Critical Issues in Mathematics Education

Philip Clarkson · Norma Presmeg
Editors

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Major Contributions of Alan Bishop

Springer

Editors
Philip Clarkson
Faculty of Education
Australian Catholic University
Fitzroy VIC 3065
Australia
p.clarkson@patrick.acu.edu.au

Norma Presmeg
Illinois State University
Department of Mathematics
313 Stevenson Hall
Normal IL 61790-4520
USA
npresmeg@msn.com

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Contributors

Bill Barton
Dept. of Mathematics, The University of Auckland, Private Bag 92019, Auckland Mail Centre, Auckland 1142, New Zealand, b.barton@auckland.ac.nz

Alan J. Bishop
Faculty of Education, Monash University, Wellington Road, Clayton, Victoria 3168, Australia, alan.biship@education.monash.edu.au

Hilda Borko
School of Education, Stanford University, 485 Lasuen Mall, Stanford, CA 94305-3096, USA, hildab@suse.stanford.edu

Philip C. Clarkson
Faculty of Education, Australian Catholic University, Fitzroy VIC 3065, Australia, p.clarkson@patrick.acu.edu.au

M. A. (Ken) Clements
Department of Mathematics, Illinois State University, Normal, IL 61790-4520, United States of America, clements@ilstu.edu

Christine Keitel
Fachbereich Erziehungswissenschaft und Psychologie, Freie Universität Berlin, Habelschwerdter Allee 45, 14195 Berlin, Germany, keitel@zedat.fu-berlin.de

Jeremy Kilpatrick
105 Aderhold Hall, University of Georgia, Athens, GA 30602-7124, United States of America, jkilpat@uga.edu

Frederick Leung
Chair Faculty of Education, The University of Hong Kong, Pokfulam Road, Hong Kong, hraslks@hkucc.hku.hk

Norma Presmeg
Illinois State University, Department of Mathematics, 313 Stevenson Hall, Normal IL 61790-4520, USA, npresmeg@msn.com

Sarah A. Roberts
School of Education, 249 UCB University of Colorado, Boulder, CO 80309-0249, USA, Sarah.A.Roberts@Colorado.edu

Kenneth Ruthven
Faculty of Education, University of Cambridge, 184 Hills Road, Cambridge CB2 8PQ, United Kingdom, kr18@hermes.cam.ac.uk

Wee Tiong Seah
Faculty of Education, Monash University (Peninsula Campus), PO Box 527, Frankston, Vic. 3199, Australia, WeeTiong.Seah@Education.monash.edu.au

Richard Shavelson
School of Education, 485 Lasuen Mall, Stanford University, CA 94305-3096, USA, richs@stanford.edu

Renuka Vithal
Dean of Education, Faculty of Education, University of KwaZulu-Natal, Edgewood Campus, Private Bag X03, Ashwood 3605, South Africa, vithalr@ukzn.ac.za

Section I
Introduction

Chapter 1
Developing a Festschrift with a Difference

Philip Clarkson and Norma Presmeg

A Festschrift is normally understood to be a volume prepared to honour a respected academic, reflecting on his or her significant additions to the field of knowledge to which they have devoted their energies. It is normal for such a volume to be composed of contributions from those who have worked closely with the academic, including doctoral students, and others whose work is also known to have made important contributions within the same areas of research.

It was the dearth of volumes of this type in the area of mathematics education research that Philip Clarkson and Michel Lokhorst, then a commissioning editor with Kluwer Academic Publishers, started to discuss some 5 years ago. This discussion point was embedded in a broader conversation that lamented the fact that little was published that kept a trace of how ideas developed over time in education, and in mathematics education in particular. Associated with this notion was how we as a community were not very good at linking the development of ideas with the people who had worked on them, and the individual contexts within which their thinking occurred. We wondered whether something should be done to draw attention to this issue. One way to do that was to begin the task of composing a Festschrift, but with a difference.

In thinking through the implications of this proposition, it seemed useful to structure the volume in such a way that perhaps more could be achieved than by just initiating a call for contributions to honour a colleague who had made a long and important contribution to mathematics education. We wondered whether a structure could be developed for the proposed volume that emphasised the following:

- the ideas of the honoured academic that she or he had developed,
- where and how they were developed, and
- what became of those ideas once they were published and taken up, or not taken up, by the community of scholars that were working in that particular area, in this case mathematics education.

P. Clarkson
Faculty of Education, Australian Catholic University, Fitzroy VIC 3065, Australia
e-mail: p.clarkson@patrick.acu.edu.au

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We decided that indeed such a project should be initiated. It was relatively easy to decide to focus on Alan Bishop's contributions to mathematics education over the last 40 years, which are still continuing. This, then is the goal of this volume.

The purpose of this volume is twofold, each part of equal weight, although the second component has given the impetus and structure for the volume. The first is to put into perspective the contribution that (now Emeritus) Professor Alan Bishop has made to mathematics education research beginning in the 1960s. The other is to review six critical issues that have been important in the establishment of mathematics education research over the last 50 years, including updating to some extent current developments in each of these areas. The volume was planned to make a valuable contribution to the ongoing reflection of mathematic education researchers world wide, but also to address topics relevant to policy makers and teacher educators who wish to understand some of the key issues with which mathematics education has been and still is concerned. However all ideas develop within an historical context. Hence in various places within this volume comment is made with regard to the contexts within which Bishop's contributions to these research issues were made.

