Ze Tang · Dong Ding · Yan Wang ·
Zhicheng Ji · Ju H. Park

Impulsive Synchronization of Complex Dynamical Networks

Modeling, Control and Simulations

Springer

Impulsive Synchronization of Complex Dynamical Networks

Ze Tang · Dong Ding · Yan Wang · Zhicheng Ji ·
Ju H. Park

Impulsive Synchronization of Complex Dynamical Networks

Modeling, Control and Simulations

Springer

Ze Tang
School of Internet of Things Engineering
Jiangnan University
Wuxi, Jiangsu, China

Dong Ding
School of Internet of Things Engineering
Jiangnan University
Wuxi, Jiangsu, China

Yan Wang
School of Internet of Things Engineering
Jiangnan University
Wuxi, Jiangsu, China

Zhicheng Ji
School of Internet of Things Engineering
Jiangnan University
Wuxi, Jiangsu, China

Ju H. Park
Yeungnam University
Kyongsan, Korea (Republic of)

ISBN 978-981-16-5382-7 ISBN 978-981-16-5383-4 (eBook)
https://doi.org/10.1007/978-981-16-5383-4

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd.
The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

This book is dedicated to our families. With tolerance, patience, and wonderful frame of mind, they have encouraged and supported us for many years.

Preface

In the concept of network science, a complex dynamical network is a connected graph with nontrivial topological characteristics, like high cluster effects, the different degree distributions of each node, homogeneous and heterogeneous among different nodes, different types of hierarchical structures and different community structures. These characteristics do not always appear in simple graphs like random graphs and lattices, but often appear in some graphs when modelling the real systems. As one of the most important collective behaviors, the investigation on synchronization of the complex networks has been lasting for several decades until now. In fact, the synchronization of complex networks has been applied in many fields, such as medical treatment, parallel image grabbing and processing, scientific index networks, smart grid and so on. Specially, due to the importance not only in the theoretical analysis but also in practical applications, the study on impulsive synchronization of complex dynamical networks has attracted engineers and scientists from various disciplines, such as electrical engineering, mechanical engineering, mathematics, network science and system engineering.

Pursuing a holistic approach, this book introduces emergent problems, models and issues in impulsive synchronization of complex networks. It establishes a fundamental framework for this topic, while emphasizing the importance of network synchronization and the significant influence of impulsive control in the design and optimization of complex networks. This book is mainly focused on the global impulsive synchronization of complex dynamical networks with different types of couplings, such as general state coupling, nonlinear state coupling, time-varying delay coupling, derivative state coupling, proportional delay coupling and distributed delay coupling. Correspondingly, different types of control methods, especially the impulsive control and pinning control will be introduced in order to derive sufficient conditions for the global synchronization. The primary audience for the book would be the scholars and graduate students whose research topics including the network science, control theory, applied mathematics, system science and so on. The prerequisite knowledge of this book could be the differential dynamic systems, the complex systems theory, the advanced and modern control theory and some basic knowledge of mathematics such as linear algebra and matrix theory.

The main outline of this book is to introduce some recent research works on analysis, control and applications of impulsive synchronization on complex networks. The book is organized as follows:

Chapter 1: The background of synchronization on complex dynamical networks is introduced, as well as the organization of this book, and some important definitions and useful lemmas are also provided in this chapter.

Chapter 2: This chapter aims to study the cluster synchronization for a kind of dynamical complex networks containing nonidentical nonlinear Lur'e systems and proportional delay, which has the features of unbounded and time-varying. Considering different functions that the impulsive effects play, some sufficient criteria for the cluster synchronization of the nonidentically coupled Lur'e dynamical networks are derived by applying the extended parameters variation formula, the impulsive comparison principle and the concept of average impulsive interval.

Chapter 3: This chapter discusses the global exponential synchronization for a class of delay derivative coupled neural networks with multiple time-varying delays and stochastic disturbance. To broaden the fields of synchronization applications in network science, consider cluster-tree topology structure of the coupled neural networks, a novel impulsive pinning control strategy is proposed, which skillfully considered the neural networks in current cluster that directly linked to the neural networks in other clusters.

