Lecture Notes in Electrical Engineering 809

Alfonso Bachiller Soler
Ramón Cano Gonzalez
Miguel Angel González Cagigal

# Solved Problems for Transient Electrical Circuits

Springer

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Volume 809

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Miguel Angel González Cagigal

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Alfonso Bachiller Soler
Department of Electrical Engineering
University of Seville
Seville, Spain

Ramón Cano Gonzalez
Department of Electrical Engineering
University of Seville
Seville, Spain

Miguel Angel González Cagigal
Department of Electrical Engineering
University of Seville
Seville, Spain

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*To our parents*

# Foreword

The title of this book, *Solved Problems for Transient Electrical Circuits*, clearly indicates that its contents deal with one of the fundamental themes of Electrical Circuit Theory, as does that of “Three-Phase Circuits” and “Dependent Sources”, in the context of Electrical Engineering. However, in my opinion, the importance of transient circuit behaviour surpasses the limits of electrical engineering, since it also constitutes a fundamental component of electronic engineering, mainly in its digital, power, and telecommunications areas. The study of the circuits under a dynamic regime enables the correct interpretation to be attained of certain types of electrical behaviours that remain elusive, even to engineers, in a first reasoning. Examples of such behaviours include the emergence of voltages far higher than those of the generators themselves in circuits and in electrical networks, with destructive effects; the untimely firing of differential switches in homes; and the sporadic performance of protection during transformer commissioning.

Transients are also present in digital electronic circuits, since their binary nature obliges transistors to work as ideal switches at high speeds, thereby generating a transient period in each switching. The same is true of power electronics, where, for performance reasons, semiconductors also work as switches by switching thousands of amps into microsecond fractions. In these cases, the permanent regime of the circuits becomes a continuous sequence of transient regimes.

Transient operation in electronic devices involves rapid variations in voltage and current that can cause electromagnetic disturbances in the circuits. Electromagnetic Compatibility regulatory requirements constitute one of the key points of electronic design and have emerged as a discipline in telecommunications and industrial engineering studies.

The importance of the study of the dynamic regime of circuits, both electrical and electronic, should be borne in mind for future professionals of these subjects.

The study of transients is addressed in this book by first stating the fundamental theoretical concepts, which are subsequently consolidated with the help of solved and annotated problems of increasing difficulty. Although, in professional practice, complex circuits are often solved by simulation and not by mathematical tools such as differential equations and the Laplace transform, the interpretation and valuation of numerical results remains essential, and this is only possible if the necessary theoretical knowledge is available.

The text is organised into three chapters: First-Order Transients, Second-Order Transients, and the Laplace Transform. Each chapter begins with the corresponding theory, supported by application examples, followed by a collection of resolved and annotated problems. In all three parts, a suitable balance is struck between the theory necessary to understand the concepts and their application through problem-solving. The authors, as in all their publications, have managed to provide the reader with a book containing figures and typography both in harmony and of the highest quality.

Finally, with the help of this book, the reader, as a student, is in possession of the tools necessary to successfully master the subjects related to electrical circuits, and, as a professional, can preserve this volume in order to extract the fundamentals of its content.

Sevilla, Spain
June 2021

Vicente Simón Sempere

# Preface

The text is intended for the first course on electric circuits. The focus is on the transient response of linear circuits. The analysis of this type of circuits is generally carried out in the second year of electrical engineering studies and related fields.

The book has been divided into three large chapters that progressively address the study of the transient response of first-order and second-order circuits and, finally, circuits of any order through the use of the Laplace transform.

Each block begins with a detailed study of the theoretical knowledge and the resolution techniques necessary to obtain the transient response of the different types of circuits. This is followed by a significant number of solved problems. The resolution of each exercise has been carried out in detail and with the support of more than 300 figures. For a better understanding of the transitory phenomenon, the evolution of the voltages and currents of the elements during the transient period has been graphically represented in the cases considered relevant. The exercises have been ordered from the most elementary to the most complex, which allows progressive learning.

In this book, only circuits with linear elements are considered where the origin of the transient is caused by the opening or closing of switches. This model responds to maneuvering operations in high and low voltage electrical networks. It is also the model that is analysed when considering the main defects in this type of installations, such as short circuits and insulation defects. Finally, the same model is obtained in power electronics topics, where electronic devices (BJT, MosFet,…) have the behaviour of switches that switch several times per second.

