

History of Mathematics Education

Nerida F. Ellerton
M. A. (Ken) Clements

Toward Mathematics for All

Reinterpreting History of Mathematics in North
America 1607–1865

 Springer

History of Mathematics Education

Series Editors

Nerida F. Ellerton, Mathematics Department, Illinois State University, Normal, IL, USA

M. A. (Ken) Clements, Mathematics Department, Illinois State University, Normal, IL, USA

Springer's History of Mathematics Education Series aims to make available to scholars and interested persons throughout the world the fruits of outstanding research into the history of mathematics education, provide historical syntheses of comparative research on important themes in mathematics education; and establish greater interest in the history of mathematics education.

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Nerida F. Ellerton • M. A. (Ken) Clements

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America 1607–1865

Nerida F. Ellerton
Department of Mathematics
Illinois State University
Bloomington, IL, USA

M. A. (Ken) Clements
Department of Mathematics
Illinois State University
Normal, IL, USA

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Dedicated to the memories of our parents, Lydia Ruth Gersch (Meyer), Paul Johannis Gersch, Brenda Margaret Clements (Fleming), and John Albert Clements who gave us opportunities that they themselves never had.

Preface to this Book

In November 2019, when we had almost finished our first draft of this book, we learned that another book on the history of mathematics in North America, authored by David E. Zitarelli, had just been released. That book was published by the MAA Press and was endorsed as an imprint of the American Mathematical Society. Its title was *A History of Mathematics in the United States and Canada (Volume 1: 1492–1900)*. Not much later we found that David Roberts’ (2019), *Republic of Numbers* had appeared. We had not been aware that these books were being prepared.

This was the second time in recent years that this had happened to us. The first time occurred in 2016 when, just as we were completing the first draft of our book *Samuel Pepys, Isaac Newton, James Hodgson and the Beginnings of Secondary School Mathematics: A History of the Royal Mathematical School at Christ’s Hospital 1673–1868* (which would be published by Springer in 2017), we learned that at the end of 2015 Clifford Jones had had his book *The Sea and the Sky: The History of the Royal Mathematical School of Christ’s Hospital* published.

On that first occasion we took the opportunity to read Clifford’s book carefully, and that caused us to change parts of what we had written. A similar thing happened this time, although we were saddened to hear of David Zitarelli’s passing late in 2018. We would have enjoyed discussing with him how he framed his history. We were particularly impressed with his well-documented account of developments relating to mathematics at Harvard College in the seventeenth century. One thing which quickly became clear to us was that he had prepared his book from a mathematician’s perspective, and we had prepared ours from a different vantage point. We were more interested than he in accounting for the history of mathematics in the 13 colonies and in the United States (to 1865) as that mathematics was experienced by people across the full spectrum of ages and places—by young children (aged less than 10), by children aged between 10 and 15, by pre-college students aged between about 15 and 18, by college students, and by others, including adults outside of formal education institutions. And, of course, like both David Zitarelli and David Roberts, we were also interested in the perspectives of

“mathematicians”—scholars who worked in the field and had an enduring interest in any of the branches of mathematics.

When people look at the same set of events from different vantage points they see mostly similarities, but also differences. As we were writing this book we were struck by the fierce inequalities of opportunity which developed with respect to the possibilities of learning Western Mathematics. On a related matter, we were also interested to note that at the end of the seventeenth century only the most elementary aspects of the Hindu-Arabic numeration system, which we regard as the most transformative theoretical position in the history of mathematics, was not known to the majority of inhabitants in North America. As you read that sentence did you think—“That can’t be true”? Well, it *is* true, and that fact provides an interesting starting point for the history to be presented in this book.

In 1992 the second author of this book (Ken Clements) authored a book titled *Mathematics for the Minority: Some Historical Perspectives on School Mathematics in Victoria*. In that book he argued that the history of school mathematics in the state of Victoria (Australia) had been controlled by politicians and mathematicians—initially British politicians and British mathematicians. After the Federation of Australian states was achieved and the nation of Australia was born in 1901 academics at the University of Melbourne, and education administrators employed by the Victorian government, believed that the main task of school mathematics was to prepare students for scientific, technical and mathematical studies at the University, at the Working Man’s College, and at the Ballarat School of Mines—which at that time were the State’s only higher education institutions. In other words, school mathematics was aimed at the needs and aspirations of a minority. In this present book we present a similar line of argument. This book tells the story of how European-background forms of mathematics were translated from “home” to the education institutions being established in the North American New World. By and large, the main idea was to make the mathematics studied in the New World identical with the mathematics taught in respectable “home” education institutions. This continued to be the case throughout the period 1607–1865—from the beginnings of the first permanent European-background settlement at Jamestown, Virginia, through to the end of the Civil War.

