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Foreword by Otto Jager

For Everyone on This Planet

President John F. Kennedy in 1961 proclaimed that the United States would place a man on the moon by the end of the decade. In 1969, Neil Armstrong took his 'one small step' on July 20th—the Americans made it with five-and-a-half months to spare. After a devastating flood in 1953 that took 1836 lives, the Dutch decided to mostly close the North Sea delta and build a storm surge barrier. Twelve building projects and 43 years later, the coastline is reduced from 700 km to only 80 km.

The energy transition is often compared to the man on the moon-mission or to the Delta works—mostly to emphasize the vastness of the challenge we are now working on. Exceptional and impressive as those achievements forever will be, the energy transition surpasses them by far. The change to a renewable energy system will have serious impact on every economical sector and on the life of every individual in our society. Armstrong's 'small step for a man' needs an upgrade: 'The energy transition will be an upswing for everyone on this planet'.

It is exactly that aspect that makes this transition such an immense challenge: it depends on the cooperation of all people. More than ever before, decisions on remodelling the energy system will be affected by social, technological and economic trends. The world of energy will be more dynamic, changes will be more drastic and people will be more involved in renewable solutions. Extra challenging, but also an open door for new chances—illustrated by the success and potential of green bonds in financial markets.

Of course, the energy sector is used to looking forward for at least a decade and anticipate on substantial changes. In this process, we need all the information and analyses available to make the right decisions. These studies on Applied Operations Research and Financial Modelling in Energy contribute to a better understanding of policy implications of the proposed or applied methodologies. They also show the value of using the right models and methods for decision making. I'm convinced that these perspectives from Operations Research and Finance will have a positive effect on the quality of our decisions—strategic, tactical, and operational.

> Otto Jager Chief Financial Officer TenneT TSO B.V. Arnhem, The Netherlands

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Introduction: Applied Operations Research and Financial Modeling in Energy

André B. Dorsman, Kazim Baris Atici, Aydin Ulucan, and Mehmet Baha Karan

1 Introduction

Decisions in the energy sector are generally complex with multiple conflicting objectives. Accumulating demand, increasing competition, rising awareness on environmental issues, together with evolving rules and regulations are all binding the energy sector with environmental, social, and financial pressure to keep the production and distribution processes under control. The planning in the sector usually involves many sources of uncertainty and risk, varying time frames, and a large number of stakeholders with different views, which makes the application of Operations Research (OR) methods particularly suitable. There exists a vast literature on energy sector applications of OR methodologies. This is due to fact that optimization and rational decision-making are vital to building up more sustainable energy management systems in such a dynamic and competitive business environment. In this vein, financial decisions are one of the main legs to be handled since energy investments are usually capital intensive.

Financial Modelling (FM) is a scientific approach for decision making and its methods are capable of supporting financial decision making at different levels of various sectors as well as the energy sector. It is possible to identify a wide spectrum of application areas in energy finance that may include but not limited to pricing

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and hedging decisions, understanding the market dynamics and managing demand, measuring the effectiveness of regulations, assessing the feasibility and efficiency of investments, supervising cash flows, capital budgeting, allocating resources and evaluating risk. This is achieved by the employment of an extensive range of quantitative tools such as deterministic and stochastic optimization, multiple criteria, multiple objective and fuzzy decision-making methods, simulation, econometric modeling, statistical inference, and contemporarily by application of learning algorithms.

Applied Operations Research and Financial Modelling in Energy (AORFME) aims to contribute to the both academic and practitioner sides of the energy sector by offering several modeling applications followed by policy implications to aid various decisions faced in the sector. The chapters of the book mainly aim to focus on a variety of energy decisions, to present a quantitative perspective on these decisions, and to provide policy implications of the proposed or applied methodologies. For this purpose, we bring a group of OR and Finance researchers together and present a collection of chapters that contribute to the applications in the field of Energy.

2 Operations Research and Financial Modelling in Energy

The volume comprises twelve chapters on the application of Operations Research and Financial Modelling in the energy sector grouped into three main parts. Within the scope of the book, a vast array of problems devoted to energy markets is addressed such as *electricity generation & transmission planning*, *location and price decisions*, *smart energy management systems, efficiency evaluation, price & volatility forecasting*, *power plant pricing*, *the potential of renewable investments, valuation issues, consumer decisions,* and *financial risk analysis*. The book presents research on both macro and micro levels as well as research focus on market dynamics, renewable energy, pricing, and capacity decisions.

The above problem areas have been undertaken by the authors of the chapters with a variety of methods. Methods implemented involve a range of techniques of Operations Research and Financial Modelling from optimization to forecasting and from conventional statistical methods to machine learning methods. Examples include *Mixed Integer Programming, Simulation-based Optimization, Data Envelopment Analysis*, *Time Series Forecasting, Malmquist Productivity Indices*, *Arithmetic Brownian Motion, Principal Component Analysis*, *Logistic regression*, *Deep Learning Methods*, *Support Vector Regression*, and *Random Forest*.

Regarding the areas of application/industries, the volume presents research devoted to different types of energy problems. The application areas also involve variety with respect to means of production and networks. The chapters include research on *integrated natural gas & power networks*, *hydroelectricity power plants*, *electricity distribution companies*, *electric vehicles*, *wind-hydro energy technologies*, and *smart energy management systems in microgrids*. There also exist chapters on *energy options*, *electricity markets*, *attitudes of industrial consumers*, and *valuation of clean innovation*.

3 Issues Covered in This Book

The book is divided into three main parts listed below, each presenting a number of chapters that focus on the abovementioned problem instances, methods, and application areas of research:

Part A. Applied OR I: Optimization Approaches.

Part B. Applied OR II: Forecasting Approaches.

Part C. Financial Modelling: Impacts of Energy Policies and Developments in Energy Markets.

Part A of the book consists of four chapters in which several optimization related applications on energy are addressed: (i) electricity generation and transmission expansion planning, (ii) demand-driven electricity supply options of electric vehicles, (iii) smart energy management systems in microgrids, (iv) efficiency and productivity change in the electricity distribution sectors. The methods vary from optimization to simulation as well as their integrated use.