Although several circuits have been included that contain controlled sources, which may lead to instability, the parameters have been selected to obtained stable circuits in all cases. The analysis of unstable circuits is carried out initially with the same techniques, but it leads to non-real results if the non-linearity of the elements is not considered.

In the problems, voltages and currents of the elements have been the variables under study. Power and energy can be easily obtained from them. However, it has been preferred not to include them in the calculations to obtain a more fluent reading of the text.

Sevilla, Spain
June 2021

Alfonso Bachiller Soler
Ramón Cano González
Miguel Angel González Cagigal

# Contents

# Chapter 1
# First Order Transients

**Abstract** This chapter covers the first order circuits, beginning with a theoretical introduction of the concepts required to correctly address each of the subsequent problems. A total of 31 fully solved problems with explanatory comments are included.

## 1.1 Introduction

In circuits with resistors as sole passive elements, voltages and currents respond immediately to the changes in the sources of excitation. In such circuits, known as static circuits, the voltage and current values of the involved elements are given by a set of algebraic equations, meaning that each instant of time can be independently analyzed. This is not the case in circuits containing energy storage elements, i.e. inductors or capacitors, where the voltage is related to the current through a differential equation, resulting in a dynamic response of the circuit. In this type of circuits (dynamic circuits), information on the past is necessary to determine the response at any time.

In dynamic circuits excited with dc or ac sources, after a period of time has elapsed (transient regime) the so-called steady-state regime is reached, where the response is stabilized at a constant value (dc excitation) or a periodic wave (ac excitation). As an illustrative example, the response of an RC circuit is represented in Figs. 1.1 and 1.2, including respectively a dc voltage source and an ac voltage source. It can be seen how, after the transient period, the steady-state regime is reached in both cases.

In general terms, the transition from a steady-state regime to a different one involves a transient period. The origin of these transient processes can be related to various actions, including opening and closing of switches, short circuits or any other variation in the circuit topology or parameters.

 
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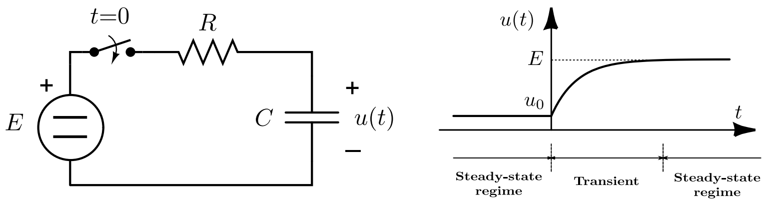



**Fig. 1.1** Connection of an RC circuit to a dc source

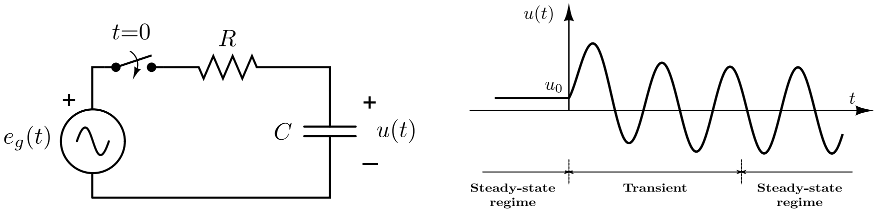



**Fig. 1.2** Connection of an RC circuit to an ac source

## 1.2 First Order Circuits

First order circuits are defined as those where any voltage or current can be obtained using a first order differential equation. Some examples of first order circuits are:

- Circuits with a single electrical energy storage element: inductor or capacitor, Fig. 1.3.

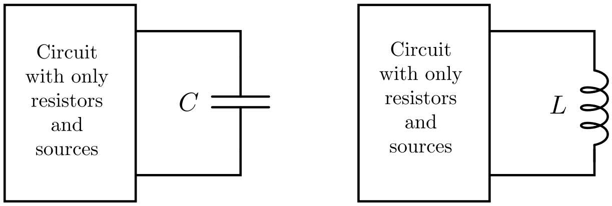



**Fig. 1.3** First order circuits with one energy storage element

- Circuits including multiple energy storage elements of the same type, which can be combined into a single equivalent element, Figs. 1.4 and 1.5.

