today's industrialized countries can be estimated with a demand for building materials of 1700 Gt. This amount of building material represents twice the world built today. The climate-damaging emissions associated with the production of these building materials would make the earth uninhabitable for mankind. It is therefore evident that we will not be able to raise the total population of this earth to the building level of today's industrialized countries nor will timber as a building material be able to play a significant role in this context in the short to medium term. Timber will be an important building component in some parts of the world, especially in the Northern Hemisphere, in the short to medium term. Not more. Other building materials, such as clay or natural stone, will also increasingly find their way into construction. However, none of these materials will be able to replace concrete as a building material.

But what should the builders, the architects, the engineers, and the executing companies do, on the one hand, to fulfill their responsibility to provide a built home, including all the necessary infrastructural construction measures, for more and more people and, on the other hand, to make their enormously important contribution to limiting, even reducing, global warming? Since an ideal way has not yet been identified and the marvel material that solves all problems has not yet been found, the solution to the overall problem will consist of a sum of components. One of these components is the restriction of construction activities to what is actually necessary, the appropriate amount. This is often referred to as the principle of sufficiency. The principle of sufficiency includes the requirement not to demolish buildings or parts of them and replace them with new buildings until this is really unavoidable.

Another component of sustainable construction is the revolutionization of construction technology to the effect that in the future only recycling-oriented planning and construction will be permitted. In this way, the extraction of new building materials from the upper layers of the earth can be increasingly reduced in the medium to long term. This will also diminish, if not solve, the availability problems of individual building materials. It is common knowledge that enormous quantities of sand and gravel are required, especially for the production of concrete, and that sand has already become an extremely rare resource in some regions of our planet. The same applies to gravel and crushed stone. Immense availability problems are also expected for tin, zinc, and copper. For the construction industry, being the largest consumer of resources of all, it is therefore a matter of dramatically reducing the "consumption" of primary materials in the future and of using secondary materials where they are

actually unavoidably needed. If we consider the availability of resources in addition to the emissions, it can be seen that the local and regional production of secondary material is associated with significantly lower climate-damaging emissions compared to primary material that is often delivered over long transport routes.

While the implementation of the closed-loop principle reduces the amount of "consumed" primary building materials, the complementary implementation of lightweight construction technologies can reduce the amount of consumed material and the amount of climate-damaging emissions during its production and distribution. This is where this book comes in. The introduction of state-of-the-art optimization methods and the resulting minimum-material component shapes, which also have a minimized need for reinforcing steel due to optimized reinforcement design, promote construction with concrete that is characterized by considerable material savings and thus considerable emission savings for the same utility value and durability. Supported by clearly understandable descriptions and a large number of examples, readers will find their way around quickly and easily. This makes it much easier to understand the subject matter, which is not always simple.

This book provides a significant contribution to establishing a new foundation for building with concrete, this wonderful building material for everyone and for almost everything. This foundation is characterized by the application of highly developed calculation methods and technologies that lead to material-minimized components and thus also to emission-minimized components. Both will be an essential part of tomorrow's construction, a construction that, like other sectors such as transportation or energy, must reduce its emissions by more than 50% by 2030. No one knows today how this will ultimately be achieved. However, the paths outlined in this book represent a valuable and indispensable tool on the way to achieving these goals.

Stuttgart, June 2021

Werner Sobek

Prof. em. Dr. Dr. E.h. Dr. h.c. Werner Sobek. From 1995 to 2021 successor of Frei Otto, and at the same time, from 2000 to 2021 also successor of Jörg Schlaich at the University of Stuttgart. Founder of the ILEK Institute of lightweight structures and conceptual design at the University of Stuttgart. From 2008 to 2014 professor at the Illinois Institute of Technology in Chicago in the succession of Ludwig Mies van der Rohe. Founder of the Werner Sobek group. Founder and co-founder of several charitable foundations and non-profit associations in the field of construction.

Preface

This book is based on over 15 years of research work at the Institute of Concrete Structures (Ruhr University Bochum, Germany) on topics related to structural optimization and lightweight concrete structures. The motivation, then and now, derives from two fundamental reasons. First, the climate challenge and the related necessity for lower material consumption. Second, modernizing the construction industry through new technologies aiming at more sustainable design and construction methods. The concepts evolved from the research work are combined in this book into an enhanced design approach, which we call *Optimization Aided Design* (OAD).

From students to researchers and practitioners, this book addresses everyone involved in structural engineering. Although the concepts primarily focus on concrete structures, they are generally adaptable to a wide range of further applications regardless of the material used. Numerous computational examples serve for a better comprehension of the methods and invite to discover the potential of OAD. Applications that have been successfully implemented further demonstrate transferability in practice and intend to provide inspiration for future projects.

Apart from the introduction, the book consists of two parts. The first part serves as introduction to the fundamentals of reinforced concrete design, on the one hand, and structural optimization on the other. Chapters 2 and 3 provide a general basis for understanding the methods presented subsequently. In no case do they claim to be exhaustive. For a more in-depth study of both topics, many excellent books from other colleagues already exist. In this regard, reference is made to the bibliography. The second part of the book introduces OAD for concrete structures. The methods are presented structurally from the outside in. In doing so, first, approaches for identifying the external structural shape are presented, followed by methods for designing the inner one (reinforcement layout), and finally techniques for the optimization of cross-sections.

Each of the OAD chapters is divided into three parts. They begin with a brief topical description supported by a representative overview figure, allowing the reader to decide whether the subsequent content has relevance for her or him. This is followed by the main section, in which the methods are discussed exhaustively and are supplemented with recommendations for their practical application. Numerous computation examples, to which reference is made in the respective main sections, provide the conclusion. They are further enhanced by application examples which

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have already been realized, for example ultra-light beams, extremely thin shells of solar thermal power plants or optimized reinforcement layouts for segmental tunnel linings.

OAD offers the possibility to enhance the daily engineering work and increase its efficiency. Our ambition is to highlight the great potential of the approach and thereby contribute to a modern, sustainable, and transparent way of designing and dimensioning reinforced concrete structures in the future. However, this can only succeed if we open the door for modern approaches and thus prove to the new generation, that the construction industry is able to adapt to the modern age. Considering the global challenges, let us be part of the solution, not part of the problem.

Bochum, Germany April 2021

Georgios Gaganelis Peter Mark Patrick Forman

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