The book starts with the research of Mahdi Noorizadegan and Alireza Shokri (Chap. "Optimization Methods on Electricity Generation and Transmission Expan[sion Planning Problem"\) on energy generation and transmission line expansion plan](#page-14-1)ning. After carefully reviewing the expansion problem domain in terms of problem setting & modeling, types of uncertainty, and the solution methods, Noorizadegan and Shokri propose a simulation-based optimization framework to handle the complex problem of generation and transmission problems (GTEP) with key features and methods inspired by their review. Their framework aims to capture the uncertainty of both the electricity load and the power generation by renewable resources. The framework suggests starting with an initial problem that leaves the complex components out and simplifies the model. Then, simulation is suggested for improving the constraints and the solution to this initial optimization problem. The authors point out that the resulting framework is advantageous because instead of searching the entire feasible region in a large-scale problem, it relies on decomposing it and introducing the complexities of the problem to a simpler initial model step-by-step.

The next Chap. "Demand-Driven Electricity Supply Options of Electric Vehi[cles: Modelling,Simulation, and Management Strategy of Public Charging Stations"](#page--1-2) presents research on a contemporary topic: Electric Vehicles (EVs) and their charging stations (CSs). Elvin Coban and Gokturk Poyrazoglu discuss the challenges and research opportunities in the demand-driven electricity supply options of electric vehicles at public charging stations. This is a comprehensive research that covers different aspects of the electric vehicle charging stations from location to pricing. After reviewing the strategic, tactical, and operational level problems related to CSs, a discussion on the existing modeling approach to locate CSs and their potential extensions are presented. Following that, a simulation framework is described to decide the number and type of chargers. Finally, the authors discuss different pricing policies and potential future problems related to CSs. Last but not least, Coban and Poyrazoglu's research provides an extensive look at one of the important concerns of the future: public charging networks.

This chapter is followed by research on another contemporary matter: Smart Energy Management Systems (SEMS). Ozgur Ican and Taha Bugra Celik, in their [Chap. "A Review on Smart Energy Management Systems in Microgrids Based On](#page--1-2) Power Generating and Environmental Costs", offer a review on renewable energy, microgrids, and Smart Energy Management Systems. They rely on the optimization methods used in improving SEMS and investigate the common grounds for computational frameworks employed within these systems. They present a comprehensive discussion and table on the previous research on SEMS, their methods, and a comparison of their results in terms of power generating and environmental costs.

The next chapter presents an application of efficiency measurement based on linear programming. Yetkin Cinar and Tekiner Kaya's Chap. "Measuring Efficiency [and Productivity Change in the Turkish Electricity Distribution Sector" looks at the](#page--1-2) efficiency of the Turkish electricity distribution sector and its change over time using well-known efficiency measurement methods of OR literature: Data Envelopment Analysis and Malmquist Productivity Index. Relying on the large-scale privatization experienced by the Turkish electricity sector starting from 2013, Cinar and Kaya evaluate the efficiency in the post-privatization period. After a detailed review of DEA applications in electricity distribution sectors, they measure the efficiency levels of Turkish distribution companies, investigate the relationship of the efficiency levels with several exogenous factors and assess the level of change over 5 years. The chapter serves as a compact application of DEA and MPI supported by a review of the related research which has been an interest since the 1990s.

Part B of the book is designed to present research on the forecasting methods applied in the energy sector. Main research interests are (i) price and volatility forecasting in electricity markets, (ii) forecasting of the hydro inflow and optimization of virtual power plant pricing, (iii) comparison of renewable energy technologies via forecasting, (iv) valuation of energy real options with regime shifts. In this part, the methods vary between the conventional econometric models to learning methodologies.

Part B starts with a Chap. "Price and Volatility Forecasting in Electricity with [Support Vector Regression and Random Forest" by Mahmut Kara, Kazim Baris](#page--1-2) Atici, and Aydin Ulucan. The chapter aims at contributing to the contemporary research stream of machine learning applications to electricity markets for forecasting prices and volatility. The chapter serves as a neat application of learning tools to the electricity markets. The authors present two types of forecasting schemes (Price & Volatility) using two types of modeling approaches (Support Vector Regression & Random Forest) in Turkish day-ahead electricity markets. Within the scope of the research, utilizing the hourly Euro prices January-2013 and September-2019 period, a rolling data scheme is designed to produce hourly prices for 340 weeks considering 16 features. The volatility forecasting covers realized volatility values comprising more than 2000 days and 10 features. The metrics of SVR and RF are compared with each other in terms of each scheme as well as with the metrics of the naive estimations.

The next Chap. "Forecasting the Hydro Inflow and Optimization of Virtual Power [Plant Pricing" is on hydroelectricity and features two-part research focusing on hydro](#page--1-2) inflow forecasting and virtual power plant pricing. The chapter combines forecasting and optimization methodologies within a well-designed framework. The authors, Sezer Cabuk, Ozenc Murat Mert, A. Sevtap Selcuk-Kestel, and Erkan Kalayci propose a multiple-stage framework for hydroelectricity power plants that every stage's output is input to the next stage resulting in virtual power plant pricing. The hydro inflow forecasting is accomplished by utilizing Seasonal ARIMA with eXogenous variables (SARIMAX). The output of this stage, the forecasts, is used as the input for the hydro optimization model to forecast the water capacity for the future. On the other hand, they determine the price behavior using Monte Carlo simulations. Once capacity and price have been modeled, the virtual hydropower plant values are estimated.

The following Chap. "Comparing the Renewable Energy Technologies via Fore[casting Approaches" by Fazil Gökgöz and Fahrettin Filiz focuses on evaluating](#page--1-2) wind and hydro energy potentials through forecasting tools as conventional regression methods and deep learning methods as Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU). Utilizing Turkish wind and hydropower electricity generation data, Gökgöz and Filiz discuss the strengths and weaknesses of several forecasting methods in predicting electricity generation using renewable resources with an emphasis that forecasting tools may serve as an effective tool for policymakers in the sector.

Part B ends with the Chap. ["Valuing Energy Real Options with Regime Shifts"](#page--1-2) by Turalay Kenc and Mehmet Fatih Ekinci. The authors focus on the real options approach to value energy projects since these investments possess a high level of macroeconomic risk. After introducing, a basic real options valuation model with regime shifts, they derive a framework based on the Arithmetic Brownian motion (ABM) process with regime shifts for valuing the energy real options. The proposed model is illustrated in numerical analysis with a detailed discussion of its implications.

Finally, *Part C* is devoted to Financial Modelling. The part consists of three chapters that financial modeling related applications on energy are presented: (i) analysis of electricity switching behavior of industrial consumers, (ii) feasibility and potential of renewable investments in Tanzania, (iii) valuation of clean innovation, (iv) the power grid as a technical to a finance issue. The methods vary from Brownian Motion to statistical tests to validate the various hypothesis.