Bishop's contributions can be conveniently outlined through a consideration of the following six issues as they relate to mathematics education research:

- Teacher decision-making
- Spatial abilities, visualization and geometry
- Cultural and social aspects of mathematics education
- Socio political issues for mathematics education
- Teachers and research
- Values and teaching mathematics.

The structure of the volume has been developed around these six issues, each issue being the focus of a section of the volume. Each section has three or four components. The first component of each section is a brief introduction that positions and gives a context for the Bishop article reprinted in the section.

The second component of each section is a reprint of a particular "key" journal article or book chapter that Bishop published. Each key article has been chosen to typify his contribution to the ongoing research on that issue. These articles were selected in conversation with Bishop.

The final component of each section consists of one or two invited chapters from selected authors. We chose authors who had either worked directly with Bishop, or had worked with the ideas canvassed in their section.

Authors were asked to use the Bishop key article for their section as a focus for a commentary on that issue in mathematics education. We anticipated that the authors would use the key articles in different ways: perhaps as a starting point to develop a dialog with the article in some way, or to take the key article and map out how the ideas have or have not been taken up in succeeding years, or to look back to what preceded the publication of the article and place it in an historical context, or to start in a completely different place and come back to the notions discussed in the key

article. The aim was for the ideas embedded in the key Bishop article to be central in the formation of each contributed chapter. We hoped that a number of approaches would be used which would give the volume a feel of variety and surprise, bound together by the brief introductory components of each section. We believe this has been achieved.

When colleagues who have worked directly with Bishop in some way, or have worked with his ideas, are asked to contribute to such a volume as this, there is a danger of the volume becoming just a set of personal reflections about him. At times documenting publicly the appreciation of and esteem in which we hold colleagues is most appropriate, and perhaps not done often enough. But more than this was envisaged for this volume. We were also aiming for a scholarly contribution to the literature. We thought that this was the best way we could honour Bishop's legacy. Hence we wanted to do both; record a little of the community's personal appreciation of Bishop's contributions over many years, but also try to make some scholarly advances in our thinking.

We had originally envisaged having two separate authors contributing to each section, at first working independently and then commenting on each other's chapters. We thought that in this way we would have to some extent a divergent yet focussed commentary on each issue, and indeed Bishop's contributions to mathematics education research. However, as can be seen, this did not always prove feasible. At times we took up the suggestion from particular colleagues that they develop a joint chapter. We would also note with appreciation that although our friend and colleague (and one of Bishop's doctoral students) Chien Chin had agreed to contribute a chapter for the last section, illness in the end prevented him from doing so.

We also suggested to the authors that inclusion in their chapter of pertinent anecdotal and/or biographical comments on Alan and his contribution to mathematics education research would not be out of place. This has been done in different ways by different authors, and enlarges the understanding of the contexts in which Bishop worked through his own ideas. As noted in the introduction component of the section dealing with teacher decision-making, Bishop firmly believed that research in education is not a disembodied objective process. Rather the researcher is intimately contained within the research process in various ways, whether those ways are immediately clear to the researcher and others involved in a particular project or not. Hence knowing more about Bishop allows us to know more and understand in different ways his contributions to the research of mathematics education.

Just as it is important to know something of the contexts within which Bishop worked while contributing to these different issues in the ongoing research of mathematics education, it is also useful to know something of the authors who were kind enough to contribute to this volume. The following paragraphs give brief introductions to each author who has contributed to the volume either individually or as part of a team. We also need to acknowledge the help, support and guidance of our editors at Springer, Marie Sheldon and Kristina Wiggins-Coppola, who have worked with us from a very early stage in the process of publishing this volume, and without whose support and insights we would not have made it through the publishing process.

Contributing Authors

Bill Barton

Bill Barton is Head of Mathematics at The University of Auckland, having come to university after a secondary teaching career including bilingual Maori/English mathematics teaching. His research areas include ethnomathematics and mathematics and language. Bill has known Alan since the early 1990s, and regards him as being one of the key influences on his mathematics education research.

Hilda Borko

Hilda Borko is Professor of Education, School of Education, Stanford University. Dr. Borko's research examines the process of learning to teach, with an emphasis on changes in novice and experienced teachers' knowledge and beliefs about teaching and learning, and their classroom practices as they participate in teacher education and professional development programs. Currently, her research team is studying the impact of a professional development program for middle school mathematics teachers which they designed, on teachers' professional community and their knowledge, beliefs, and instructional practices. Many of Alan Bishop's ideas about teacher decision making and the use of video as a tool for teacher learning are reflected in that work.

Philip Clarkson

Philip is Professor of Education at Australian Catholic University where he has taught since 1985. This followed nearly 5 years as Director of a Mathematics Education Research Centre at the Papua New Guinea University of Technology, and prior to that as a lecturer at Monash University and tertiary colleges in Melbourne. He began his professional life as a teacher of mathematics, chemistry, environmental science and physical education in secondary schools. At present he is the Deputy Director of the Mathematics and Literacy Education Research Flagship at Australian Catholic University, the State Coordinator of graduate research programs, teaching general education and mathematics education units in these programs, and tutoring in first year mathematics. He has served as President, Secretary and Vice President (Publications) of the Mathematics Education Research Group of Australasia (MERGA), and was the foundation editor of the association's research journal *Mathematics Education Research Journal*. Major funded research projects in the last 10 years have been: "A longitudinal evaluation of the teacher education programs in Papua New Guinea"; "An evaluation of the computer Navigator Schools Project"; "The impact of language on mathematics learning, particularly for bilingual students"; and "Globalisation and the professional development of mathematics

educators". Philip met Alan in 1977, and from then on our paths have regularly crossed. They have had a mutual interest in education in Papua New Guinea, a particular context for discussions of language and cultural issues. They have also worked together in various ways, particularly on the project "Values and Mathematics" and in the running of the 1995 ICME Regional Conference.