The world of publishing has changed considerably over the past few decades, and that has had an impact on how we have asked authors to prepare chapters for books in Springer’s History of Mathematics Education series. In the past, authors and series editors could assume that a whole book, or at least quite a few chapters in it, would be read by interested persons. But now, e-books and individual chapters of a book in digital form are readily available, and that has affected how we have written individual chapters of this book. Thus, for example, a careful reader of this book might notice that, occasionally, there is repetition of points made in earlier chapters. Obviously, because some readers will have access to just one of the chapters in the book, it made sense for us to repeat material covered in earlier

chapters. We have attempted to limit such repetition to cases where what is being repeated represents essential knowledge if the present chapter is to be understood as a stand-alone document. Another sign of the times is that there is a reference list at the end of *each* chapter, *and* a *composite* reference list after all nine chapters have been presented. The reason for that is simple: readers who have access to just one chapter are likely to want to have access to a fully documented statement setting out the works to which reference is made in the chapter.

We wish to thank librarians, archivists and the staff at the Phillips Library at the Peabody Essex Museum, Salem, Massachusetts, the Butler Library at Columbia University, New York, the Clements Library at the University of Michigan, the Houghton Library at Harvard University, the Library of Congress (in Washington DC), the Wilson Library at the University of North Carolina at Chapel Hill, the Beinecke Library at Yale University, the Winterthur Museum in Delaware, the Lilly Library at the University of Indiana, the Special Collections Research Center in the Swem Library at the College of William and Mary and at the Rockefeller Library (both in Williamsburg, Virginia), the New York Public Library, the British Library (London), Guildhall Library, London Metropolitan Archives, the Royal Observatory and the National Maritime Museum at Greenwich, the Bodleian Libraries at the University of Oxford, the Cambridge University Library, the Pepys Library at Magdalene College within the University of Cambridge, the State Library of Victoria (Australia), and the Milner Library at Illinois State University, for locating relevant manuscripts, artifacts, and books for us.

We could not have been more pleased with the cooperation we received by our publisher, Springer Nature, especially Melissa James and Nick Melchior. We always felt that we were being supported in the best possible ways. We would also like to thank Dr George Seelinger, the Head of the Mathematics Department at Illinois State University (in which we both worked until our recent retirements) for encouraging us in our research endeavors.

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- Clements, M. A. (1992). *Mathematics for the minority: Some historical perspectives on school mathematics in Victoria*. Geelong, Australia: Deakin University.
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- Zitarelli, D. A. (2019). *A history of mathematics in the United States and Canada*. Volume 1: 1492–1900. Washington, DC: American Mathematical Society.

Preface to the Series

Books in Springer Nature's series on the history of mathematics education comprise scholarly works on a wide variety of themes, prepared by authors from around the world. An important aim of the series is to develop and report syntheses of historical research which have already been carried out in different parts of the world with respect to important themes in mathematics education—like, for example, “Historical Perspectives on how Language Factors Influence Mathematics Teaching and Learning,” and “Historically Important Theories Which Have Influenced the Learning and Teaching of Mathematics.”

The mission for the series can be summarized as:

- To make available to scholars and interested persons around the world the fruits of outstanding research into the history of mathematics education;
- To provide historical syntheses of comparative research on important themes in mathematics education; and
- To establish greater interest in the history of mathematics education.

In this present book we offer a history of mathematics in North America between 1607 and 1865, as told from a mathematics-for-all vantage point. As far as we know, no other writers have addressed that theme. As the text proceeds, readers are invited to think about how mathematics in North America (excluding Canada and Alaska) emerged during a period when curricula of education institutions were controlled by what we have called the “classics stranglehold.” Of special interest are the profound effects this background had on fundamental questions like: “What should be the intended mathematics curricula in schools?” “Should the intended curricula be the same for all learners?” And “Who should be responsible for bringing about changes to implemented mathematics curricula in schools and colleges?”