Part C starts with the Chap. "Understanding the Electricity Switching Behavior [of Industrial Consumers: An Empirical Study on An Emerging Market" by Murside](#page--1-2) Erdogan, Selin Metin Camgoz, Mehmet Baha Karan, and M. Hakan Berument. The chapter is focusing on the supplier switching behavior of industrial electricity consumers. The authors present empirical research based on survey data consisting of items for risk of switching, cost of switching, the attractiveness of switching, perceptions of the service quality, and market competition. The relation between these items and the probability of switching from suppliers is established using a

binary logistic regression model. The results of the research aim to shed light on the decisions of electricity suppliers, regulatory agencies, and policymakers.

[In Chap. "Does the Market Value Clean Innovation? Evidence from US Listed](#page--1-2) Firms", Antoine Dechezleprêtre, Cal B. Muckley, and Parvati Neelakantan aim at bringing new insights to the corporate environmental-financial performance debates. Utilizing the US patent data for the period 1995 to 2012, econometric modeling is used to disaggregate the innovation measurements into clean, dirty, and other components. The analysis reveals an important finding that environment-friendly innovation pays off.

[The final chapter of the book \(Chap. "The Power Grid: From a Technical to a](#page--1-2) Finance Issue. Who Bears the Financial Risk?") is written by André B. Dorsman and Kees van Montfort. In their chapter, the authors provide insight into financial relations between various stakeholders of the Dutch electricity market. The authors provide an understanding of the Dutch energy sector dynamics of clearing and the margin requirements in financing. After establishing the key parties and system players, the chapter discusses the quantification and bearing of the financial risks on the future cash flows in the energy sector.

4 Concluding Remarks

To conclude, with a wide range of look into the energy sector decisions from the perspective of OR and Finance perspectives, we hope that *Applied Operations Research and Financial Modelling in Energy* (*AORFME*) contributes to the applied research on energy-related issues and reaches its audience from the both academic and practitioner sides of the energy sector.

This is the eighth volume in a series on energy organized by the *Centre for Energy and Value Issues* (*CEVI*). In this volume, CEVI collaborates with *Hacettepe University Energy Markets Research and Application Center*. The previous volumes in the series were: Financial Aspects in Energy (2011), Energy Economics and Financial Markets (2012), Perspectives on Energy Risk (2014), Energy Technology and Valuation Issues (2015), Energy and Finance (2016), Energy Economy, Finance and Geostrategy (2018) and Financial Implications of Regulations in the Energy Industry (2020).

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Optimization Methods on Electricity Generation and Transmission Expansion Planning Problem

Mahdi Noorizadegan and Alireza Shokri

1 Introduction

Electricity is considered as the heart of modern economies and is predicted to have a significant increase in its share in the global energy mix i.e., twice the rate of primary energy demand (IEA, [2019\)](#page--1-3). Solar and wind will have the highest growth rates among other electricity resources from 2018 to 2040 (IEA, [2019\)](#page--1-3). According the same report, in a sustainable development scenario where electricity plays a larger role in energy demand, renewable resources will account for two thirds of global electricity demand. Therefore, given the energy mix transition towards electricity (particularly renewable sources), energy planning studies mainly focus on power Generation Expansion Planning (GEP). In general, GEP seeks an optimal investment of power generation units over a planning horizon to meet predicted/projected energy consumption (load) subject to a variety of constraints and considerations. Moreover, transmission facilities play important economic and technical roles in GEP as their installation cost and technical constraints could have substantial impacts on generation expansion decisions. Therefore, many studies combine these two problems and reviewed an integrated generation and transmission expansion planning problem (GTEP). Whilst there are different versions of GEP, the main decision variables include investment schedule for generation units. While transmission expansion planning is included in the problem, location of new power generation units and decisions for transmitting power from generating locations to demand points/areas are also decided.

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However, decision variables are not limited to only these generation units and transmission lines/corridors. Depending on the setting and assumptions, a problem may include many other types of decision variables such as decommission decisions, power generated by units, phase angles of voltages and currents, etc. For instance, Micheliet al. [\(2020\)](#page--1-4) consider decommissioning variables. Direct Current (DC) load flow is an approximation of Alternating Current (AC) load flow and has been considered in some studies (Caunhye & Cardin, 2018) while it has been ignored in many GTEP related studies. This involves the computation voltage angle which depends on geographical properties and technical characteristics of transmission lines. Some studies (Coester et al., [2018\)](#page--1-6) incorporated less technical details and instead focused more on economic analysis and environmental aspects of GTEP and GEP.

In recent years, technological and economic advancements in renewable sources as well as environmental requirements have directed the focus of energy planning problems towards GTEP with high renewable energy penetration. Despite their advantages, renewable sources impose considerable complexities to power supply. Although cost of renewable sources has substantially declined (e.g., 70% for solar from 2010 to 2018), it seems that renewables still cannot effectively compete with thermal technologies as their cost has also reduced (Feldman & Margolis, [2019;](#page--1-7) Fu et al., [2018\)](#page--1-8). Therefore, governments designed attractive incentive schemes to encourage companies for investing in renewable sources. Levin et al. [\(2019\)](#page--1-9) studied incentives mechanisms under four categories: (1) investment support, (2) generation support, (3) quantity targets, and (4) carbon policies. There are various studies for further investigation of incentive mechanisms (Alolo et al., [2020;](#page--1-10) Newbery, [2016\)](#page--1-11). Because of their uncertain power generation, integrating renewable sources into existing power systems which mainly consist of thermal units, is complex and requires sophisticated planning and scheduling. Moreover, technical restrictions such as rampage constraints of thermal units limit the utilisation of renewable sources. For instance, Duck curve is a critical concept to address the impact of power generation by solar units on power systems (Denholm et al., [2015\)](#page--1-12). Capacity factors of renewable sources are another important component that can have a considerable impact on economic and technical analysis in GTEP. Capacity factors are usually estimated for an entire year. In fact, this type of complexity and limitation makes renewable sources more expensive. The wide use of renewable sources imposes another complication to GEP. The consumption of natural gas by gas-fired units is significantly affected by the uncertainty of' power generated by renewable sources. In other words, the production of gas turbines needs to be adjusted with respect to power generation changes of renewable sources to satisfy demand. As a result, the uncertainty of renewables is transferred to the gas network. In order to maintain gas pressure at a safe level in a gas network for other usage (e.g., residential and industrial sectors), gas and electricity storage devices and gas compressors need to be installed. Integrated gas and power networks have been studied in operational level (Fallahi & Maghouli, [2020a\)](#page--1-13). However, recently there has been an interest for this integration in planning problem (Conejo et al., [2020\)](#page--1-14). Such problems are in general non-convex non-linear mixed integer problems (Esmaili et al., [2020\)](#page--1-15). The source of non-linearity is the gas flow equation known as Weymouth equation.

