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and Philosophy of Science

Stefania Centrone

Pierluigi Minari

Oskar Becker,
On the Logic
of Modalities
(1930): Translation,
Commentary and
Analysis



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of Modalities (1930):
Translation, Commentary
and Analysis

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*This book is dedicated to Prof. Dr Volker
Peckhaus*

Preface

The history of twentieth-century modal logic is all too often presented as the American success story that started with the work of Clarence Irving Lewis, while prewar modal logic research in Europe is passed off as a side-show of well-intended failures. This book is intended as a first attempt to drastically correct such a picture. It is related to the research project *Modal Logic and Austro-Polish Philosophy* awarded to the first author by the German Research Association (DFG) in Summer 2017, which started in October 2018. The preparatory work has enormously profited from the first author's 1-year research (2017–2018) within the research program *Modalities and Conditionals: Systematic and Historical studies* at the University of Helsinki, whose principal investigator was Sara Negri.

The core of the project *Modal Logic and Austro-Polish Philosophy* consists in formally exploiting ideas, attempts, and solutions emerging from prewar modal logic with the aim to open up unexpected, radically new paths in various areas of contemporary research in modal logic.

The starting point of the research should have been, and indeed it has been, to make the key works in modal logic that appeared in prewar continental Europe accessible to a wider audience. Three works in particular come to the mind when one thinks of European research in modal logic prior to the war: Jan Łukasiewicz's *O logice trójwartościowej* (Łukasiewicz 1920), Ernst Mally's *Grundgesetze des Sollens* (Mally 1926), and Oskar Becker's *Zur Logik der Modalitäten* (Becker 1930).

The present book focuses on Oskar Becker's essay *On the Logic of Modalities*.

The decision to begin therewith was, in part, due to the fact that Becker seemed to have been deeply influenced by his teacher, Edmund Husserl, and an in-depth knowledge of Husserl's work appeared to be essential to correctly interpret certain choices of Becker about modalities. Such background had been elaborated in several volumes and articles that have appeared mostly in the Springer series *Synthese Library* in recent years, for instance, the monograph *Logic and Philosophy of Mathematics in the Early Husserl* (*Synthese Library* 2010) (Centrone 2010) or the volume *Essays on Husserl's Logic and Philosophy of Mathematics* (*Synthese Library* 2017) (Centrone 2017), which contains our joint work on Husserl and the

algebraists of logic, *Husserl and Boole* (Centrone and Minari 2017a) and *Husserl and Schröder* (Centrone and Minari 2017b). On the other side, two important volumes have paved the way for an adequate understanding of the philosophical significance of Oskar Becker's contribution to the history and philosophy of mathematics, namely *Die Philosophie und die Wissenschaften. Zum Werk Oskar Beckers* edited by Annemarie Gethmann-Siefert and Jürgen Mittelstraß (Gethmann-Siefert and Mittelstraß 2002) and *Oskar Becker und die Philosophie der Mathematik* edited by Volker Peckhaus (Peckhaus 2005). At a variance with the wide spectrum of topics dealt with in these volumes, our choice has been to focus exclusively on Oskar Becker's pioneering contributions to modal logic in 1930, with the aim to evaluate Becker's scientific accomplishments on this topic and possibly to make them, so to say, bear fruit.

We first decided to give an idea of the work that had to be done on this aspect of Oskar Becker's research in a booklet titled *Oskar Becker on Modalities*, which appeared in 2019 within the series *Philosophische Hefte* edited by Axel Gelfert and Thomas Gil for the *Logos Verlag* in Berlin (Centrone and Minari 2019). On this basis, we then turned to a deeper examination and a rigorous proof of all the claims made in the booklet, but not only that.

The present work is the result of two years of intense work on Oskar Becker's *On the Logic of Modalities*. In the first place, we try to give an idea of the reasons that led Becker to deal with modalities, first of all, the birth of intuitionistic logic, two years earlier, with the publication of Arend Heyting's seminal work *The Formal Rules of Intuitionistic Logic* (Heyting 1930). It may perhaps turn out to be surprising that Oskar Becker is the first philosopher ever to have put forward the idea of a translation of intuitionistic logic into modal logic, even if it is Kurt Gödel that actually realized it shortly later.

This book provides the reader with the basic modal logical tools necessary to read *On the Logic of Modalities* and gives a sketch of Lewis's Logic **S3**, which is the target of Becker's essay, in the modal logical symbolism that is current nowadays. Next, two extensions of **S3** proposed by Oskar Becker are presented and all objections raised by Kurt Gödel in his *Review of Becker 1930: On the Logic of Modalities* (Gödel 1931) are considered with the aim of correcting some historical misunderstandings related to them.