We hope that the series will continue to provide a multi-layered canvas portraying rich details of mathematics education from the past, while at the same time presenting historical insights that can support the future. This is a canvas which can never be complete, for today's mathematics education becomes history for

tomorrow. A single snapshot of mathematics education today is, by contrast with this canvas, flat and unidimensional—a mere pixel in a detailed image. We encourage readers both to explore and to contribute to the detailed image which is beginning to take shape on the canvas for this series.

Any scholar contemplating the preparation of a book for the series is invited to contact Nerida Ellerton (ellerton@ilstu.edu) in the Department of Mathematics at Illinois State University or Melissa James, at the Springer Nature New York office.

List of Publications in the Springer Nature History of Mathematics Education Series

- Barbin, É., Guichard, J-P., Moyon, M., Guyot, P., & Morice-Singh, C. (2017). *Let history into the mathematics classroom*
- De Bock, D. (Ed.), (2022). *Modern mathematics—An international movement?*
- De Bock, D., & Vanpaemel, G. (2019). *Rods, sets and arrows: The rise and fall of modern mathematics in Belgium*
- Ellerton, N. F., & Clements, M. A. (2017). *Samuel Pepys, Isaac Newton, James Hodgson and the beginnings of secondary school mathematics: A history of the Royal Mathematical School within Christ's Hospital 1673–1868*
- Garnica, A. V. M. (2019). (Ed.), *Oral history and mathematics education*
- Kanbir, S., Clements, M. A., & N. F. Ellerton (2018). *Using design research and history to tackle a fundamental problem with school algebra*
- Owens, K. D., Lean, G. A. Paraide, P., & Muke, C. (2017). *History of number: Evidence from Papua New Guinea and Oceania*
- Ravn, O., & Skovsmose, O. (2018). *Connecting humans to equations: A reinterpretation of the philosophy of mathematics*
- Ellerton, N. F., & Clements, M. A. (2022). *Toward mathematics for all: Reinterpreting history of mathematics in North America 1607–1865*
- Paraide, P., Owens, K. D., Clarkson, P. C., Owens, C., & Muke, C. (2022). *Mathematics education in Papua New Guinea: A case study of colonial and postcolonial influences on mathematics education*. New York, NY: Springer.

**Recent Springer Books on the History of Mathematics Education
by Nerida F. Ellerton and M. A. (Ken) Clements**

- (2012). *Rewriting the history of mathematics education in North America 1607–1861*
- (2014). *Abraham Lincoln’s cyphering book and ten other extraordinary cyphering books*
- (2015). *Thomas Jefferson and his decimals 1775–1810: Neglected years in the history of U.S. school mathematics*
- (2017). *Samuel Pepys, Isaac Newton, James Hodgson and the beginnings of secondary school mathematics: A history of the Royal Mathematical School within Christ’s Hospital 1673–1868*
- (2022). *Toward mathematics for all: Reinterpreting the history of mathematics in North America 1607–1865*

Also Note:

- Clements, M. A., Keitel, C. Bishop, A. J., Kilpatrick, J., & Leung, F. (2013). From the few to the many: Historical perspectives on who should learn mathematics. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 7–40). New York, NY: Springer.
- Ellerton, N. F., & Clements, M. A. (2022). Australian school mathematics and “colonial echo” influences, 1901–1975. In D. De Bock (Ed.), *Modern mathematics—An international movement?* New York, NY: Springer.
- Singh, P., & Ellerton, N. F. (2013). International collaborative studies in mathematics education. In M. A. Clements, A. J. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 827–860). New York, NY: Springer.

Mathematics in the eighteenth century . . . did not originate generally in the schools of this country . . . If we except such mechanical features as the elementary operations of arithmetic and algebra and consider the progress of real mathematics, neither the elementary schools of this country nor the colleges were much concerned in that period with the subject.

(Smith & Ginsburg, 1934, p. 16)

I should rejoice to see . . . Euclid honourably shelved or buried “deeper than did ever plummet sound” out of the schoolboys’ reach.

(Statement by J. J. Sylvester, 1870, p. 261).

Entrance requirements for 1786 at Columbia College (New York) were specified as follows:

“No candidate shall be admitted into the College . . . unless he shall be able to render into English Caesar’s Commentaries of the Gallic War; the four orations of Cicero against Catiline; the four first books of Virgil’s *Aeneid*, and the gospels from the Greek; and to explain the government and connection of the words, and to turn English into grammatical Latin, and shall understand the four first rules of arithmetic, with the rule of three.”