This book chapter provides a relatively comprehensive overview on GTEP and suggests an optimisation modelling framework for GTEP that includes important features. The rest of this chapter is organised as follow. In Sect. [2,](#page-17-0) we focus on the mathematical modelling of a general GTEP where various types of objective functions and constraints are discussed. In Sect. [3,](#page-24-0) we discuss two types of uncertainties: demand and power generation of renewable resources, and equipment failure. We suggest Interval Optimisation to deal with demand and generation of renewable resources and a cutting plane-based method for equipment failure. The both approaches are conservative and consider the worst possible situations. In Sect. [4,](#page-27-0) we briefly review the solution methods and suggest a simulation-based optimisation framework for solving practical GTEP problems. Finally, we provide a summary of this chapter in Sect. [5.](#page--1-2)

Notation—In this chapter, we use a simple notation for simplicity. Bold face characters and symbols indicate vector i.e., $\mathbf{x}_t^D := [x_{ik}^D]_{ik}$. Subscripts denote indices while superscripts denote the type of variables (e.g., *D* stands for decommissioned equipment). The main sets, indices, decision variables and parameters are defined as per below. The rest of variables and parameters are defined where needed.

Sets, indices, superscripts

I: the set of locations (nodes),

t: index for time period (year),

 T_t : a subset within time period t ,

i, *j*: location indices,

h: index for hours of a day,

k: index for technology type (either power generation unit or transmission line),

l: index for fuel type,

G: used to denote the natural gas,

st: used to denote storage devices,

dis: used to denote discharging storage devices,

ch: used to denote charging storage devices,

D: used to denote decommissioned equipment,

N: used to denote new equipment.

Decision variables

 $\mathbf{x}_t^N := [x_{ikt}^N]_{ik}$: binary variables representing whether at location *i*, a unit of technology *k* at time period *t* is installed,

 $\mathbf{x}_t^D := [x_{ikt}^D]_{ik}$: binary variables representing whether a unit of technology *k* located at location *i* at time period *t* is decommissioned,

 $\mathbf{y}_t^N := \left[y_{i,jkt}^N \right]_{ijk}$: binary variables representing whether between locations *i* and *j*, a transmission line of technology *k* at time period *t* is installed,

 $\mathbf{p}_{ht} := [p_{ikht}]_{ik}$: continuous variables representing power generation at location *i* with technology *k* at hour *h* in time period *t*,

 $\mathbf{f}_{ht} := [f_{ijklh}]_{ijk}$: continuous variables representing power flow between locations *i* and *j* using transmission line technology *k* at hour *h* in time period *t*,

 $I_{ht} = [I_{ikh}]$: continuous variables representing inventory of storage at location *i* with technology *k* at hour *h* in time period *t*,

 θ_{iht} : continuous variables representing the phase angle at location *i* at hour *h* in time period *t*,

 $dis_{ht} := [dis_{ikht}]$ _{ik}: continuous variables representing discharge of storage at location *i* with technology *k* at hour *h* in time period *t*,

 ch_{ht} := $[ch_{ikht}]$: continuous variables representing discharge of storage at location *i* with technology *k* at hour *h* in time period *t*,

 π_{iht} : continuous variables representing gas pressure at location *i* at hour *h* in time period *t*,

 $\mathbf{g}_{ht} := [g_{ijht}]_{ij}$: continuous variables representing natural flow between locations *i* and *j* at hour *h* in time period*t*,

M, M^G : large enough numbers.

Parameters

 \mathbf{Q} := $[Q_k]_k$: maximum capacity of power generation unit of technologyk,

 $\mathbf{Q}^{\min} := \left[\mathcal{Q}_k^{\min} \right]_k$: minimum level of power generation unit of technology*k*,

: maximum capacity of power flow for line of technologyk,

 $\mathbf{r} := [r_k]_k$: increasing ramp rate for power generation unit of technologyk,

 L_{IT} : maximum available fuel of type *k* in time period T_t

 $\alpha_h := [\alpha_{kh}]_k$: capacity factor for hourly power generation unit of technology *k*,

 $\hat{\mathbf{x}} := [\hat{x}_{ik}]_{ik}$: indicator for existing unit at location *i*, a unit of technology *k* in the beginning of the planning horizon,

 $\hat{\mathbf{y}} := [\hat{y}_{ijk}]_{ijk}$: indicator for existing transmission line between locations *i* and *j* of technology \vec{k} in the beginning of the planning horizon,

 π_i^{\min} , π_i^{\max} : minimum and maximum permitted gas pressure at location *i*.

Note that $\mathbf{x}_t = \hat{\mathbf{x}} + \mathbf{x}_t^N$ and $\mathbf{y}_t = \hat{\mathbf{y}} + \mathbf{y}_t^N$. We also eliminate the transpose sign in the notation.

2 Mathematical Model

Key components of GEPR include but not limited to objective function, environmental impacts of sources of power generation, reliability, resiliency, uncertainty, operational restriction and consideration, and impact of power generated by gas network. GTEP decides on facilities (generation units and transmission lines) with effective lives of more than 30 years. Therefore, similar to long term planning problems, some technical details such as daily operational details are replaced with their approximations or ignored. However, this could affect the electricity mix leading to higher costs or in some severe cases infeasible situations in reality. On the other hand, building and solving a GTEP with many details are formidable tasks. Therefore, a

reasonable trade-off between operational details and the problem complexity and computational challenges is usually sought.

2.1 Objective Function

Whilst majority of studies consider cost-based models, some (Allahdadi Mehrabadi et al., [2020;](#page--1-16) Lohmann & Rebennack, [2017\)](#page--1-17) study a welfare or profit-based objective functions. Electricity markets will have to be simulated to detect the electricity price to model a maximisation of GTEP model. Simulating electricity markets with reasonable details is a complex topic. However, some studies (Coester et al., [2018\)](#page--1-6) applied a rather simple methods such as Merit Order Curve. In such models, regulators also play an important role in setting electricity markets. We refer to Cramton et al. [\(2013\)](#page--1-18) for further discussion on energy and capacity markets. The main components of cost for GTEP include investment, decommission, operation, and fixed maintenance costs.