M. A. (Ken) Clements

Since 2005, M. A. (Ken) Clements has been a Professor within the Mathematics Department at Illinois State University. He was in charge of mathematics education at Monash University between 1974 and 1982, and subsequently held positions in mathematics education at Deakin University (1987–1993), the University of Newcastle (NSW) (1993–1997), and Universiti Brunei Darussalam (1997–2004). Ken has authored and/or edited many articles, chapters, and books on mathematics education. Ken first met Alan Bishop when Alan came to Monash University as a visiting Research Fellow during the second half of 1977. He has subsequently worked with Alan on many projects, including as co-editor of two international handbooks on mathematics education.

Christine Keitel

Christine Keitel is Professor for Mathematics Education at Free University of Berlin. At present she is serving a second term as Vice-President (Deputy Vice Chancellor) of the university, responsible for restructuring of teaching and research. Her major research areas are comparative studies on the history and current state of mathematics education in various European and Non-European countries, on social practices of mathematics, on values of teachers and students, on "mathematics for all" and "mathematical literacy", on equity and social justice, on learners' perspectives on classroom practice, and on internationalization and globalization of mathematics education.

She was a member of the International Group BACOMET (Basic Component of Mathematics Education for Teachers) 1985–2005 and its director 1989–1993, director of the NATO- Research Workshop on "Mathematics Education and Technology" 1993–1994, a member of the Steering Committee of the OECD-project "Future Perspectives of Science, Mathematics and Technology Education" (1989–1995), Expert Consultant for the Middle-School Reform Project in PR China in 1990, for the Indonesian Ministry of Education in 1992, and for the TIMSS-Video-Project and Curriculum-Analysis-Project (1993–1995). She is member of editorial boards of several journals for curriculum and mathematics education and on the Advisory Board of Kluwer's Mathematics Education Library. Together with David Clarke and Yoshinori Shimizu she started the international LPS-project "Mathematics Classroom Practice: The Learners' Perspective" in

1999, which represents a collaboration of academics of 15 countries around the world (www.edfac.unimelb.edu.au/DSME/lps/). She is leader of the German team of LPS.

She was a founding member, National Coordinator, and Convenor/President of IOWME (International Organisation of Women and Mathematics Education) 1988–1996; Vice-president, Newsletter Editor and President of CIEAEM (Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques) 1992–2004; and member of the International committee of PME (International Group for Psychology and Mathematics Education) 1988–1992. As a guest professor she has lectured and researched at research institutions and universities around the world, in particular in Southern Europe, USA, Australia and South Africa. In 1999 she received an Honorary Doctorate of the University of Southampton, UK and the Alexander-von-Humboldt/South-African-Scholarship Award for undertaking capacity building in research in South Africa.

Jeremy Kilpatrick

Jeremy Kilpatrick is Regents Professor of Mathematics Education at the University of Georgia. He holds an honorary doctorate from the University of Gothenburg, is a National Associate of the National Academy of Sciences, received a 2003 Lifetime Achievement Award from the National Council of Teachers of Mathematics (NCTM), and received the 2007 Felix Klein medal from the International Commission on Mathematical Instruction. His research interests include teachers' proficiency in teaching mathematics, mathematics curriculum change and its history, assessment, and the history of research in mathematics education. He and his family have known Alan Bishop and his family for more than a third of a century, and when their boys were young, each family spent some months in or near the other family's hometown. A treasured memory is of the four boys and four parents walking the South Downs near Eastbourne during the summer of 1976.

Frederick Koon-Shing Leung

Born and raised in Hong Kong, Frederick Leung is Professor of Mathematics Education in the Faculty of Education, at the University of Hong Kong. Frederick obtained his B.Sc., Cert.Ed. and M.Ed. from the University of Hong Kong, and subsequently his Ph.D. from the University of London Institute of Education. Alan Bishop was an external examiner for his Ph.D. thesis. Frederick's major research interests are in the comparison of mathematics education in different countries, and in the influence of different cultures on teaching and learning. He is the principal investigator of a number of major research projects, including the Hong Kong component of the Trends in International Mathematics and Science Study (TIMSS), the TIMSS 1999 Video Study, and the Learner's Perspective Study (LPS).