(Quoted in Broome, 1903, p. 34)

[When Benjamin Silliman (aged 13) entered Yale College (in 1792)] “the entrance requirements might also have been appropriate for a ‘school of Plato.’ Candidates for admission to the Freshman Class were examined in Cicero’s Select Orations, Virgil, Sallust, the Greek Testament, Dalzel’s *Collectanea Græca Minora*, Adam’s Latin Grammar, Goodrich’s Greek Grammar, Latin Prosody, Writing Latin, Barnard’s or Adams’ Arithmetic, Murray’s English Grammar, and Morse’s Worcester’s or Woodridge’s Geography. Jacob’s Greek Reader and the four Gospels were admitted as a substitute for *Græca Minora* and the Greek Testament.”

(Fulton & Thomson, 1947, p. 9)

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Abstracts

Abstract for the Book

The 104 men and boys who arrived in Jamestown, Virginia, in 1607 heralded the first permanent settlement by European-background persons on territory now part of the United States of America. This book provides a history of mathematics in North America (excluding Canada, Alaska, and Mexico) between 1607 and the end of the Civil War, in 1865. The position taken is that all people engaged actively, in some way, with mathematics, and therefore the history of mathematics should tell the story of how mathematics emerged, for all, in the New World.

In this book, we take a mathematics-for-all perspective by considering the history of early-childhood mathematics, elementary-school and secondary-school mathematics, college mathematics, mathematics employed outside of formal education institutions, and attempts to create “new” forms of mathematics, mainly by “mathematicians.” Furthermore, the mathematics developed by and applied within different social groups—like, for example, females, Native Americans, African-American slaves, rural families, and persons engaged in specific employment areas (such as business, teaching, navigation, surveying, building construction, astronomy, and local and other forms of government administration)—should also be part of the story.

Today, in the 2020s, it is assumed that everybody should be offered the opportunity to learn mathematics. However, it was not until well into the twentieth century that “mathematics for all” became a recognizable and achievable goal in much of North America. Before then, the geographical location of schools in relation to children’s homes, the availability (or non-availability) of plantation workers and of teachers capable of teaching mathematics, the attitudes within families—especially parental attitudes—to schooling, economic circumstances of families, and social and psychological presuppositions and prejudices about mathematical ability or giftedness all influenced greatly the amount and type of mathematics a person would have the opportunity to learn. Moreover, in many societal subcultures, the perceived difference between two social functions of mathematics—its utilitarian, modeling function and its capability to sharpen the mind and induce logical

thinking—generated mathematics curricula and forms of teaching in local schools which met the needs of some learners more than others.

This book identifies a historical progression towards the achievement of mathematics for all: from schooling for all to quantitative literacy for all, to basic mathematics for all, to secondary mathematics for all, to college mathematics, to mathematics research, and to mathematical modelling in order to solve real-life problems. As much as has been possible, arguments have been based on data available in primary sources, and on interpretative analyses of those data.

Abstracts for the Nine Individual Chapters

Abstract for Chapter 1: “The Scope of the Book”

This book presents a history of mathematics between 1607 and 1865 in that part of North America which is the present United States of America (excluding Alaska), and this first chapter begins with some discussion of the meanings which could be given to the title of the book. During most of the seventeenth century the number of European-background settlers was always small in comparison with the number of Native American peoples, and the struggles by the settlers to survive meant that any desire to study higher forms of mathematics, or to conduct research in mathematics, was virtually non-existent. It was difficult for them even to provide ways and means by which young children could learn the Hindu-Arabic methods of counting or calculating. Products of technology like paper, slate, and ink were not readily available, and very few mathematics-knowledgeable teachers were available. The situation improved during the period 1700–1865, but even during the first half of the nineteenth century most young children did not have ready access to mathematics textbooks. In this introductory chapter, issues associated with the education of Native American children, and of children of indentured European-background workers and African American slaves are also considered. Toward the end of the chapter, six research questions are stated, and summaries of what will be investigated in the remaining eight chapters of the book are given.