Investment cost—The investment for power system equipment is capital intensive and usually involves long-term financial arrangements. Uncertainty of demand and power generation by renewable sources complicates the risk assessment for investors. Hence, some studies (de Oliveira et al., [2017;](#page--1-19) Simo et al., [2015\)](#page--1-20) formulate GTEP as a dynamic program. GTEP can be considered as a facility location problem with fixed cost. This approach essentially requires a long-term planning horizon (to include the full effective lifecycle of all equipment) in order to make a right balance between operational and investment costs. However, due to complications such as disparity of lifecycle of different equipment, it is not always possible. Instead of total fixed investment costs, an equivalent annual cost for each equipment is computed. In this situation, additional constraints are required to ensure of availability of a selected facility for its entire lifetime. The investment cost of facility is computed towards the end of planning horizon (Caunhye & Cardin, [2018;](#page--1-5) Ding et al., [2018\)](#page--1-21).

Let $\mathcal{F}^{inv}_t(\mathbf{x}_t^N, \mathbf{y}_t^N)$ denote the investment cost function at time period *t*.

Decommission and upgrade cost—The main reasons to retire a unit are its high maintenance and operational cost and its high rate of failure and unreliability. In practice, even units may be used beyond their nominal effective lifetime when they are properly maintained and looked after. Decommissioning some units such as nuclear units incurs cost while decommissioning units such as gas turbines may lead to profit as they have salvage values. In some cases, old units may not be decommissioned and may be used to cover limited peak loads as an alternative for installing new units for this purpose. The decision to keep old units or to decommission them should be made through GTEP models. Moreover, units can be upgraded in an extra cost. In some cases, upgrading old units can be more cost effective than investing in new ones. For instance, gas turbines may be converted into combined cycle units or an overhaul may significantly increase the efficiency of steam units. Upgrading a unit usually involves considerable modelling complexities. Related constraints need to be defined based on the type of upgrade. A simple solution is to define two new variables

one for investment in the unit upgrade and one for decommissioning the old unit. We use $\mathcal{F}^{D,U}_t(\mathbf{x}_t, \mathbf{y}_t)$ to denote the cost function for decommissioning and upgrading power generation units and transmission line at time period *t*. It is worth mentioning that some units such as distributed generators have shorter effective lifecycle and may be installed and also decommissioned within the planning horizon.

Operation cost—Alongside the investment cost, operation costs (i.e., variable cost) comprise the main part of electricity cost. The operation cost includes fuel cost, water supply, pollution and emission cost, start-up cost. All components of operation cost may depend on the age of units. Thermal units can usually work with more than one type of fuel, which increases the availability of units. However, the efficiency, and emission produced by units depend on the type of fuel. For instance, due to restriction of natural gas network in winter as the result of higher level of consumption, other fuels such as Mazut are used in power plants. As such periods are usually short, modelling other fuels can have a considerable impact on the electricity mix. The reason is that when the natural gas limitation is enforced, a single-fuel GTEP model would change the mix e.g., installing sufficient fuel-efficient units to ensure the feasibility. Once fuel-efficient units are installed, the power generation plan and consequently the operation cost would change. However, due to modelling and computational complexities, alternative fuels are generally ignored in GTEP. Temperature and altitude also affect power generation by almost 10 percent (Sen et al., [2018\)](#page--1-22). Whilst it is not difficult to incorporate temperature and altitude in GTEP models, their impacts are usually neglected. It is worth mentioning that the fuel consumption function is not linear; but, a linear approximation is usually studied for more simplicity. Start-up cost is often considered in GTEP models. Modelling start-up requires constraints that link power generation in different hours. Such constraints are complicating constraints and increase the computational complexity.

Although water supply is crucial for thermal units, it is not included in GTEP studies as it is considered water is available everywhere. However, this is not a correct assumption. Water supply at certain dry locations can be quite expensive or in some areas impossible. We carried out a simple experiment to investigate the impact of water supply cost and restrictions. We noticed that water supply in areas with particular restrictions could play an important role in determining the electricity mix.

Environmental consideration is among the most influential factors to shift electricity mix towards renewable sources. The emission cost is now a key part of operation costs, and is mainly considered for NOx, SO_2 , CO, SPM, CO_2 , CH₄and N₂O (Li & Taeihagh, [2020\)](#page--1-23). It is worth mentioning that the penalty for each one is different. In addition to penalising, the production of some pollutants may be restricted. The emission cost depends on several factors such as distance between power plants and cities, population of cities, type of emission and pollution and type of fuel. Moreover, some environmental consideration may forbid installing power plants in special areas. This is important when the decision for transmission lines/corridors in included.

Let $\mathcal{F}^{opr}_t(\mathbf{x}_t, \mathbf{y}_t)$ denote the operation cost function at time period *t*.

Fixed and maintenance cost—There is usually a schedule for power plants and unit maintenance, which depends on hours that each unit produces electricity in each year. Some types of maintenance activities are short, but some are longer. The cost for each type is therefore different. But it is common to consider a fixed cost for annual maintenance for each unit depending on the technology of units. A fixed cost is also considered for each unit, which does not depend on its performance. We use $\mathcal{F}^{fix}_t(\mathbf{x}_t, \mathbf{y}_t)$ to denote the fixed and maintenance cost function at time period *t*.

2.2 Constraints and Technical Conditions

We classify the constraints and technical conditions of a GTEP model into three groups: (1) investment related constraints, (2) capacity constraints, and (3) technical constraints. These constraints are related to power generation units, transmission lines, and gas network. A key factor in formulating a GTEP is time interval which is usually hourly based intervals. But depending on the problem, 24 h in a day could be split into 6 intervals. This will significantly reduce the number of variables and constraints. In the following constraints, we consider hourly interval and present a brief mathematical model for a general GTEP.