Norma Presmeg

Jeremy Kilpatrick introduced me to Alan Bishop at a conference on the recently translated publications of Krutetskii, in Athens, Georgia, USA, in 1980. At the time I was completing a Master of Education thesis on Albert Einstein's creativity, in the Department of Educational Psychology at the University of Natal in South Africa. The heart and soul of Einstein's creative thought, by his own admission, lay in his proclivity for visualization. I had been teaching high school mathematics for 12 years, and noticed that there were students of high spatial abilities who were not succeeding in mathematics in their final year of school. All three of the boys singled out wished to pursue careers that involved visualization, namely, architecture, structural engineering, and technical drawing. The current state of their mathematics achievements would not permit these aspirations to be realized. A research goal was born, namely, *To understand more about the circumstances that affect the visual pupil's operating in his or her preferred mode, and how the teacher facilitates this, or otherwise*. Alan Bishop encouraged me to undertake this research on the strengths and pitfalls of visualization in the teaching and learning of mathematics. My 3 years at Cambridge University (1982–1985) pursuing doctoral research under the able and caring supervision of Alan Bishop remain a highlight of my life. The results of this research on visualization in mathematics education were exciting and fascinating. But the association with Alan opened up another significant field. In 1985, Alan was working on the first three chapters of his book, *Mathematical enculturation*, and it was my privilege to serve as a sounding board for his ideas while I waited to defend my dissertation during those summer months. When I returned to South Africa and worked at the University of Durban-Westville for five years (before immigrating to the USA in 1990), the role of culture in mathematics education became a central topic of my concern. Alan Bishop's influence in my professional career has been a significant one. After 10 years at The Florida State University, I moved to Illinois, where I am currently a Professor in the Mathematics Department at Illinois State University.

Sarah Roberts

Sarah Roberts is a doctoral candidate in mathematics curriculum and instruction at the University of Colorado at Boulder. Her research interests include pre-service and inservice teacher education, equity in mathematics, and issues related to English language learners.

Kenneth Ruthven

After working as a secondary-school teacher in Scotland and England, and completing doctoral research at Stirling University, Kenneth Ruthven was appointed

to Cambridge University where he worked closely with Alan Bishop for nearly 10 years. It was during this period that Ken joined the Editorial Board of Educational Studies in Mathematics of which Alan was then Editor-in-Chief; some years later Ken was to take on that senior role; and both currently continue to serve as Advisory Editors. Now Professor of Education at Cambridge, Ken's research focuses on issues of curriculum and pedagogy, especially in mathematics, and particularly in the light of social and technological change and of deepening conceptualisation of educational processes. Recent projects have examined technology integration in secondary subject teaching; future commitments include a major project on principled improvement in STEM education.

Wee Tiong Seah

Wee Tiong Seah is a Lecturer in the Faculty of Education, Monash University, Australia. Amongst his several research interests, he is particularly passionate about researching and facilitating effective (mathematics) teaching/learning through promoting teacher/student's socio-cognitive growth (e.g. values) and through harnessing their intercultural competencies. Wee Tiong completed his doctoral research study in 2004 under the supervision of Alan Bishop and Barbara Clarke. If Alan's migration to Australia from Britain in the early 1990s had been a motivation for Wee Tiong to migrate from Singapore in the late 1990s, then he has also been instrumental in socialising Wee Tiong to the mathematics education research community. Over the years, Alan has also become colleague, mentor and friend to Wee Tiong.

Richard J. Shavelson

Richard J. Shavelson is the Margaret Jacks Professor of Education, Professor of Psychology (courtesy), and former I. James Quillen Dean of the School of Education at Stanford University and Senior Fellow in the Woods Institute for the Environment at Stanford. He served as president of the American Educational Research Association; is a fellow of the American Association for the Advancement of Science, the American Psychological Association, the American Psychological Society, and a Humboldt Fellow. His early research focused on teachers' decision making at the same time Alan Bishop was working in this area. His current work includes the assessment of science achievement and the study of inquiry-based science teaching and its impact on students' knowledge structures and performance. Other current work includes: studies of the causal impact of computer cognitive training on working memory, fluid intelligence and science achievement; assessment of undergraduates' learning with the Collegiate Learning Assessment; accountability in higher education; the scientific basis of education research; and new standards for measuring students' science achievement in the National Assessment of Educational Progress (the nation's "report card"). His publications include Statistical

Reasoning for the Behavioral Sciences, Generalizability Theory: A Primer (with Noreen Webb), and Scientific Research in Education (edited with Lisa Towne). He is currently working on a book titled, The Quest to Assess Learning and Hold Higher Education Accountable.

Renuka Vithal

Renuka Vithal is Professor of Mathematics Education and Dean of the Faculty of Education at the University of KwaZulu-Natal, South Africa. She has taught at all levels from preservice to inservice mathematics teacher education programs, in postgraduate studies in mathematics education, and in educational research. Her research interests are in the social, cultural and political dimensions of mathematics education theory and practice.

Chapter 2
In Conversation with Alan Bishop

Philip Clarkson

> Doing a graduate psychology course with Jerome Bruner switched me on. I thought to myself, we should be doing more of this stuff (research) in education, and in mathematics education. Gee! You know! Why are just psychologists doing this stuff? Soooo I took on various tutoring jobs just to check out some things. I tutored at a mental hospital. I taught and then tutored in schools in a black part of Boston in a program that Harvard ran with gifted black kids. I also taught in 'normal' classes in middle years. This really got me interested in research on teachers in the classroom.
>
> (Bishop reflecting on his time in Boston in the mid 1960s)

Alan was born in 1937, just before the Second World War commenced. His father was a mathematics teacher, who progressed to be a foundation principal of a new Grammar School in London. Alan's mother was a seamstress, who – not unusual for that time – concentrated on making a home for her husband and only child. One of the great joys of the family was music. His father played the violin for public performance in a trio, and his mother played the cello. Both gave Alan much active encouragement to develop his own musicality.