We dwell at length on the controversy between Oskar Becker and Ernst Cassirer with respect to the meaning and the legitimacy of the use of *ideal elements* in logic and mathematics. This results in an interesting multi-voice canon whose main actors are none other than Husserl, Weyl, Hilbert, and Brouwer.

We then provide the English translation of Becker's essay, accompanying the whole translation with footnotes that explain step by step, in current symbolism, and sometimes even in words, what is going on in the text, in order to allow even a reader not versed in logic to read the text easily.

The Appendix at the end of the book contains the detailed proofs of our assertions about Becker's logical accomplishments, conjectures, and mistakes as well. It is preceded by a systematic presentation of the Kripke-style semantics for **S3**—of the

completeness theorems and other related results—in a comprehensive way which, to our knowledge, is not yet to be found in the literature.

We are particularly indebted to our teacher, Ettore Casari, who aroused our interest in logic, mathematics, and in Husserl's and Bolzano's writings and made us realize that restriction to one single field of research can be more of a hindrance than a help for original work. Very special thanks go to Klaus Mainzer, emeritus of excellence at the Technical University of Munich, and Niklas Hebing, Head Office of the Section of Humanities and Social Science by the German Research Association (DFG), who both strongly supported this work. The encouragement at a decisive moment and the friendly advice we received from Otavio Bueno, editor in chief of Synthese Library, were truly invaluable.

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Chapter 1

Introduction



Oskar Becker was involved with Nazism—he was indeed, as is written in an official file of the SS Security Service,¹ “not a party member but a loyal to National Socialism, who tries to consolidate the National Socialistic ideology”.

We wish to make fully explicit that we completely distance ourselves from Becker’s political views, and that our book, which deals exclusively with Becker’s accomplishments in modal logic and its philosophy, neither endorses nor defends in any way Becker’s political and moral failings.

* * *

This book contains the first English translation of Oskar Becker’s essay *On the Logic of Modalities* (*Zur Logik der Modalitäten*)² that appeared in 1930 in the *Yearbook for Philosophy and Phenomenological Research*.³ Our commentary aims to present, to contextualize and to evaluate the pioneering contributions to modal logic contained in this work.

¹ Cp. Leaman and Simon (1992).

² Becker (1930). Henceforth cited using the page number(s) of the original pagination and, in brackets, of the English translation contained in the present volume (the original pagination is reproduced in the margin).

³ Becker (1930). The *Yearbook for Philosophy and Phenomenological Research* (*Jahrbuch für Philosophie und phänomenologische Forschung*) was founded by Edmund Husserl in 1912 and served the Husserl’s circle as an important organ during Husserl’s Freiburg period (1916–1938). The first issue of the journal was published in 1913 and contains Husserl’s *Ideas for a Pure Phenomenology and Phenomenological Philosophy* (Husserl 1913). Volume 8 includes Heidegger’s masterpiece *Being and Time* (Heidegger 1927) as well as Oskar Becker’s famous investigation on the logic and ontology of mathematical phenomena “*Mathematical Existence* (*Mathematische Existenz*)” (Becker 1927).

Oskar Becker (Leipzig 1889–Bonn 1964) was a German philosopher, logician, mathematician and historian of mathematics. He is often remembered, together with Martin Heidegger, for being one of the most prominent students of Edmund Husserl (1859–1938). He was, together with Moritz Geiger (1880–1937), Martin Heidegger (1889–1976), Alexander Pfänder (1870–1941), Adolf Reinach (1883–1917) and Max Scheler (1874–1928), one of the members of the editorial board of the *Yearbook*.

Oskar Becker got his PhD in mathematics in 1914 with a work⁴ entitled *On the Decomposition of Polygons in non-intersecting triangles on the Basis of the Axioms of Connection and Order (Über die Zerlegung eines Polygons in exclusive Dreiecke auf Grund der ebenen Axiome der Verknüpfung und Anordnung)*. In 1922 he wrote under Husserl’s supervision his *Habilitationsschrift, On Investigations of the Phenomenological Foundation of Geometry and their physical Application (Beiträge zur phänomenologischen Begründung der Geometrie und ihrer physikalischen Anwendungen)*.⁵ In 1927 Oskar Becker published in the *Yearbook* his masterpiece *Mathematical Existence*,⁶ where he uses the Husserlian phenomenology to clarify the process of counting. In 1952—when the study of modal logic was already well beyond its pioneering era—Becker came back to the subject publishing a monograph, *Investigations on the Modal Calculus (Untersuchungen über den Modalkalkül)*, perhaps too old-fashioned for the time.⁷

* * *

The essay *On the Logic of Modalities* represents an attempt to treat modal logical issues with a phenomenological method. This enterprise appeared from the outset not to be easy at all, for logic and phenomenology are completely different disciplines. Depending on the way in which it constructs its formal systems, formal logic can be seen as *the theory of the correct inferences*, or alternatively, as the *theory of purely formal truths*, that is, as the theory of those truths that hold without any condition. Phenomenology, instead, deals with the description of lived experiences.