Chapter 4: This chapter studies the adaptive control and the exponential synchronization problem of a kind of derivative coupled complex dynamical networks with proportional delay. Sufficient criteria of the exponential synchronization on complex networks are given based on impulsive control and adaptive pinning control strategies jointly applying the proportional delayed impulsive comparison principle, the extended parameters variation formula and the definition of average impulsive interval.

Chapter 5: This chapter concentrates on the exponential synchronization of nonlinear coupled Lur'e networks with proportional delay. For the sake of the heterogeneities existed in different Lur'e systems, the investigation on the global quasi-synchronization instead of complete synchronization for the coupled Lur'e networks is given. Comparing to pervious general time delay, as a type of unbounded time-varying delays, the proportional delay largely increases the challenge on network synchronization.

Chapter 6: This chapter is mainly focused on the global and exponential synchronization problem for a class of complex dynamical networks with nonidentically coupling and time-varying delays. Since the mismatched parameters in different systems, the quasi-synchronization issue instead of complete synchronization is investigated based on the application of impulsive control schemes. In view of the concept of average impulsive interval, the extended comparison principle for impulsive systems and some matrix calculation techniques, sufficient conditions for the achievement of quasi-synchronization of coupled complex neural networks are obtained.

Chapter 7: This chapter is devoted to investigating the exponentially cluster synchronization of a class of nonlinearly and nonidentically coupled Lur'e networks with multiple time-varying delays. A novel impulsive pinning control strategy is introduced, which are imposed on the Lur'e systems which have directed connections with any other clusters. Based on the Lyapunov stability theorem, mathematical induction method and the average impulsive interval, the conditions for successful cluster synchronization of Lur'e networks are derived.

Chapter 8: This chapter mainly studies the leader-following synchronization issue for a class of Lur'e networks with nonlinear couplings and multi-delay with various sizes. A kind of impulsive pinning controllers is designed for achieving exponential synchronization. By utilizing the definition of average impulsive interval, parameter variation method and contradiction method, sufficient synchronization criteria are derived. Noticeably, convergence rates of impulses with different functions of impulsive effects are discussed specifically.

Wuxi, China — Ze Tang
Wuxi, China — Dong Ding
Wuxi, China — Yan Wang
Wuxi, China — Zhicheng Ji
Gyeongsan, Republic of Korea — Ju H. Park
July 2021

Acknowledgements

This book is supported by the National Natural Science Foundation of China with Grant No. 61803180, the Natural Science Foundation of Jiangsu Province with Grant No. BK20180599, the China Postdoctoral Science Foundation funded Project with Grant No. 2021T140280, 2020M681484, the Postdoctoral Science Foundation of Jiangsu Province with Grant No. 2021K408C, the 111 Project with Grant No. B12018, the National Key Research and Development Program of China with Grant No. 2018YFB1701903 and the National Research Foundation of Korea with Grant No. 2019R1A5A808029011.

I'd like to express my appreciation to my postgraduate students Miss Yue Gao, Mr. Deli Xuan, Mr. Chenhui Jiang, Mr. Jiafeng Wang and Mr. Kunpeng Wang who have unselfishly given their valuable time in arranging these raw materials into something I'm proud of. Last but not the least, I want to give the most special appreciation to Prof. Jianwen Feng from Shenzhen University, Shenzhen, China for his greatly and kindly supporting to me.