Alan sat for his 11 plus examination and scored enough to go to the University College School in London, a public school linked, originally, to London University. At school he chose to take a lot of mathematics and science, a lot of music and sport, all of which he has continued with throughout his life. Towards the end of secondary school, Alan successfully auditioned and subsequently played the bassoon for 2 years in the National Youth Orchestra. Clearly he had a wonderful, although for a young man, a difficult decision to make in those final years of schooling: would he concentrate on his music or mathematics? Taking the advice of a visiting musician from Holland, "Do you really want to enjoy your music? Then stay an amateur",

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This chapter is mainly based on a number of conservations I had with Alan Bishop during April and May of 2008. But my conversation with Alan started with a brief question to him at a seminar he gave at Monash University in 1977. It continues through to today, in many and various locations including on golf courses, although those times should happen more regularly. Clearly the assertions and interpretations in this chapter are mine, although the dates and events have been checked with Alan.

P. Clarkson
Faculty of Education, Australian Catholic University, Fitzroy VIC 3065, Australia
e-mail: p.clarkson@patrick.acu.edu.au

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Alan choose to continue his studies in mathematics, with music and of course sport as his second level studies.

At the conclusion of his secondary education in 1956, Alan chose to complete 2 years of national service. He entered the air force and spent most of that time as an air-radar fitter, which essentially meant trouble shooting the huge analogue computers then in use for navigation. This was Alan's first introduction to computers, and since this was 20 years or more before computer technology became widely available in society, he was considerably ahead of the game. On completing national service he presented himself for an interview at Southampton University, a normal part of the selection process. During the 30 minute interview, the Professor of Physics was far more interested in learning what Alan knew about computers, regarding his application for selection as a mere formality.

Alan had chosen to apply for Southampton since while concentrating on mathematics in his program, there would also be opportunity for music and sport as well. During his first year of study, he had the great fortune to meet up with Jenny, a talented linguist. They subsequently married, and still are supporting each other. His tutor turned out to be Bill Cockcroft, well known later for writing the Cockcroft Report in 1982, which advised the British government on strategies for revamping school mathematics. Interestingly it was just as much their common interest in jazz that sealed the beginning of a long friendship between Bill and Alan.

The notion of becoming a teacher had formed for Alan in his senior years in secondary school. He chose to pursue this interest by moving to Loughborough College on graduation from Southampton, since there he could undertake a 1 year Diploma in Education, not just for mathematics teaching but also in Physical Education. Alan was still in contact with Bill Cockcroft who suggested on the completion of his Diploma that he should apply for scholarships that would allow him to study in the United States, and incidentally get to know something of the interesting curriculum moves being made there with the so called "new math". Alan did win a scholarship through the Ford Foundation, so he and Jenny, now married, were off to Harvard University in the United States to complete an MA in Teaching. Although the scholarship was for 1 year, they stretched it out for 2 years, supplementing the scholarship monies with tutoring. They managed to stay for a third year by taking on full time school teaching in a local high school. Hence while taking classes with the likes of Jerome Bruner, Alan was teaching the new School Mathematics Study Group (SMSG) mathematics in high school, a wonderful preparation for his then glimmering idea of becoming a researcher in education. This glimmer of an idea is captured by the statement from Alan at the head of this chapter. It was at Harvard he started to see the possibility, and the excitement that can be generated, of doing good research.

Heading back to England after their stay in the United States, Alan rejected various school teaching jobs at top public schools, some of whom were teaching the new School Mathematics Project (SMP) mathematics curriculum, which would have ensured him a stable and well provided professional life. He was clearly well qualified for such jobs. But he rejected these lucrative offers, preferring instead to pursue this dream of researching in education. Hence he applied for and was appointed to

a full time research fellow position at University of Hull working with Professor Frank Land. Unbeknown to Alan, Bill Cockcroft had moved to Hull, taking up the position of Dean of Science and Warden of one of the University Halls. Alan was delighted to take up the offer to be Deputy Warden to Cockcroft for his first 2 years at Hull. Apart from anything else, it provided him and Jenny with a free flat in which to live.

Land's 4 year project on which Alan was to work was centred on visualisation and the impact of this on mathematics learning. Although the project was very much in the psychology mould of doing research, nevertheless it was a project that was being carried out from within education. It was this subtle change that had excited Alan's interest at Harvard. Here at last he was starting to act out the idea. The project was basically assessing secondary school students on a range of visualisation and spatial ability measures, and on a number of attitude scales to do with mathematics. The students were also asked which primary schools they attended. At that time the primary education these secondary students had experienced in mathematics, formed a naturalistic but classic design for a research study. By ascertaining which primary school they had attended, the secondary students could be grouped into one of three groups: those who had completed their primary mathematics learning with the use of Cuisenaire materials; those who had used material devised by Dienes such as his MAB blocks and his logic blocks; and a third group who had experienced a traditional textbook resourced program. Interestingly those students who had used the various block materials in primary school, either Cuisenaire or Dienes materials, did much better on the spatial ability and visualisation tests, and had a much better attitude to geometry. The crucial aspect however of the study was later seen to be that the apparatus that the students had used in primary school was developed to help teach number concepts, not geometrical concepts, nor spatial abilities, nor visualisation. However it was in geometry that the real impact was made: this result seems obvious today, but in those days it was not so. These notions clearly linked with ideas that Alan had come across in the classes he had attended given by Bruner some years earlier. For Alan a real interest in visualisation and indeed spatial abilit*ies* of children grew, and this interest actively engaged him for the next 15 years or more. More comment is made on this focus in Section 3 of this volume.