### *1.2.1 RC Circuits*

First order circuits will be considered with one or more capacitors that can be combined into a single equivalent one. The rest of the circuit, composed by electrical

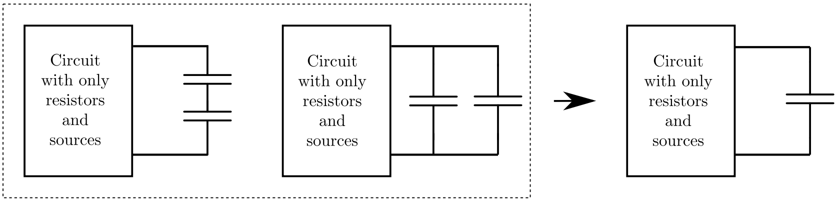



**Fig. 1.4** Circuit with two capacitors connected in series and in parallel

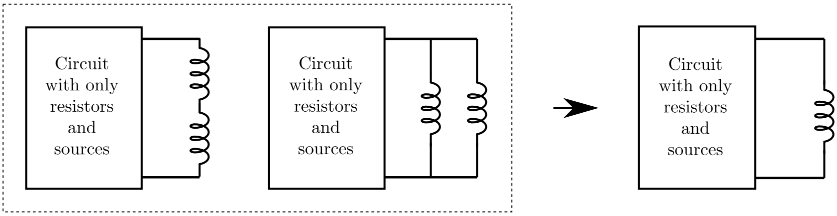



**Fig. 1.5** Circuit with two inductors connected in series and in parallel

sources and resistors, can be replaced by its Thévenin equivalent as shown in Fig. 1.6. This way, the study of the RC series circuit excited by a voltage source encompasses all the first order circuits whose storage element is a capacitor.

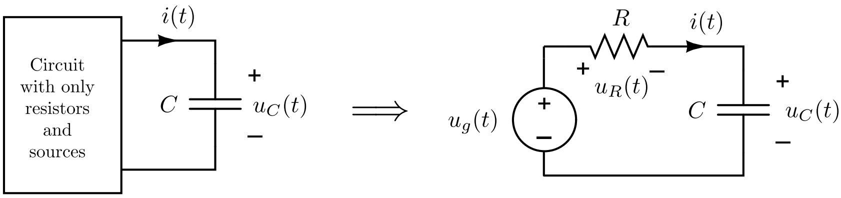



**Fig. 1.6** RC circuit and its Thévenin equivalent

The differential equations that defines the behavior of the variables involved in the circuit represented in Fig. 1.6 will be subsequently obtained.

**Differential equation of capacitor voltage**

Applying Kirchhoff's voltage law:

$$u_C(t) + u_R(t) = u_g(t) \tag{1.1}$$

The resistor voltage can be substituted using Ohm's law:

$$u_C(t) + R \cdot i(t) = u_g(t) \tag{1.2}$$

The capacitor defining equation is now considered

$$i(t) = C\frac{du_C(t)}{dt} \tag{1.3}$$

and after rearranging terms, the final equation yields:

$$\frac{du_C(t)}{dt} + \frac{1}{RC}u_C(t) = \frac{u_g(t)}{RC} \tag{1.4}$$

**Differential equation of current**

Differentiating Eq. (1.2) gives:

$$\frac{du_C(t)}{dt} + R\frac{di(t)}{dt} = \frac{du_g(t)}{dt} \tag{1.5}$$

The defining equation of the capacitor (1.3) is considered again,

$$\frac{i(t)}{C} + R\frac{di(t)}{dt} = \frac{du_g(t)}{dt} \tag{1.6}$$

and rearranging terms:

$$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = \frac{1}{R}\frac{du_g(t)}{dt} \tag{1.7}$$

**Differential equation of the resistor voltage**

If Eq. (1.1) is differentiated, the following expression is obtained:

$$\frac{du_C(t)}{dt} + \frac{du_R(t)}{dt} = \frac{du_g(t)}{dt} \tag{1.8}$$

and the capacitor defining equation (1.3) is used, yielding:

$$\frac{i(t)}{C} + \frac{du_R(t)}{dt} = \frac{du_g(t)}{dt} \tag{1.9}$$

Finally, applying Ohm's law on the resistor and rearranging terms:

$$\frac{du_R(t)}{dt} + \frac{1}{RC}u_R(t) = \frac{du_g(t)}{dt} \tag{1.10}$$

It can be verified from the differential equations obtained for each variable, (1.4), (1.7) and (1.10), that all of them have the same coefficients and differ only in the independent term.