Contents

Symbols

N	1, 2, 3, ...
$\mathbb{R}$	Field of real numbers
$\mathbb{R}^n$	Field of n-dimensional real vector space
$\mathbb{R}^{m\times n}$	Field of $m \times n$ real matrices space
L^T	The Transpose of matrix L
L^{-1}	The inverse of matrix L
$\otimes$	Kronecker product of two matrices
$*$	The symmetrical part in a matrix
$diag\{\cdots\}$	Block-diagonal matrix
$\|\cdot\|$	Euclid norm of the matrix or the vector
$\lvert\cdot\rvert$	Absolute value
$\lambda_{\max}(L)$	The largest eigenvalue of matrix L
$\lambda_{\min}(L)$	The smallest eigenvalue of matrix L
$\max_{1\le i\le N}\{\cdot\}$	The maximum value
$\min_{1\le i\le N}\{\cdot\}$	The minimum value
$L > 0$	The matrix L is positive definite
$L \ge 0$	The matrix L is positive semi-definite
$I_{n\times n}$	The $n \times n$ real identity matrix
$D^+u(t)$	$D^+u(t) = \overline{\lim}_{h\to 0^+} \frac{u(t+h)-u(t)}{h}$
$u(t_k^+)$	$u(t_k^+) = \lim_{h\to 0^+} u(t_k + h)$
$u(t_k^-)$	$u(t_k^-) = \lim_{h\to 0^-} u(t_k + h)$
sup	Supremum
inf	Infimum

Acronyms

CDNs	Complex dynamical networks
CNNs	Coupled neural networks
Lur'e DNs	Lur'e dynamical networks
NNs	Neural networks

Chapter 1
Introduction

1.1 Background

In the concept of network theory, a complex dynamical network is a connected graph with nontrivial topological characteristics, like high cluster effects, the different degree distributions of each node, homogeneous and heterogeneous among different nodes, different types of hierarchical structures and different community structures. These characteristics do not always appear in simple graphs like random graphs and lattices, but often appear in some graphs when modelling the real systems. As one of the most important collective behaviors, the investigation on synchronization of the complex networks has been lasting for several decades until now. In fact, the synchronization of complex networks has been applied in many fields, such as medical treatment, parallel image grabbing and processing, scientific index networks, smart grid and so on. Specially, due to the importance not only in the theoretical analysis but also in practical applications, the study on impulsive synchronization of complex dynamical networks have attracted engineers and scientists from various disciplines, such as electrical engineering, mechanical engineering, mathematics, network science, system engineering.

Complex dynamical networks have been widely studied by researchers from different fields in the past two decades. Initially, it was discovered that the states of certain systems would have abrupt changes when facing complicated working conditions. Under such a circumstance, impulses are introduced to describe the special phenomenon. Since impulses can be regarded as disturbances with some conditions, the circumstances under that the impulses will have a beneficial effect on synchronization have attracted scholars to study. In Lu et al.'s work [1], the definition of desynchronizing impulses and synchronizing impulses was standardized for the first time, and the corresponding ranges of these two functions of impulsive effects were discussed. In the case of synchronizing impulses [2], Zhao et al. designed a single impulsive controller and successfully achieved exponential synchronization. In [3], the continuous feedback controller was utilized to overcome the desynchronizing

Z. Tang et al., *Impulsive Synchronization of Complex Dynamical Networks*,
https://doi.org/10.1007/978-981-16-5383-4_1

impulses to achieve synchronization. However, it is impractical to deploy controllers globally for all the nodes in a complex network. Therefore, it is particularly critical to give an impulsive pinning control scheme. Notably, for some special network synchronization goals such as cluster synchronization, how to determine the controlled nodes is also a key issue. Designing a control law that can balance control costs and control effects is the core problem we hope to solve.