At the conclusion of the project, and the completion of his doctoral studies in Hull, Alan moved to Cambridge University to take up a lectureship in the Faculty of Education that lasted for the next 23 years. He notes that he was regarded as an unusual appointment, because he did not come with the then normal 15 years or more of school teaching experience. Fortunately Richard Whitfield had gained an appointment in science education in this Faculty just before Alan's appointment. Whitfield also came with a research background rather than many years of school teaching experience. Interestingly Whitfield had been 1 year behind Alan at the same secondary school. Hence it is no surprise that once Alan had accepted the offer of an appointment, he and Whitfield joined forces to try to enliven the Faculty with a research program of their own.

The key to their project was to focus on the teacher in the classroom. Alan comments that then there were psychologists of various hues interested in studying the

learner, often in "controlled" conditions out of the classroom, but gradually more and more working with the learners in the normal classroom situation. There were also curriculum colleagues more interested in *the* mathematics, thinking through what topics should be taught, in what order they should be taught, and since the break with the ossified traditional curriculum had been made, what resources could be brought in to help students learn. Many of the curriculum workers started to become aware of the psychologists and their findings on learning. But very few researchers were prepared to focus on the teacher in this mix.

The other critical ingredient that made this type of research possible at Cambridge was that they had access to video tape and video recorders. The video equipment was located in a suit of rooms in the Engineering Department. Hence bookings for it and relocation of students from their normal classrooms became a necessity. But nevertheless this apparatus gave the possibility of recording teachers teaching in situ, and then later replaying the recording and stopping the action at critical points to ask what became Alan's central question; "What might the teacher do next?" In listing possibilities of action before knowing what actually did occur, discussing them, and then evaluating these possible actions, Alan found a very powerful way to engage both practicing and beginning teachers in analysing their own and other's teaching. Hence this aspect of his research became known as the "teacher decision-making" phase. This became the enduring focus for Alan throughout his research career. In one way or another he has been asking, "And how will the good experienced (not the ideal) teacher teach the mathematics?"

As Borko, Roberts and Shavelson note in their chapter (this volume), the research on teacher decision-making did not take root in England to spawn an enduring research agenda. They go on to examine what then happened in the USA. However the echoing legacy of this research in England was not recorded in the research literature. In many tutorial rooms, both in England and parts of Australia used for pre service programs, video recordings of teachers are still being used in the way that was thought of in Cambridge in the early 1970s, the aim being to foster in inexperienced teachers, the ways of doing that experienced teachers just seem to know is correct for this moment and context. More comment is made on these research activities in Section 2 of this volume.

Clearly "doing research as educationists" was a novel idea at Cambridge at that time, as it was up to the early 1970s in Australia and elsewhere. Bishop and Whitfield were challenging a very fixed idea. It was all right for other disciplines to research learning, teaching and indeed all aspects of education. But those who practiced education as a craft really had no role as researchers. That notion seems quite quaint today.

During his time at Cambridge his engagement with a broad range of activities and people grew considerably, so by the time he moved to Monash University in 1992 he was a well known international academic with a rounded research pedigree. At Cambridge he was active in various ways within mathematics education in England, becoming a frequent speaker and convenor of workshops. He was active in various professional associations, including the Association of Mathematics Teachers (at one point Chair), the Mathematics Association (President for some years), and the

British Society for Research into Learning of Mathematics. One incident is instructive concerning his involvement with such associations. Alan tells of his attempts, alone and with others, to try to integrate the various professional mathematics associations during the 1980s, but to no avail. His concern was to have a strong united front, as mathematics education, as well as education generally, came under ever increasing pressure during the Thatcher years. To hear him speak of this time is to sense a deep regret that he and colleagues had not been able to make more headway on this political agenda.

However, working with individual teachers and small groups of teachers Alan always found profitable and exciting. He recounts a story of events that happened after he gave a talk for the Association of Mathematics Teachers on research in the early 1970s. Someone asked him at the conclusion whether ordinary teachers could engage in research themselves. Alan replied that essentially yes, although there were some protocols and procedures with which one should become familiar, and work within. He was then challenged directly after the talk by a small group of teachers who wanted to get going with some of their own research. From this interaction a small informal group of teachers grew, who did continue to engage in research in their own schools on their own teaching, with Alan as a mentor. The group included people like Geoff Giles, Kath Cross, and Bob Jeffreys. It began in 1972, developing a small but interesting series of studies using what would today be called action research.

His work gradually broadened on to the international scene during the 1970s. Part of this was through the people he had opportunity to meet. For example, the beginning of a long friendship, as well as opportunity for a rich academic partnership began on meeting Jeremy Kilpatrick for the first time at an invited working group in France in 1971 (see Kilpatrick's chapter 14, this volume).

These opportunities expanded when Alan, with others, developed and then began to teach an M Phil research degree program in mathematics education in the early 1980s at Cambridge, and also at about the same time began supervision of doctoral students. To comment on this today seems to be noting not much out of the ordinary, but it was then quite different. The earlier battles for engaging directly in research within education were starting to bear fruit, but even so there was still the lingering notion that practice was the normal and perhaps only aim of education, with research in education to be conducted by other more qualified social scientists rather than educationists. This meant another interesting difference, compared to the environment of today. Then there was much less pressure for tertiary education staff to have a coterie of research students. Alan notes that from time to time he would advise potential candidates to enroll elsewhere when he knew that they would be supervised by someone who had a deep interest in their particular set of research questions, rather than "grabbing" all candidates that came one's way, which is a tendency for some staff today. This mutual trust of colleagues across universities within Britain also helped meld the small but growing community of mathematics education tertiary staff into a very active supportive research group.