Investment related constraints—As mentioned in the objective function description, when the investment decisions are annually modelled, additional constraints are required to ensure that once a unit is installed, it will be available for the rest of the planning horizon. Analogously, we need to make sure once a unit is decommissioned, it will be no longer available for production.

$$
\mathbf{x}_t^N \le \mathbf{x}_{t+1}^N \tag{1}
$$

$$
\mathbf{y}_t^N \le \mathbf{y}_{t+1}^N \tag{2}
$$

$$
\mathbf{x}_t^D \ge \mathbf{x}_{t+1}^D \tag{3}
$$

$$
\mathbf{y}_t^D \ge \mathbf{y}_{t+1}^D \tag{4}
$$

Capacity constraints—These constraints enforce capacity constraints for new and existing power generation units and transmission lines.

$$
\sum_{l \in \mathcal{L}} \mathbf{p}_{htl} \le \alpha_h (\mathbf{Q} \mathbf{x}_t + \mathbf{Q} (1 - \mathbf{x}_t^D) \tag{5}
$$

$$
|\mathbf{f}_{ht}| \leq \mathbf{F} \mathbf{y}_t + \mathbf{F} (1 - \mathbf{y}_t^D) \tag{6}
$$

$$
\sum_{l \in \mathcal{L}} \eta \mathbf{p}_{\text{htl}} \leq L_{lT_t} \tag{7}
$$

where *η* is the vector of fuel consumption rate for 1MWh corresponding to units in vector \mathbf{p}_{htl} . When hydropower units are included in GTEP, additional constraints for their power generation should be considered such as intakes and reservoir levels. Since other entities (agricultural related organizations) are involved, hydropower units may not be always available in particular during pick times.

Technical constraints—In an accurate model, all technical constraints in a Security Unit Commitment (SUC) problem will have to be considered. However, as GTEP is a long-term planning problem, only important conditions are studied. Given their complexities, effective approximations for some constrains are developed and used in GTEP. Here, we consider ramp rate constraints, start-up related constraints, and DC power flow requirements. In order to formulate rampage constraints, additional binary variables are needed for each unit. However, it may not be vital to include such details for a GTEP. We suggest using the following ramp rate constraints:

$$
\mathbf{p}_{ht} - \mathbf{p}_{h+1,t} \leq \min\{\mathbf{r}, \alpha_h(\mathbf{Q}\mathbf{x}_t + \mathbf{Q}(1-\mathbf{x}_t^D) - \mathbf{p}_{ht}\}\
$$
 (8)

The above inequality only enforces increasing ramp rate limits. A similar inequality can be used for modelling decreasing ramp rate; but it is not crucial to add the latter to the model. Equivalent ramp rate functions can be used to further simplify the ramp rate constraints. Lohmann and Rebennack (2017) proposed an efficient way of modelling unit start-up. They divided the power generation of each unit to two parts: \mathbf{p}^L_{ht} is a vector of variable for power generation of units up to their \mathbf{Q}^{\min} , and \mathbf{p}^U_{ht} is another vector of variables for power generation between \mathbf{Q}^{\min} and \mathbf{Q} . Then, the difference between $\mathbf{p}_{h-1,t}^L$ and \mathbf{p}_{ht}^L approximates the start-up variable u_{ht} . The below inequalities compute the start-up variables

$$
\mathbf{Q}^{\min}\mathbf{p}_{ht}^L + (\mathbf{Q} - \mathbf{Q}^{\min})\mathbf{p}_{ht}^U = \mathbf{p}_{ht}
$$
 (9)

$$
\mathbf{p}_{ht}^L + \mathbf{p}_{h-1,t}^L \le \mathbf{u}_{ht} \tag{10}
$$

$$
\mathbf{p}_{ht}^U \le \mathbf{p}_{ht}^L \tag{11}
$$

The last inequality ensures that the load below minimum generation always exceeds the load above minimum generation. It is trivial that without this inequality, the start-up variable may be zero. AC load flow equations involve non-linear and complex terms. Even in SUC problems which needs to be accurate, they are approximated by DC load flow equations. In a long-term planning, a good approximation may be sufficient. Therefore, we only enforce the key equations for load flow as follows.

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$$
-M(1 - y_{ht}) \le f_{ht} - S\hat{\theta}_{ht} \le M(1 - y_{ht})
$$
\n(12)

where **S** is the matrix of reactance of lines and $\hat{\theta}_{ht}$ is the vector of difference of phase angles at two ends of each line. We misuse the notation in the above inequality for the notation brevity and simplicity. But it is worth mentioning that it should be constructed so that we have $f_{i} = s_{i}(\theta_{i} - \theta_{i} + \theta_{i})$ if a line is installed or exists. We have observed that when the above inequality is removed, the solution of GTEP significantly changes and may not be feasible for a real situation.

Storage Constraints—Storages substantially complicate the problem; because it includes binding constraints that connect power generation of different hours (similar to rampage constraints). Therefore, although they have become vital in power systems with high renewable penetration, many studies still do not explicitly formulate them (Chen et al., [2019\)](#page--1-24). They are very important for technical purposes such as helping to cover load rampage, stability of power network, and variation of renewables' power generation. Storages are especially useful when the difference of the maximum and minimum electricity loads is very high. In this case, there will be enough idle units to charge storages during off-peak times to be used in pick times. Storages conventionally are batteries and pumped-storage hydroelectricity. Recently, Powerto-Gas (PtG) systems are used to produce gas during off-pick times, in order to be used to generate power when required (Ban et al., [2017;](#page--1-25) Fallahi & Maghouli, [2020b\)](#page--1-26).

$$
\mathbf{I}_{h+1,t} = \mathbf{I}_{ht} - \gamma^{dis} \mathbf{dis}_{ht} + \gamma^{ch} \mathbf{ch}_{ht}
$$
 (13)

$$
\mathbf{I}_{ht} \leq \mathbf{Q}^{st}(\mathbf{Q}\mathbf{x}_t^{st} + \mathbf{Q}(1 - \mathbf{x}_t^{D, st})
$$
\n(14)

$$
\mathbf{dis}_{ht} \leq t^{dis} (\mathbf{Q} \mathbf{x}_t^{st} + \mathbf{Q} (1 - \mathbf{x}_t^{D, st})
$$
(15)

$$
\mathbf{ch}_{ht} \leq \iota^{ch}(\mathbf{Q}\mathbf{x}_t^{st} + \mathbf{Q}(1 - \mathbf{x}_t^{D, st})
$$
\n(16)

where γ^{ch} and γ^{dis} are charging and discharging efficiency vectors, respectively. Also, *ι ch* and *ι dis* are charge and discharge rate vectors, respectively. Equations [\(13\)](#page-22-0) state the storage balance equation between two hours. Constraints (14–16) enforce inventory, discharging and charging restrictions based on the existing, installed and decommissioned capacity.