Abstract for Chapter 2: “Young Children’s Introduction to Mathematics in North America Between 1607 and 1865”

In this chapter we consider the mathematics studied by young children—not yet 10 years of age—the eastern colonies during the seventeenth and eighteenth centuries. We draw special attention to the hornbook—the artifact which most influenced intended, implemented and attained curricula for young European-background children during the period 1607–1799—and provide details on what is possibly the

earliest extant hornbook constructed and used in North America during that period. During the seventeenth century there was little opportunity for most young children to advance their understandings of mathematics. Evidence is put forward that as late as the beginning of the nineteenth century most children aged less than 10 years were not given any opportunity to study any form of mathematics beyond counting verbally and learning to read and write the Hindu-Arabic numerals. It was not until the early 1820s that the idea began to be accepted by some North American educators. that all young children from about the age of 6 should learn to read and write Hindu-Arabic numerals, and to develop other elementary arithmetical concepts and skills. Once that idea was put forward, initially by Warren Colburn, it steadily gathered momentum among scholars, educators, and the society at large.

Abstract for Chapter 3: “The Influence of the Cyphering Tradition on North American Elementary- and Middle-School Mathematics Between 1607 and 1865”

Commercially-published textbooks do not offer the most important data for those interested in the histories of mathematics and mathematics education in North America during the period 1607–1865. In fact, until well into the nineteenth century most North American schoolchildren who were learning mathematics did not own a mathematics textbook, and many teachers of mathematics did not own one either. By contrast, almost all students aged from 10 to 16 years who studied any branch of mathematics prepared handwritten cyphering books, and often their teachers made available to them the cyphering books that they had prepared in their own school days. In this chapter we summarize our previous work on the cyphering tradition, drawing attention to theoretical bases, and also to the way the tradition controlled both the implemented and the attained mathematics curricula in grammar schools and in other pre-college institutions. Summaries of curriculum content and of the teaching and learning patterns which were an inherent part of the cyphering tradition are given. The discussion is based on our analyses of about 1500 extant North American cyphering books from the period.

Abstract for Chapter 4: “Mathematics Textbooks and the Gradual Decline in the Use of Middle- to Advanced-Level *Abbaco* Arithmetic 1607–1865”

This chapter focuses on the influence of textbooks and textbook authors on the teaching and learning of middle- to more advanced-level *abbaco* arithmetic in North America during three sub-periods—from 1607 to 1776, from 1776 to 1825, and from 1825 to 1865. During the first sub-period, from 1607 to 1776, there were relatively few students who concentrated on learning any form of mathematics beyond low-

level *abbaco* arithmetic. Those who prepared cyphering books copied statements of rules, cases and model examples from “parent” cyphering books or directly from textbooks. During the second sub-period, from 1776 to 1825, textbooks by North American authors were increasingly used to assist students preparing cyphering books, the most popular authors being Thomas Dilworth, Nicolas Pike, Nathan Daboll, Daniel Adams, Michael Walsh, Stephen Pike, and Warren Colburn. Although algebra and geometry were more studied than in the previous sub-period, any movement away from traditional *abbaco* arithmetic to other forms of mathematics tended to be resisted in the schools. The third sub-period, 1825–1865 witnessed a struggle between those who wanted to revolutionize and expand the teaching and learning of mathematics in the United States of America and those who clung to the content and pedagogical approaches associated with traditional *abbaco* arithmetic intended curricula. In this chapter we concentrate on showing that although initially in school mathematics textbooks were used to complement cyphering, ultimately they came to play a more decisive role.

Abstract for Chapter 5: “The Struggle for Algebra”

This chapter focuses on the emergence of algebra in the intended and implemented curricula of U.S. schools between 1607 and 1865. Before 1776 only a tiny proportion of school-age children, in what is now the mainland part of the United States, studied algebra. The chapter begins by providing evidence that until about 1820 the study of mathematics other than *abbaco* arithmetic was not something seriously engaged in by most young people in North America. Very few textbooks on any branch of mathematics other than arithmetic were suitable for school children, and relatively few cyphering books which focused on mathematics other than arithmetic were prepared. That changed in the early 1820s, after the first public high schools were opened, and after colleges began to require prospective students to demonstrate a knowledge of algebra. Nevertheless, even in the 1850s less than 10% of school-age North American children studied any of algebra, geometry, trigonometry, surveying, navigation, or calculus. The cyphering tradition was strongly linked to both *abbaco* arithmetic and algebra, but algebra was much less studied. In 1730 a Dutch-language textbook, by Pieter Venema, on arithmetic and algebra, was published in New York, but at that time there was little demand for it and a second edition never appeared. Documentary evidence—never before available to historians—from a “precursor” document prepared by Venema in New York in 1725, is discussed and analyzed. In that document Venema demonstrated how algebra could be used to prove and to generalize. Venema was ahead of his time and offered North American mathematics education an opportunity which it failed to grasp. Venema was ahead of his time and offered North American mathematics education an opportunity which it failed to grasp.