Different from the analyzing procedures for the continuous systems, the analysis for complex networks with impulsive controllers not only needs to build the relationship between the Lyapunov function and the error vectors of each node, but also needs to discuss the relationship between left and right limitations of the Lyapunov function at the impulse instants. As a result, the selection of the Lyapunov function will greatly affect the complexity of the calculation. Initially, a simple Lyapunov function was constructed to obtain the synchronization conditions of the complex network with nonlinear coupling [4]. In order to solve the synchronization issue for complex networks with time-varying delays, in the work of [5], the integral terms and multiple integral terms related to the delay sizes are introduced in the Lyapunov function. For various complex network models, different types of Lyapunov functions are proposed. However, as the Lyapunov function continues to become more and more complicated, the mathematical proof steps becomes more and more cumbersome. Through the work in [6], an impulsive synchronization theorem was summarized. As long as the conditions of the theorem are satisfied, the synchronization conclusion could be obtained. However, the obtained theorem was a kind of delay-independent theorems, and the model was limited to complex networks with time-varying delays. On the other hand, even though the less conservative theorems can be achieved by constructing some more complicated Lyapunov functions, the difficulties of analyzing the relationship between left and right limits will be enhanced. To tackle such an issue, the parameter variation method is proposed to solve the above contradiction between computational complexity and conservativeness of the theorem. By establishing a more appropriate comparison system, only a simple Lyapunov function is needed to obtain the synchronization conditions, and the exponential synchronization convergence can be accurately estimated. Further, with the in-depth studies of complex network coupling types, the comparison system built in such as [7] has been unable to meet the demand for complex models, which also motivates us to carry out this work.

Based on all discussed above, in this book, we are committed to analyzing the following issues:

1. Since the complex network model is undoubtedly the key point to the synchronization issue, this book will analyze a variety of models including CNs with proportional delays, derivative coupled CNs, CNNs with cluster-tree topology. We will show how parameter variation method is extended and appropriate Lyapunov functions are selected.
2. The impulsive control schemes are applied to synchronize CNs. In consideration of control cost and control efficiency, impulsive controllers will cooperate with pinning negative controllers in some situations. Particularly, for those network

structures with clustering topology, the pinning strategy of impulsive controllers will be introduced.

3. Synchronization theorems in regard to synchronizing impulses and desynchronizing are given. In addition, the exponential convergence rates with respect to different functions of impulsive effects are precisely estimated.

In next section, we will briefly introduce the content of each chapter.

1.2 Book Organization

Chapter 2 aims to study the cluster synchronization for a kind of dynamical complex networks containing nonidentical nonlinear Lur'e systems and proportional delay, which has the features of unbounded and time-varying. Based on the topological structure of nonidentically coupled Lur'e network, an effective impulsive pinning controller is proposed and placed on the Lur'e system with directional path to other clusters. Considering different functions that the impulsive effects play, some sufficient criteria for the cluster synchronization of the nonidentically coupled Lur'e dynamical networks are derived by applying the extended parameters variation formula, the impulsive comparison principle and the concept of average impulsive interval.

Chapter 3 discusses the global exponential synchronization for a class of delay derivative coupled neural networks with multiple time-varying delays and stochastic disturbance. To broaden the fields of synchronization applications in network science, consider cluster-tree topology structure of the coupled neural networks, a novel impulsive pinning control strategy is proposed, which skillfully considered the neural networks in current cluster that directly linked to the neural networks in other clusters.

Chapter 4 studies the adaptive control and the exponential synchronization problem of a kind of derivative coupled complex dynamical networks with proportional delay. Sufficient criteria of the exponential synchronization on complex networks are given based on impulsive control and adaptive pinning control strategies jointly applying the proportional delayed impulsive comparison principle, the extended parameters variation formula and the definition of average impulsive interval.

Chapter 5 concentrates on the exponential synchronization of nonlinear coupled Lur'e networks with proportional delay. For the sake of the heterogeneities existed in different Lur'e systems, the investigation on the global quasi-synchronization instead of complete synchronization for the coupled Lur'e networks is given. Comparing to previous general time delay, as a type of unbounded time-varying delays, the proportional delay largely increases the challenge on network synchronization.

Chapter 6 is mainly focused on the global and exponential synchronization problem for a class of complex dynamical networks with nonidentically coupling and time-varying delays. Since the mismatched parameters in different systems, the quasi-synchronization issue instead of complete synchronization is investigated based on the application of impulsive control schemes. In view of the concept of average impulsive interval, the extended comparison principle for impulsive systems and some matrix calculation techniques, sufficient conditions for the achievement of quasi-synchronization of coupled complex neural networks are obtained.