In taking these steps of engaging with teaching in research programs, Alan was brought into contact with colleagues from a number of countries. His first two

doctoral students were Lloyd Dawe from Australia, and Norma Presmeg then from South Africa. The variety of students who enrolled in the 1 year M.Phil. program is also impressive: many have gone on to hold various positions in their own national professional education associations, as well as on the international stage. For example, Fou-Lai Lin, who was already a highly qualified mathematician and highly placed in the research administration in Taiwan, enrolled in the M.Phil. as his ideas turned to mathematics education. From the early days there was also Bill Higginson from Canada, and Renuka Vithal and Chris Breen from South Africa.

Alan also became active in international organisations. He attended the first International Congress on Mathematics Education (ICME) in 1969, and has since convened various groups for these conferences through the years. He was a founding member and co-director for 5 years of BACOMET (Basic Components of Mathematics Education for Teachers), an invitational international and hence multicultural research group that began in 1980 and continued to meet for more than 10 years. At times Alan held various positions in the International Group for the Psychology of Mathematics Education (PME) including being a member of the International Committee.

An important event that typified his work within these organisations concerned the year that PME was to meet in London during the mid 1980s. This was the time that world attention had finally turned to the apartheid question in South Africa. In line with a boycott of all things South African, there was a move to ban South African academics from attending the PME conference that year. After much arguing, the ban on the South African attendance was lifted, although the question was raised at the annual general meeting of the organisation. At Alan's suggestion, PME decided from then on not to ban attendance at the conference of any identifiable group of mathematics educators, even if such a ban could be seen as support of an acceptable political stance. Rather PME should find ways to support the attendance at its conferences of colleagues who are disadvantaged because of political situations, and such like. Putting this notion into action was another matter. An approach to UNESCO through Ed Jacobson by Alan to fund the publication of a book proved fruitful: the profit from the book was directed to PME. These monies became the founding amount for what has become the PME Skemp Fund, which continues to support the travel of colleagues who otherwise would not be able to attend PME conferences.

One of the mathematics educators that was influential in Alan's thinking was Hans Freudenthal. Freudenthal had founded what became one of the important international research journals in mathematics education, the *Educational Studies in Mathematics*. Alan was invited to become the second editor of this journal in the late 1970s (see Clements' chapter 7, this volume, for more discussion). He remains an advisory editor to this journal. This began for Alan a long association with the Kluwer Academic (now Springer) publishing house. In 1980 he founded and became the series editor for their Mathematics Education Library book series, a most highly regarded series that is still attracting authors. Within this series first in 1996, and again in 2003, two important two-volume handbooks were published that canvassed the state of mathematics education research worldwide.

However the most significant event that occurred during his time at Cambridge was in 1977. During the previous year Glen Lean from the Papua New Guinea University of Technology had visited Alan in Cambridge wishing to discuss with him the spatial abilities research that Alan had been involved with for 10 years or more. Glen's aim was to elicit support for the university students he was teaching who seemed to have great difficulty in mastering and understanding the geometry in the first year mathematics they had to study. Glen left with a parting invitation to Alan to visit sometime. Glen's visit certainly intrigued Alan. As it happened, Alan was planning to undertake a year of sabbatical through the 1977 academic year. An invitation had arrived from Professor Peter Fensham to spend some time at Monash University to work with Ken Clements. There was also an invitation to go to University of Georgia at Athens, USA, to link up with Jeremy Kilpatrick. Hence a year long round the world trip was planned for the family (by then Jenny and Alan had two sons) starting with 3 months in Papua New Guinea, then moving south to spend 5 months at Monash in Melbourne, Australia, and then finally travelling across the Pacific to spend time at the University of Georgia. It was the 3 months in Papua New Guinea that made the difference.

> "He changed" Ah ha! Yes he did.
>
> (Alan commenting on the first paragraph of Section 4 Introduction, this volume)

Ken Clements comments in his chapter (this volume) on the aftermath of Alan's Papua New Guinea visit in some length. This visit refocused Alan's interests in mathematics education away from his work with spatial abilities on to work with the impact of the social, cultural and political aspects on the teaching of mathematics. It seems, however, that this was not the first time that Alan had considered these other factors (in the traditional research way of thinking), or aspects of the educational environment, to say it a different way. A diagram first used by Bishop and Whitfield in the early 1970s, and reproduced in this volume by Borko et al. as Fig. 4.1 (see the introduction to Section 2, this volume), clearly has rectangles that suggest that during the 1970s Alan was well aware that the social, cultural and political aspects were important in understanding how teachers teach. His own experiences of school teaching in deliberately varied environments in Boston in the mid 1960s also alerted him to their individual and collective importance. A somewhat different experience in 1969 had also given Alan pause for thought. This concerned cross-cultural issues and forewarned him in part of the intricacies in trying to understand what was happening in such contexts. This experience was a keynote address at the first ICME conference given by Professor Hugh Philps from Australia, who reflected on his research conducted in Papua New Guinea. Philps' discussion of cross-cultural issues, which were mainly anchored in his Piagetian psychological studies with school students learning mathematics, fascinated Alan at the time. He spent some time talking with Philps at that conference. But even given these precursor experiences, it was his own experiences while living in Papua New Guinea that transformed Alan's thinking. No longer for him were the social, cultural and political issues of some importance; they became *the* important issues with which he needed to try and come to grips, as far as teaching mathematics was concerned.