### 1.2.2 RL Circuits

First order circuits with one inductor or a group of them that can be combined into a single equivalent are now considered. The rest of the circuit is exclusively made up of electrical sources and resistors, without energy storage elements, so that it can be replaced by its Norton equivalent, which consists of a current source in parallel with a resistor, as shown in Fig. 1.7.

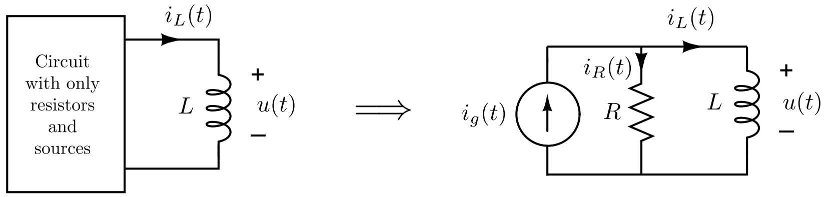



**Fig. 1.7** RL circuit and its Norton equivalent

The differential equations that defines the behavior of the variables involved in the parallel RL circuit represented in Fig. 1.7 will be subsequently obtained.

**Inductor current equation**

Applying Kirchhoff's current law:

$$i_L(t) + i_R(t) = i_g(t) \tag{1.11}$$

The resistor current is substituted using Ohm's law

$$i_L(t) + \frac{u(t)}{R} = i_g(t) \tag{1.12}$$

and considering the inductor defining equation

$$u(t) = L\frac{di_L(t)}{dt} \tag{1.13}$$

the final expression yields, after rearranging terms:

$$\frac{di_L(t)}{dt} + \frac{R}{L}i_L(t) = \frac{R}{L}i_g(t) \tag{1.14}$$

**Circuit voltage equation**

Differentiating Eq. (1.12) gives:

$$\frac{di_L(t)}{dt} + \frac{1}{R}\frac{du(t)}{dt} = \frac{di_g(t)}{dt} \tag{1.15}$$

Using the inductor defining equation (1.13)

$$\frac{u(t)}{L} + \frac{1}{R}\frac{du(t)}{dt} = \frac{di_g(t)}{dt} \tag{1.16}$$

and rearranging terms:

$$\frac{du(t)}{dt} + \frac{R}{L}u(t) = R\frac{di_g(t)}{dt} \tag{1.17}$$

**Equation of resistor current**

Equation (1.11) is differentiated

$$\frac{di_L(t)}{dt} + \frac{di_R(t)}{dt} = \frac{di_g(t)}{dt} \tag{1.18}$$

and the inductor defining equation (1.13) is used as follows:

$$\frac{u(t)}{L} + \frac{di_R(t)}{dt} = \frac{di_g(t)}{dt} \tag{1.19}$$

Finally, Ohm's law is applied on the resistor and the terms are rearranged, resulting:

$$\frac{di_R(t)}{dt} + \frac{R}{L}i_R(t) = \frac{di_g(t)}{dt} \tag{1.20}$$

It can be noticed that the coefficients of the previous differential equations, (1.14), (1.17) and (1.20), are the same in all cases, only differing in the independent terms. This fact allows the derivation of a generic equation for all the variables.

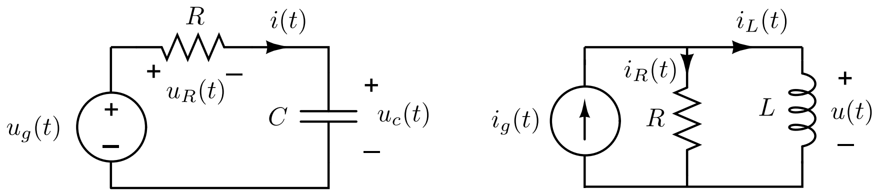
### *1.2.3 Generic Differential Equation of a First Order Circuit*

Figure 1.8 shows the differential equations of all the variables involved in the RC and RL circuits, as obtained in the previous sections.