Indeed, we might better say that in his investigations Becker pursued two loosely related goals. The first, more technical in character, was to find axiomatic conditions that reduced to the finite the number of logically non-equivalent combinations arising from the iterated application of the operators “not” and “it is impossible that (...)” in Lewis’s modal system, as we will explain in details below. The second,

⁴ Becker (1914). For a complete bibliography of Becker’s works see Zimny (1969).

⁵ Becker (1923). For a precise placement of Oskar Becker in the political and philosophical panorama of his time see Wolters (2002).

⁶ Becker (1927). Hereto see at least: Gethmann (2003), Peckhaus (2005), Gethmann-Siefert and Mittelstraß (2002), and Mancosu (1998). A very important contribution for a correct understanding of Oskar Becker’s position in the history of modal logic is, in our opinion (Peckhaus 2002).

⁷ Becker (1952). It is worth noticing that Becker proposes here a Leibnizian semantics for the modal operators, which he calls the “statistical interpretation of the modal calculus”, cp. Martin (1969) and, in particular, Peckhaus (2002), 174 and Copeland (2002), 107–108.

more philosophically oriented and in a sense much more ambitious, was to treat the logic of modalities from a phenomenological perspective and to understand, from this perspective, the philosophical and logical-ontological problems underlying, and posed by, Intuitionism.

On the Logic of Modalities consists of two parts, loosely related as the above mentioned corresponding goals are. Part I opens with a general Introduction that shortly reviews the Aristotelian conception of modalities as well as Hugh MacColl's pioneering modal logical investigations in his *Symbolic Logic and its Applications*⁸ of 1906. It then focuses on C. I. Lewis's *Survey of Symbolic Logic*⁹ of 1918. The latter work contains the first presentation of the so-called "Survey system", known since 1932 as "modal system **S3**."¹⁰

De facto, **S3** is the actual object of the investigations contained in Part I of Becker's essay. As pointed out by Emil L. Post, the system Lewis presents in 1918 *collapses* into classical logic. Lewis corrects it in a paper entitled *Strict Implication: An Emendation*¹¹ and published in 1920, where the system effectively becomes the logic we nowadays know as "**S3**."¹² In his essay Becker faithfully reports both that the original version of the "Survey system" proves the *collapse* of modalities, as well as Lewis's amendment thereof. Incidentally, "*collapse of modalities*" is a customary expression in the modal-logical jargon. It means that a modal logical system proves that *necessity* and *truth* are one and the same, or equivalently (as it is the case in the "Survey system") that *impossibility* and *falsity* are one and the same. Obviously, such a system is *trivial* from a modal point of view.

Becker's Introduction touches on the paradoxes of *material* and *strict implication* and sets out to establish a propositional modal logic that is *decidable* as the classical propositional logic is:¹³

The aim of the present essay is now closely related to both MacColl's and Lewis's investigations. The ultimate goal of our investigations is to develop an elementary logical calculus that takes adequately into account the *modalities* of the statement, namely in such a way that *the so-called elementary decision problem is solvable*, as in the ordinary propositional calculus.

Part I, *On the Rank Order and the Reduction of Logical Modalities*, is specifically devoted to the problems of ranking and iteration of modalities. Becker sets out to modify **S3** by means of some additional axioms effecting the reduction of complex modalities to simple ones in order to obtain two new modal systems—he calls them "*the six modalities calculus*" (henceforth denoted here by **S3'**) and "*the ten modalities calculus*" (henceforth **S3''**) with the following properties:

⁸ MacColl (1906).

⁹ Lewis (1918).

¹⁰ The name appears for the first time in Appendix II of Lewis and Langford (1932).

¹¹ Lewis (1920).

¹² Cp. Cresswell et al. (2016), 281 f.

¹³ Becker (1930), 4 (58).

- (i) *the number of irreducible modalities is finite,*
- (ii) *the positive (and by consequence the negative) modalities are arranged in a linear order with respect to logical strength.*

He believes that, since the “System of Strict Implication” has the conjunction, the negation and the impossibility as primitive logical constants, it is possible to generate within it infinitely many non equivalent nested modalities through the iteration of the negation and the impossibility operators. Such modalities, as Kurt Gödel (1906–1978) puts it in his *Review of Becker 1930*, “cannot even be linearly ordered according to their logical strength in the sense that, of any two affirming modalities, one will imply the other, and similarly for negating ones.”¹⁴ Otherwise said, there are modalities that are incomparable in Lewis’s system.