Clearly Alan's concentration on these concerns can be seen in the headings used for the last four sections of this volume. The ways he chose to be involved with various professional groups noted above also indicates his new refocusing on these issues. His thinking was also stimulated by the small but engaged group of full time international students who came to Cambridge to enroll in the 1 year M.Phil. program that Alan started (see above), and the increasing numbers of doctoral students, again many from overseas. Within such a multicultural group, with most of the members already having substantial experience in education, Alan was able to test many of his own ideas as he sought to push himself into thinking through the implications of the political, cultural and social issues that impinged on mathematics teaching.

The key output from these years of reflection emerged as two books. The first is one of the most referenced volumes on mathematics education research, *Mathematical enculturation: A cultural perspective on mathematics education* (1988). Its sequel, which many do not realize is such, was the much later edited book by Abreu, Bishop, and Presmeg; *Transitions between contexts of mathematical practices* (2002). A plan that Alan had formed in the early 1980s, prompted by his Papua New Guinea experiences, was to develop two books, one on enculturation and another on acculturation. He was going to start with acculturation, but turned from that, being undecided on just how best to deal with the core notion, since he had never had to experience it directly. He then turned his whole attention to what enculturation means for mathematics teaching. Norma Presmeg, in her biographical notes in Chapter 1 (this volume), briefly comments on being a sounding board in the mid 1980s for Alan's ideas as the book came to fruition. One is not sure whether having lived in Australia for some 6–7 years, Alan finally felt he had some experience of acculturation, and hence was in a position himself to explore the long delayed second part of this writing program. Whether or not this is so, he interestingly had come to a way of breaking his blockage on this issue. Rather than deal with the idea front on by himself as he had with enculturation, he chose to think through the nuances of the idea, with a group of colleagues, using notions of transitions and indeed conflicts between cultures.

As noted above, Alan moved to Monash University, Australia, in 1992. This was not an easy move. It meant leaving their two grown boys back in England, and an aging parent. However the idea of growing old and crusty in an English establishment university was not the way Alan wished to finish his academic career. The move to Monash was attractive. It did mean promotion to a professorship, something that is not always a given in a place where one has been a long time. Both he and Jenny had enjoyed their extended stay back in 1977, and on subsequent visits had been made most welcome. He felt there were staff in the Faculty with whom he could easily form a working relationship. By this time Ken Clements had left Monash, and was on the staff at the Geelong Campus of Deakin University, a rural city about an hour's drive from Melbourne. Another interesting connection was that Glen Lean, who had inspired Alan's first visit to Papua New Guinea, was by then also on the staff at Deakin.

During his years as a paid staff member at Monash, from which he officially retired in 2002, Alan was heavily involved in the administration of the Faculty. He avoided the role of Dean with skilful footwork, but had different roles as Associate Dean, at various times, for Research, for International Affairs, and then as Deputy Dean, as well as being Head of the Mathematics, Science and Technology Group within the Faculty for some years. This of course meant membership and chairing of various Faculty and University committees. The time devoted to such increased through the 1990s as Monash, like universities elsewhere, moved totally into the age of performitivity and the attendant "need" for documenting everyone's activity to the nth degree, so that the organisation could work within a so-called "culture of evidence". Needless to say, much time was taken away from the core work of a high profile academic.

An early project that Alan worked on soon after arriving in Australia was to initiate the planning for an international regional conference through the agency of the International Commission on Mathematical Instruction (ICMI). This notion of ICMI supporting initiatives in particular regions of the world was not new, but certainly none had been contemplated for the South East Asian/Pacific region. However support was not always forthcoming from the Australians. In fact few in Australia had active involvement with the ICMI organisation, although they were regular attendees at the International Congress on Mathematical Education (ICME) four yearly conferences. Indeed when beginning the organisation for what eventuated as the 1995 ICME Regional Conference, no one in Melbourne was quite sure who was the Australian delegate to ICMI. Although such connections had been built for and during the 1984 ICME Conference held in Adelaide, Australia, 8 years later, interest in being actively engaged with this world wide organisation for many had waned. Hence those who had promoted the 1984 conference still held positions, even though lines of responsibility for action back to the mathematics education community were by then decidedly blurred. Alan's initiative inadvertently stirred up quite some angst. However the conference itself, although not as well attended as was hoped, still proved to be a success and cemented many connections between colleagues in Australia and overseas.

Although Alan continued writing on issues that he had started to think about before moving to Australia, he initiated two crucial research decisions. First he returned to the notions of what acculturation means in the mathematics education context. As noted above, this initiative finally produced an edited book, a conclusion to his original speculation some 20 years earlier on enculturation and acculturation. The other decision was to concentrate on values. This was not something new in Alan's thinking. The term appears in the diagram he and Whitfield used to conceptualize their ideas on teacher decision-making in the early 1970s. He had also begun to write quite explicitly on values by the early 1990s. By the mid 1990s Alan was ready to actively push the door of the classroom open again, and see what impact teachers' values had on their decision-making in the act of teaching. Thus began the Values and Mathematics Project (more comment on this is made in the Section 7, this volume). An Australian Research Council (ARC) Grant funded the original