According to Fig. 1.8, it is observed how all equations can be expressed in the following generic form:

$$\frac{df(t)}{dt} + \frac{1}{\tau}f(t) = g(t) \tag{1.21}$$

where $f(t)$ denotes the considered voltage or current, $g(t)$ is a function related to the excitation source of the circuit and $\tau$ is known as the circuit time constant, whose SI unit is the second, and depends on the parameter values for the passive elements of the circuit. This constant is characteristic of each circuit and its value is given by:




$$\frac{du_c(t)}{dt} + \frac{1}{RC}u_c(t) = \frac{u_g(t)}{RC} \qquad \frac{di_L(t)}{dt} + \frac{R}{L}i_L(t) = \frac{R}{L}i_g(t)$$

$$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = \frac{1}{R}\frac{du_g(t)}{dt} \qquad \frac{du(t)}{dt} + \frac{R}{L}u(t) = R\frac{di_g(t)}{dt}$$

$$\frac{du_R(t)}{dt} + \frac{1}{RC}u_R(t) = \frac{du_g(t)}{dt} \qquad \frac{di_R(t)}{dt} + \frac{R}{L}i_R(t) = \frac{di_g(t)}{dt}$$

**Fig. 1.8** Generic differential equations of RC and RL circuits

- RC circuit: $\tau = R \cdot C$
- RL circuit: $\tau = L/R$

It should be remarked that, if there are several capacitors in the first order circuit which can be grouped into one, the value of $C$ represents the equivalent capacity. Similarly, the value of $L$ represents the coefficient of self-induction of the equivalent inductor. Finally, $R$ is the equivalent resistance of the passive circuit seen from the $L$ or $C$ terminals. Therefore, the most general expressions for the constant $\tau$ are:

- RC circuit: $\tau = R_{eq} \cdot C_{eq}$
- RL circuit: $\tau = L_{eq}/R_{eq}$

## 1.3 Transient Response of First Order Circuits

As explained before, all the voltages and currents of the first order circuits are given by the following constant-coefficient linear differential equation:

$$\frac{df(t)}{dt} + \frac{1}{\tau}f(t) = g(t) \tag{1.22}$$

The solution of this equation is used to obtain the transient response of the different variables of the circuit under study. Mathematically, the general solution of this type of equations can be expressed as the sum of the solution of the homogeneous equation plus a particular solution of the complete equation. In electrical circuits, the first term is known as the natural response of the circuit, while the particular solution is known as the forced or steady-state response. Therefore, the solution of the differential equation (1.22) can be expressed as

$$f(t) = f_n(t) + f_p(t) \tag{1.23}$$

where $f_n(t)$ is the natural response of the circuit and $f_p(t)$ represents the forced or steady-state response. The derivation of each term is described below.

### 1.3.1 Natural Response

The natural response corresponds, mathematically, to the solution of the homogeneous differential equation, that is, the independent term equal to zero

$$\frac{df(t)}{dt} + \frac{1}{\tau} f(t) = 0 \tag{1.24}$$

whose solution for $t \geq 0$ is:

$$f_n(t) = K \cdot \mathrm{e}^{-t/\tau} \tag{1.25}$$

It should be remarked that, as in the homogeneous Eq. (1.24) the term $g(t)$ (related to the excitation source) does not appear, the solution of this equation corresponds to the circuit response if the excitation sources were canceled, hence it is called the natural response of the circuit.

The natural response of a first order circuit has an exponential nature, being the rate of decrease determined by the value of the time constant, $\tau$. It can be observed in Fig. 1.9 that, after a time period equal to the value of the time constant has elapsed, the natural response has decreased from its initial value, $K$, to $0{,}368K$, that is, it has been reduced a 63,2 %. Although mathematically the natural response never disappears, in practice it can be considered that, after a time period equal to $5\tau$, the natural response is negligible, since its value has been reduced to only $0{,}007K$.

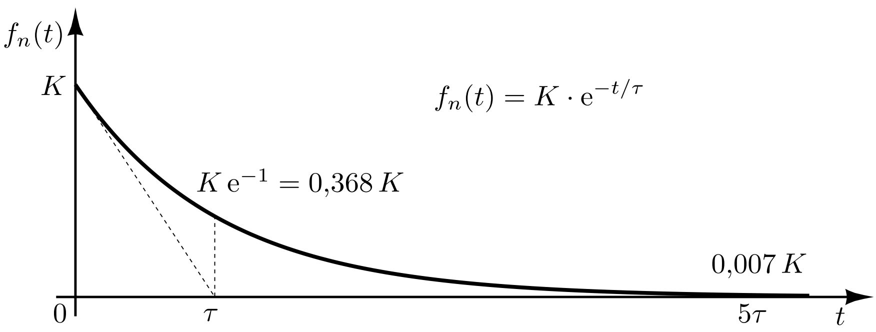



**Fig. 1.9** Evolution of the natural response

Finally, it can be noticed from Fig. 1.10 that a higher value of the time constant corresponds to a longer duration of the natural response, as expected.