Chapter 7 is devoted to investigating the exponentially cluster synchronization of a class of nonlinearly and nonidentically coupled Lur'e networks with multiple time-varying delays. A novel impulsive pinning control strategy is introduced, which are imposed on the Lur'e systems which have directed connections with any other clusters. Based on the Lyapunov stability theorem, mathematical induction method and the average impulsive interval, the conditions for successful cluster synchronization of Lur'e networks are derived.

Chapter 8 studies the leader-following synchronization issue for a class of Lur'e networks with nonlinear couplings and multi-delay with various sizes. A kind of impulsive pinning controllers is designed for achieving exponential synchronization. By utilizing the definition of average impulsive interval, parameter variation method, and contradiction method, sufficient synchronization criteria are derived. Noticeably, convergence rates of impulses with different functions of impulsive effects are discussed specifically.

1.3 Preliminaries

Definition 1.1 ([8]) The dynamic system $\dot{x}(t) = f(x(t))$ is said to be mean square stable if for any $\varepsilon > 0$, there is a $\rho(\varepsilon) > 0$ such that $\mathbb{E}\{\|x(t)\|^2\} < \varepsilon,\quad t > 0$ when $\mathbb{E}\{\|x(0)\|^2\} < \rho(\varepsilon)$. In addition, if $\lim_{t\to\infty} \mathbb{E}\{\|x(t)\|^2\} = 0$, for any initial conditions, then the system is said to be globally mean square asymptotically stable.

Definition 1.2 ([9]) Consider a complex dynamical network

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), i = 1, 2, \ldots, N, \tag{1.1}$$

where $x_i(t) = (x_i^1(t), x_i^2(t), \ldots, x_i^n(t))^T$ is the state vector of the i-th system, c is the coupling strength, $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the coupling matrix, Γ is the inner connected matrix. It is said to achieve asymptotic synchronization if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t), t \to \infty,$$

where $s(t)$ is a solution of the local dynamics of an isolated node satisfying $\dot{s}(t) = f(s(t))$.

Definition 1.3 ([10]) Consider a complex dynamical network (1.1). Synchronization manifold is defined as follows

$$M = \{[x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T \in \mathbb{R}^{Nn\times 1} : x_i(t) = x_j(t), i, j = 1, 2, \ldots, N\}.$$

Definition 1.4 ([11]) A network with N oscillators is said to be cluster synchronized if it satisfies

$$\lim_{t\to\infty} \|x_i(t) - s_{\mu_i}(t)\| = 0, i = 1, 2, \ldots, N,$$

where $s_{\mu_i}(t) \in \mathbb{R}^n$ is a solution of an isolate node and satisfies

$$\dot{s}_{\mu_i}(t) = f(s_{\mu_i}(t)), i = 1, 2, \ldots, N,$$

which describes the identical local dynamics for the nodes in the μ_ith cluster.

Definition 1.5 ([12]) Let N systems as

$$\dot{x}_i(t) = f_i(x_i, t),\ i = 1, \ldots, N,$$

then, these systems are said to achieve the asymptotical inner synchronization, if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t), as\ t \to \infty,$$

where $s(t) \in \mathbb{R}^n$ is a solution of a target node.

Definition 1.6 ([13]) Function class QUAD(Δ, P, η): Let $\Delta = \mathrm{diag}\{\delta_1, \ldots, \delta_n\}$ be a diagonal matrix and $P = \mathrm{diag}\{p_1, \ldots, p_n\}$ be a positive-definite diagonal matrix. QUAD(Δ, P, η) denotes a class of continuous functions $f(x, t) : \mathbb{R}^n \times [0, +\infty) \to \mathbb{R}^n$ satisfying

$$(x - y)^T P(f(x, t) - f(y, t) - \Delta(x - y)) \leq -\eta(x - y)^T(x - y)$$

for some $\eta > 0$, all $x, y \in \mathbb{R}^n$ and all $t \geq 0$.