Gas Network—Natural gas is the main fuel used thermal units. The variation of power generation by renewable sources changes the natural gas consumption of thermal units and consequently the gas pressure in gas network. This could affect the natural gas consumption of residential and industrial sectors. Therefore, it is important to include the gas equation into GTEP to manage the impact of power system with high renewable penetration on gas network. Below, we suggest a simplified variation of gas network modelling. We use a non-vector notation for clarity.

$$
-M^{G}(1 - \overline{y}_{ijt}^{G}) \le g_{ijht} |g_{ijht}| - \phi_{ij} (\pi_{jht}^{2} - \pi_{iht}^{2}) \le M^{G}(1 - \overline{y}_{ijt}^{G})
$$
(17)

$$
\pi_{iht} \leq \pi_{jht} \leq \Gamma \pi_{iht} \tag{18}
$$

$$
\pi_{iht}^{\min}\overline{y}_{ijt}^G \le \pi_{iht} \le \pi_{iht}^{\max}\overline{y}_{ijt}^G \tag{19}
$$

$$
g_{iht}^{\min} \overline{y}_{ijt}^G \le g_{iht} \le g_{iht}^{\max} \overline{y}_{ijt}^G \tag{20}
$$

$$
p_i^{G,\min} \le p_{iht}^G \le p_i^{G,\max} \tag{21}
$$

where ϕ_{ij} is the parameter of natural gas pipeline, Γ is the compression ratio and $\overline{y}_{ijt}^G = 1$ if there exists a pipeline between node *i* and *j*, and otherwise, $\overline{y}_{ijt}^G = y_{ijt}^G$ (i.e., a decision variable). Constraints (17) state relation between gas flow and gas pressure for new and existing pipelines. Constraints (18) model the impact of compressors on the gas pressure. Constraints (19 and 20) respectively enforce the pressure and flow restrictions on new and existing pipelines. Constraints (21) ensure gas production restrictions on gas production nodes. Adding the above set of inequalities to GTEP results in a non-linear program. There are various methods such as Newton method and decomposition-based methods to deal with the nonlinear terms. The reader is referred to Ding et al. (2018) and Fallahi and Maghouli $(2020b)$ for further topics on non-linear gas network related terms. Note that in these inequalities, we assume that the gas flow direction is known in each pipeline. We also neglected modelling line-pack and installing new compressors.

Balance Equations—Natural gas and power networks have to be separately balanced at each node

$$
\mathbf{p}_{ht} + \mathbf{dis}_{ht} - \mathbf{ch}_{ht} + \mathbf{f}_{ht}^{in} - \mathbf{f}_{ht}^{out} = \mathbf{d}_{ht}
$$
 (22)

$$
\mathbf{p}_{ht}^G + \mathbf{g}_{ht}^{in} - \mathbf{g}_{ht}^{out} = \mathbf{d}_{ht}^G + \eta \mathbf{p}_{htl}
$$
 (23)

where *l* is the fuel index for natural gas, \mathbf{f}_{ht}^{out} is a vector with element f_{iht}^{out} presented $\arg_{ht}^{out} = [f_{iht}^{out}]_i, f_{iht}^{out} = \sum_{j \in I} f_{ijht}$. Similarly, we have $f_{ht}^{in} = [f_{iht}^{in}]_i, f_{iht}^{in} =$ $\sum_{j \in I} f_{jiht}$, $\mathbf{g}_{ht}^{out} = [g_{iht}^{out}]_i$, $g_{iht}^{out} = \sum_{j \in I} g_{ijht} \mathbf{g}_{ht}^{in} = [g_{iht}^{in}]_i$, and $g_{iht}^{in} = \sum_{j \in I} g_{jiht}$.
The first balance equation ensures that electricity load at each node is satisfied. The second balance equation connects the natural gas network to the power network. The last term in this equation ($\eta \mathbf{p}_{htl}$) is the consumption of natural gas by power generating units.

2.3 Final Deterministic Model

The summary of this section is a deterministic optimisation model as presented below:

$$
\min \sum_{t \in T} \mathcal{F}_t^{inv}(\mathbf{x}_t^N, \mathbf{y}_t^N) + \mathcal{F}_t^{fix}(\mathbf{x}_t, \mathbf{y}_t) + \mathcal{F}_t^{D,U}(\mathbf{x}_t, \mathbf{y}_t) + \mathcal{F}_t^{opr}(\mathbf{x}_t, \mathbf{y}_t)
$$

s.t., (1 – 23)

The above model can be used as a base model for the next stage, which is to consider uncertain parameters. The above problem has a diagonal structure based on *t*. In other words, there is no constraint coupling variables for different *t*. The objective function is also separatable based on *t*. As it will be explained in the solution method section, decomposition methods can be applied to such a structure. In some studies (Moradi Sepahvand & Amraee, [2020\)](#page--1-27), reserve and spinning reserve are included in GTEP models. In security-constrained unit commitment problems, reserve and spinning reserve are considered to respond unforeseen events such as demand variations and equipment failure. A simple and practical way of computing reserve and spinning reserve is to consider a certain fraction of load (Moradi Sepahvand & Amraee, [2020\)](#page--1-27). As reserve and spinning reserve are mainly operational decisions, we do not independently address them in this model. In the next section, we study uncertainty in GTEP which are due to two events: net load variation and equipment.

3 Uncertainty

Electricity demand and power generation by renewable sources are two key sources of uncertainty in GTEP. Power unit and transmission line failures are also uncertain events in a power system. These two types of uncertainty are usually dealt with differently. We briefly review main approaches for both types of uncertainties in this section.

3.1 Uncertain Electricity Demand and Power Generation by Renewable Sources

Stochastic programming (Ding et al., [2018\)](#page--1-21) and robust optimisation (Jabr, [2013\)](#page--1-28) are the common approaches used to deal with power generation of renewable sources and load uncertainties. The application of stochastic programming involves scenario generation for possible electricity load and power generation by renewable sources for the planning horizon. For a GTEP problem, the planning horizon is generally more

than 15 years. Based on prediction/projection methods, a discrete set of possible realisations of each uncertain parameter is generated. Therefore, the number of scenarios for hourly electricity load and power generation by renewable sources will be exponential. As the first step to reduce the number of scenarios, only selective days are considered for modelling (e.g., few days per month, or even per season for each year). Another way of reducing number of scenarios is to merge 24 h of a day into fewer time blocks. Then, scenario reduction approaches are applied to eliminate less likely scenarios. However, solving a large multi-stage stochastic problem specially for practical cases is still very challenging.