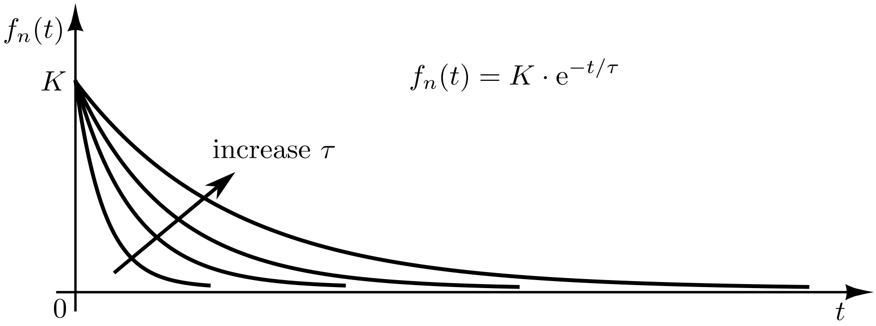



**Fig. 1.10** Influence of the time constant on the natural response

### *1.3.2 Forced or Steady State Response*

The forced response corresponds mathematically to a particular solution of the complete differential equation. The nature of this particular solution is usually the same as that of the independent term $g(t)$, that is, the same type of response as the excitation of the circuit. Since the natural response vanishes, the forced response is the only one remaining in time, hence in electrical circuits it is also known as the steady-state response.

Several methods can be used to obtain a particular solution of a constant-coefficient differential equation, such as parameter variation or undetermined coefficients, among others. However, in the particular case of electrical circuits under dc or ac supply, specific techniques have been developed to obtain the permanent regime. Therefore, the forced response will be obtained using these techniques.

### *1.3.3 Complete Response*

Once the natural and the steady-state responses are known, the complete response of the circuit under study will be:

$$f(t) = f_n(t) + f_p(t) = K \cdot e^{-t/\tau} + f_p(t) \tag{1.26}$$

When the steady-state response, $f_p(t)$, and the time constant of the circuit, $\tau$, have been obtained, the last step is the calculation of the constant $K$, in order to fully characterize the response of the considered variable.

The value of this constant is obtained from the initial value of the variable ($f(0^+)$). Thus, in $t = 0^+$ it must be verified:

$$f(0^+) = K + f_p(0^+) \tag{1.27}$$

from where it can be derived

$$K = f(0^+) - f_p(0^+) \tag{1.28}$$

yielding the final expression:

$$f(t) = f_p(t) + \left[f(0^+) - f_p(0^+)\right] \cdot \mathrm{e}^{-t/\tau} \tag{1.29}$$

This expression allows the calculation of the voltage or current of any element in a first order circuit, where:

- $f(t)$ is the considered voltage or current.
- $f_p(t)$ is the steady-state response of this variable.
- $f_p(0^+)$ is the value in $t = 0^+$ of the steady-state response.
- $\tau$ is the time constant of the circuit.
- $f(0^+)$ is the initial value of the mentioned voltage or current.

The calculation of $f(0^+)$ in first order circuits will be described in the next section.

### *1.3.4 Initial Conditions*

The transition from a steady-state regime to a different one is generally determined by a transient period involving a redistribution of the energy stored in inductors and capacitors. Additionally, a variation in the energy state of the electrical sources is also produced. Since an instantaneous energy redistribution is not possible, the following points must be verified in absence of impulse responses:

- The capacitor voltage cannot suffer discontinuities:

$$u_C(0^+) = u_C(0^-)$$

- The inductor current cannot suffer discontinuities:

$$i_L(0^+) = i_L(0^-)$$

With these premises, the initial value of any voltage or current, $f(0^+)$, can be calculated by solving a circuit where:

1. The excitation sources, $e_g(t)$ and $i_g(t)$, are replaced by two sources of constant value:

$$E_g = e_g(0^+) \;\; ; \;\; I_g = i_g(0^+)$$

2. In the case of an RC circuit, the capacitor is replaced by a constant voltage source, whose value is given by:

$$u_C(0^+) = u_C(0^-) = U_0$$

3. In RL circuits, the corresponding inductor is replaced by a constant current source, whose value is given by:

$$i_L(0^+) = i_L(0^-) = I_0$$

Figures 1.11 and 1.12 summarize the procedure for first order circuits with a capacitor and an inductor, respectively.