Lemma 1.1 ([14] Schur complements) *Given constant symmetric matrices A_1, A_2, A_3, where $A_1 = A_1^T$ and $0 < A_2 = A_2^T$, then $A_1 + A_3^T A_2^{-1} A_3 < 0$ if and only if*

$$\begin{pmatrix} A_1 & A_3^T \\ A_3 & -A_2 \end{pmatrix} < 0\ or \begin{pmatrix} -A_2 & A_3 \\ A_3^T & A_1 \end{pmatrix} < 0.$$

Lemma 1.2 ([15]) *Let $\otimes$ denotes the notation of Kronecker product. Then for a constant α and matrices A, B, C, D with appropriate dimensions, the following properties are easily established:*

1. $(\alpha A) \otimes B = A \otimes (\alpha B)$;
2. $(A + B) \otimes C = A \otimes C + B \otimes C$;
3. $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$;
4. $(A \otimes B)^T = A^T \otimes B^T$.

Lemma 1.3 ([16] Finsler's lemma) *Let $\zeta \in \mathbb{R}^n$, $\Phi = \Phi^T \in \mathbb{R}^{n\times n}$, $B^\perp$ represents a basis for the null-space of B, and $B \in \mathbb{R}^{m\times n}$ such that $rank(B) < n$. The following statements are equivalent:*

1. $\zeta^T \Phi \zeta < 0 \;\; \forall B\zeta = 0, \quad \zeta \neq 0$
2. ${B^\perp}^T \Phi B^\perp < 0$
3. $\exists X \in \mathbb{R}^{n\times m} : \Phi + XB + B^T X^T < 0$

Lemma 1.4 ([17] Jensen Inequality) *For any matrix $M > 0$, scalars γ_1 and γ_2 satisfying $\gamma_2 > \gamma_1$, a vector function $x : [\gamma_1, \gamma_2] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_{\gamma_1}^{\gamma_2} x(s)ds\right)^T M \left(\int_{\gamma_1}^{\gamma_2} x(s)ds\right) \leq (\gamma_2 - \gamma_1)\int_{\gamma_1}^{\gamma_2} x^T(s)Mx(s)ds.$$

Lemma 1.5 ([18] Lyapunov-Krasovskii Stability Theorem) *Consider the delayed differential equation*

$$x(t) = \dot{f}(t, x(t)).$$

Suppose that f is continuous and $f : \mathbb{R} \times \mathbb{C} \to \mathbb{R}^n$ takes $\mathbb{R}\times$ (bounded sets of $\mathbb{C}$) into bounded sets of $\mathbb{R}^n$, and $u, v, w : \mathbb{R}^+ \to \mathbb{R}^+$ are continuous and strictly monotonically nondecreasing functions, $u(s)$, $v(s)$, $w(s)$ are positive for $s > 0$ with $u(0) = v(0) = 0$. If there exists a continuous functional $V : \mathbb{R} \times \mathbb{C} \to \mathbb{R}$ such that

$$u(\|x\|) \leq V(t, x) \leq v(\|x\|),$$

$$\dot{V}(t, x(t, x(t))) \leq -w(\|x(t)\|),$$

where $\dot{V}$ is the derivation of V along the solutions of the above delayed differential equation, then the solution $x = 0$ of this equation is uniformly asymptotically sable.

Lemma 1.6 ([19]) *Assume that x and y are vectors, then for any positive-definite matrix P, the following inequality holds*

$$-2x^T y \leq \inf_{p>0}\{x^T Px + y^T p^{-1} y\}.$$

Lemma 1.7 ([20]) *Let $U = (u_{ij})_{N\times N}$, $P \in \mathbb{R}^{n\times n}$, $x^T = [x_1^T, x_2^T, \ldots, x_N^T]$, $y^T = [y_1^T, y_2^T, \ldots, y_N^T]$, and $x_i, y_i \in \mathbb{R}^n$, $i = 1, \ldots, N$. If $U = U^T$ and each row sum of U is zero, then*

$$x^T(U \otimes P)y = -\sum_{1\leq i<j\leq N} u_{ij}(x_i - x_j)^T P(y_i - y_j).$$