That said, it is worth to be mentioned that Oskar Becker neither shows that the two systems he sets up (and others he tentatively introduces, as we will see below) really differ from one another, nor that his additional axioms cannot be derived from those of Lewis, nor that in his own systems, with *six* and, respectively, with *ten* “irreducible” modalities, such modalities cannot be further reduced.¹⁵

Actually, nine years later, W. T. Parry will show, in a paper entitled *Modalities in the Survey System of Strict Implication*,¹⁶ that, at a variance with what Becker seems to believe, **S3** has a finite number of modalities. More precisely, Parry shows, with the help of a number of suitable theses he is able to derive in the system, that it is possible to reduce all the complex modalities in **S3** to a finite number of irreducible modalities, viz. 42. He also shows that no further reduction is possible.

Part **II** of Becker’s essay explores, more or less independently from Part **I**, the connection between modal and intuitionistic logic both from a formal and from a phenomenological perspective.

From a formal perspective, the particular interest of a (propositional) modal calculus with nested modalities that is *decidable* lies in the fact, so Becker, that Brouwer’s idea to set up a finite logic grounded on *evidence*, or—to put it with Husserl—on *evidence levels* seems to be realizable only within the framework of a modal formal system.

Indeed, Becker is the first logician and philosopher of mathematics to put forward the idea of a *modal interpretation* of intuitionistic logic,¹⁷ more precisely the idea of a possible sound and faithful translation of intuitionistic logic into modal logic. However, the first actual translation is to be found in a one-page celebrated and influential paper entitled *An interpretation of the intuitionistic propositional*

¹⁴ Gödel (1931).

¹⁵ Hereto cp. Gödel (1931).

¹⁶ Parry (1939).

¹⁷ A modal translation of intuitionistic logic was in a sense foreshadowed also by Ivan Orlov in Orlov (1928). The paper, written in Russian, remained however unknown outside the Soviet Union for a very long time. It is only since the 1990’s that Orlov’s pioneering contributions were “rediscovered” and came to be known and discussed, see Chagrov and Zakharyashchev (1992) and Došen (1992).

calculus written in 1933 by Kurt Gödel.¹⁸ The basic idea of Gödel is similar to the one Oskar Becker outlines in *On the Logic of Modalities*.

Becker suggests to add to classical logic the predicates “(...) is provable”, “(...) is such, that its negation is provable” and “(...) is undecided”. Such predicates should express Brouwer’s primitive logical concepts.

Similarly, Gödel’s idea is to extend the language of classical propositional logic with the unary operator “it is provable that (...)”, denoted by “**B**”, and to add to an axiomatic calculus for propositional classical logic *three* axiom-schemas and *one* rule of inference. The axiom-schemas are the modal schemas *K*, *T* and *4* that characterize modal logics that are nowadays standard, the rule of inference is the necessitation rule that is contained in all *normal* modal systems. We will introduce both the schemas and the rule of inference in detail later on.

Notice, incidentally, that both Becker and Gödel seem to take the predicate “(...) is provable” and the operator “it is provable that (...)” as conveying the same piece of information. Actually, the predicate “(...) is provable” denotes the property of a proposition to be provable, while the operator “it is provable that (...)” takes a proposition as input and gives a different proposition as output. (Unfortunately, such practice of systematically neglecting the difference between predicate and operator is, even nowadays, quite widespread among logicians.)

Gödel writes:¹⁹

One can interpret Heyting’s propositional calculus by means of the notions of the ordinary propositional calculus and the notion “*p* is provable” (written “**B***p*”), if one adopts for that notion the following system \mathfrak{S} of axioms:

1. $Bp \rightarrow p$
if it is provable that *p*, then it is true that *p*
2. $Bp \rightarrow ((B(p \rightarrow q) \rightarrow Bq)$
if it is provable that *p* and it is provable that *p* implies *q*, then it is provable that *q*
3. $Bp \rightarrow BBp$
if it is provable that *p*, then it is provable that it is provable that *p*

In addition, [...] the new rule of inference is to be added

$$\frac{A}{BA}$$

From *A*, it is provable that *A* may be inferred

By substituting throughout the operator “**B**” (“it is provable that (...)”) by the operator “ \square ” (“it is necessary that (...)”) one obtains one of the modal logical systems that are nowadays standard, namely Lewis’s system **S4**.

* * *

Becker’s more logically oriented investigations, mainly contained in Part **I** of *On the Logic of Modalities*, will be carefully analyzed in Chap. 2, while his more

¹⁸ Gödel (1933a).

¹⁹ Gödel (1933a), 301.

philosophically oriented discussions, to be found in Part **II**, will be examined at length in Chap. 3. Chapter 4 contains the English translation of *On the Logic of Modalities*, together with an apparatus of explanatory footnotes. The final Chap. 5 (Appendix) provides a self-contained and comprehensive survey of the most relevant “technical” issues related to Lewis’s system **S3** and other non normal modal systems in the neighbourhood.