Abstract for Chapter 6: “Pre-College Geometry, Mensuration, Trigonometry, Surveying, and Navigation 1607–1865”

This chapter analyzes pre-college education developments in geometry, mensuration, trigonometry, surveying, and navigation between 1607 and 1865, in the 13 colonies and then in the United States of America. Although throughout that period relatively few students prepared cyphering books which focused on anything other than *abbaco* arithmetic. Some school students did study one or more of algebra, geometry, trigonometry, astronomy, navigation, and surveying, but most of those who did had not previously studied topics like angles, decimals, fractions, logarithms, or elementary mechanics, and therefore it was extremely difficult for them to make good progress. Evidence will be presented showing that some students nevertheless managed to succeed. In particular, data from a cyphering book prepared by Thomas Willson in Pennsylvania in 1789 will be examined in detail, and the analysis will suggest what implemented curricula in post-*abbaco* forms of mathematics were like at that time. It has often been argued that so far as mathematics education was concerned much was achieved in the schools of that time, because there was an over-emphasis on mere memorization. In this chapter it is argued, however, that that contention rests on the untested assertion that students who prepared cyphering books did not understand and could not apply what they entered in their cyphering books. An important aim for the cyphering tradition was that students who prepared manuscripts would consult them if and when they felt the need to do so later in their lives.

Abstract for Chapter 7: “College Mathematics, 1607–1865”

Throughout the period 1607–1865 most families had very few books other than a Bible in their homes, and most people did not know much mathematics beyond reading, writing, and counting with Hindu-Arabic numerals. Between 1636 and 1865 only a tiny proportion of the population of that part of North America which is now mainland United States of America attended college and, of those who did, most had not previously studied mathematics beyond low-level *abbaco* arithmetic, elementary algebra, and the first few books of Euclid’s *Elements*. It is not surprising, therefore, that the period did not produce more than three or four scholars who, by European standards, might be considered to have been “outstanding” mathematicians. The U. S. college curriculum had its origin in the classical curriculum traditions of the medieval universities of Europe and especially of Cambridge and Oxford Universities. However, many of those who attended North American colleges did study what we have called “applied mathematics”—embracing fields like astronomy, surveying, mensuration and navigation— while they were at college, and we

argue that this aspect of the implemented curriculum had been successfully translated mainly from Great Britain.

Abstract for Chapter 8: “Different Perspectives on Mathematics in North America 1607–1865”

It would be unreasonable to expect the inhabitants of North America to have produced great works of mathematics—judging by European standards—during the period 1607–1865. At that time a New World began to be constructed in North America by the European “invaders”—houses, schools, and towns were built, administrative structures were created, and lands were cleared for farming. But very few books other than bibles and, perhaps, almanacs were to be found in homes or schools, and most of the relatively few settlers who knew enough mathematics to teach it had other things to do. It is not surprising, therefore, that the 258-year period did not produce more than three or four mathematicians who, by the European standards of the time, might be regarded as “outstanding.” Between 1775 and 1820 U.S. college curricula drew their inspiration from the classical curricular traditions of the medieval universities of Europe and especially of Cambridge and Oxford Universities. However, many students who attended the North American colleges did enroll in “applied mathematics” subjects—embracing fields like astronomy, surveying, mensuration, and navigation. Interest in those forms of mathematics had been successfully translated mainly from Great Britain.

Abstract for Chapter 9: “Toward Mathematics for All: Answers to Research Questions, Limitations, and Possibilities for Further Research”

This final chapter begins by answering the six research questions which were posed towards the end of the first chapter. Those questions were:

1. What were the intended, implemented and attained mathematics curricula for young children aged less than 10 years (in North America) (a) during the seventeenth century? And (b) during the period 1700–1865? And, to what extent did the answers to those questions vary across North America, and in different groups of children (e.g., boys versus girls, European-background children versus Native American children, and European-background children versus African-American children)?
2. What were the intended, implemented and attained mathematics curricula for North American children aged between 10 and 15 years during (a) the seventeenth century, and (b) the period 1700–1865? And,