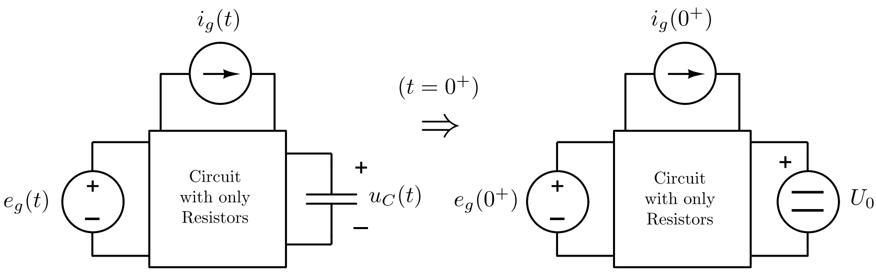



**Fig. 1.11** Derivation of the circuit at $t = 0^+$. Circuit with capacitor

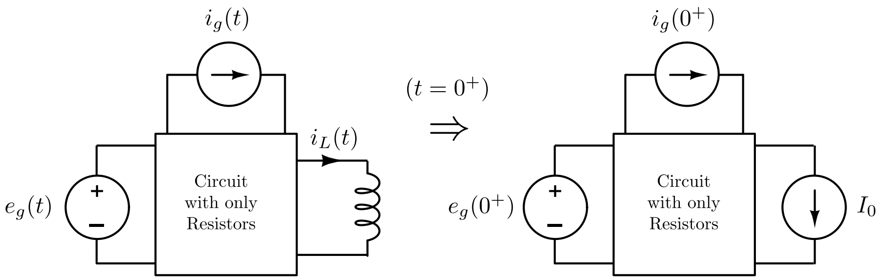



**Fig. 1.12** Derivation of the circuit at $t = 0^+$. Circuit with inductor

## 1.4 Generalization of the Transient Response

In the previous sections it was assumed that the transient period under analysis begins at instant $t = 0$. However, the obtained results can be generalized for any time $t = t_0$. This fact will be applied to the analysis of several concatenated transient periods. For time-invariant systems, if the transient period started at $t = t_0$, the complete response would be given by the following expression:

$$f(t) = f_p(t) + \left[f(t_0^+) - f_p(t_0^+)\right] \mathrm{e}^{-(t-t_0)/\tau} \tag{1.30}$$

As mentioned before, for the calculation of the initial conditions, and in the absence of impulse responses, it must be verified that:

- The voltage capacitor cannot suffer discontinuities:

$$u_C(t_0^+) = u_C(t_0^-) \tag{1.31}$$

- The current inductor cannot suffer discontinuities:

$$i_L(t_0^+) = i_L(t_0^-) \tag{1.32}$$

With these premises, the initial value of any voltage or current, $f(t_0^+)$, can be calculated by solving a circuit where:

1. The excitation sources, $e_g(t)$ and $i_g(t)$, are replaced by two sources of constant value:
$$E_g = e_g(t_0^+) \ \ ; \ \ I_g = i_g(t_0^+)$$
2. In the case of an RC circuit, the capacitor is replaced by a constant voltage source, whose value is given by:
$$u_C(t_0^+) = u_C(t_0^-) = U_{t_0}$$
3. In RL circuits, the corresponding inductor is replaced by a constant current source, whose value is given by:
$$i_L(t_0^+) = i_L(t_0^-) = I_{t_0}$$

Figures 1.13 and 1.14 summarize the described procedure to obtain the circuit at $t = t_0^+$, from which the initial value of any variable can be obtained at the considered instant.

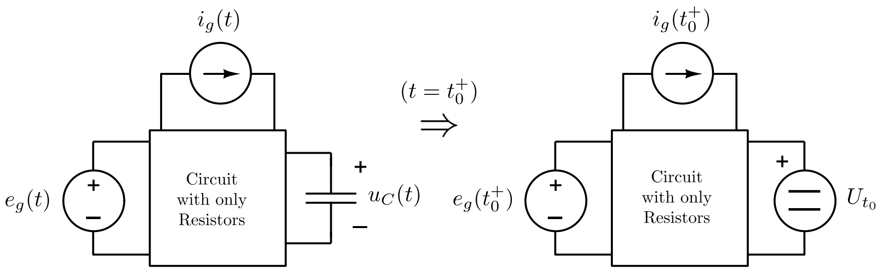



**Fig. 1.13** Derivation of the circuit at $t = t_0^+$. Circuit with capacitor