Alternatively, robust optimisation takes a less complex but more conservative approach and plans for the worst cases. The worst cases can be formed prior to the start of solution procedures. Multivariate statistical analysis based methods such as "flying-brick" have been developed to deal with variable requirements of the lookahead generation capacity, ramping capability, and ramp duration for unit commitment problems. For more details see Pourahmadi et al. [\(2020\)](#page--1-29). We focus on Interval Optimisation approach developed for unit commitment problems by Wu et al. [\(2012\)](#page--1-30). They used the concept of net load (NL) which is equal to total demand minus wind generation output minus solar output generation. Net load is commonly used because wind and solar generation, and demand have some similar characteristics such as they are non-dispatchable, they depend on the weather condition, and they deviate from forecasts (Makarov et al., [2010\)](#page--1-31). Therefore, the electricity balance equation is modified by the concept of net load. The key idea is to make sure that the installed electricity mix is capable of responding to extreme situations which are illustrated in Fig. [1.](#page-25-0) The worst situations are as follow: (1) power generation units including thermal and hydropower units are able to increase their generation to satisfy the net load from hour h where the net load is in its lowest level to hour *h* + 1 where the net load is in its highest level. (2) power generation units can deal with duck curve from mid-day towards night peak.

To this end, we need to define three power generation variable \mathbf{p}_{ht}^P , \mathbf{p}_{ht}^E , \mathbf{p}_{ht}^O , power generation for pessimistic, expected and optimistic net loads. Then, the rampage

Fig. 1 Net load uncertainty intervals for a sample daily load

constraints need to be imposed for all combinations of these new variables (e.g., ${\bf p}^P_{ht} - {\bf p}^O_{h-1,t} \le \min\{{\bf r}, {\bf \alpha}_h({\bf Qx}_t + {\bf Q}(1 - {\bf x}^D_t) - {\bf p}^O_{ht}\})$. When the ramp rate constraints are revised as explained, it can be ensured that the duck curve is also addressed.

3.2 Uncertain Equipment Failure

Security and resiliency of a power system are usually defined by uncertain equipment failures which can be due to technical failure, natural disasters, or sabotage. One solution to deal with unforeseen equipment failures is to allocate a sufficient level of spinning and non-spinning reserves, which is usually a topic for daily operation (Morales et al., [2009\)](#page--1-32). Another approach is to set the N-k security criterion. That means if for any reasons, k equipment (mainly lines) fails at the same time, there will be no power cut in the power system. This criterion can be imposed locally with different values for k. As the number of equipment is high in a power system, contingencies are limited to a pre-defined set of contingency scenarios (Qiming Chen & McCalley, [2005\)](#page--1-33). Then, binary variables or indicators and a set of related constraints are used to model the N-k security criterion. This idea is applied within bi-level programming, multi-stage robust optimisation and multi-stage stochastic programming (Wu et al., [2016\)](#page--1-34). There are also some probabilistic versions of N-k security criterion (Sundar et al., [2018\)](#page--1-35). But due to the complexity of probabilistic constraints, this approach is not popular for GTEP.

As GTEP problems expand existing electricity networks, it may not be necessary to define a binary variable for each line for the N-k security criterion. In other words, it is very likely that there are already other routes to a demand bus if one line fails. Studying the topology of the network could be very useful. Therefore, instead of initially defining binary variables for each line, cutting plane methods can be used to ensure the N-k security criterion with much less computational complexity. In a cutting plane method, the N-k security criterion is first relaxed and the problem is solved. Then, using a separation algorithm, it is checked to find a violation of the N-k security criterion for each demand bus. If found, a cutting plane is constructed to enforce the security criterion for that bus. In general, separation algorithms are usually quite fast and it is relatively simple to identify violated constraints which were relaxed (Nemhauser & Wolsey, [1988\)](#page--1-36). For a GTEP, graph-based problems such as maximum flow problem and shortest path problem could be used in designing separation algorithms. Therefore, it is expected to achieve a better computational efficiency in particular for real problems, as significantly less binary variables are required in the model.

4 Solution Method

There is a longitudinal study on GTEP in which the majority of them use Benders' decomposition-based methods to solve their problems (such as Lohmann and Reben-nack [\(2016\)](#page--1-34) and Wu et al., (2016)). Therefore, this section reviews some principles of Benders' decomposition and few important tips for implementing Benders' decomposition particularly useful for solving practical problems.

Decision variables in a GTEP problem are naturally divided into strategic decisions and operational decisions. This division paves the way for applying Benders' decomposition where the investment and operational decisions are made in the master problem and subproblems, respectively. In particular, Benders' decomposition is applied to two or multi-stage stochastic programming or robust optimisation variations of GTEP. Some studies (Lohmann & Rebennack, [2017\)](#page--1-17) have further explored the structure of their problem and proposed nested Benders decomposition reformulations. As the operational problems are independent some time intervals, they can be solved separately. Constraints such as available fuel and maximum amount of pollution produced by units are usually defined seasonally or annually. These constraints link operational variables within a season or a year. In these cases, seasonal or annual operational problems can be independently solved. Such further breakdowns can help to reduce the computational efforts.

Connecting the master problem and the subproblems is done using optimality and feasibility cuts. Optimality cuts approximate the impact of the master problem decisions on the cost of the subproblem (Conejo et al., [2006\)](#page--1-37). It is worth mentioning that if there are binary or integer decision variables in the subproblem, standard optimality which are developed for pure linear continuous subproblems cannot be used. The reason is that optimality cuts are constructed based on the dual form of the subproblem. The linear programming duality theorem does not hold for an integer program (Nemhauser & Wolsey, [1988\)](#page--1-36). This is a common mistake in studies about GTEP problems. Further details can be found in studies about the concept of value function (Guzelsoy & Ralphs, [2006;](#page--1-38) Trapp et al., [2013\)](#page--1-39). Feasibility cuts are driven when a solution of the master problem leads to an infeasible subproblem. When a subproblem is infeasible for a master problem solution, feasibility cuts driven for that solution only remove that solution from the search space. Nevertheless, it is possible that next solutions of the master problem still lead to infeasible subproblems. Significant computational efforts thus would be spent on finding master problem solutions with feasible subproblems. It will be computationally beneficial to avoid feasibility cuts, if possible. Investment decisions e.g., power generation unit installation, are made in the master problem. For a minimisation problem, in the first iteration, no new investment is made to keep the master problem cost at its minimum level. This will lead to some infeasible subproblem. To avoid dealing with feasibility cuts and their drawbacks, valid inequalities that reflect the subproblem's feasible region of subproblems can be derived and added to the master problem prior to the solution process. For power generation expansion decisions, a constraint can be formed to

enforce the sufficiency of the accumulative capacity to satisfy a selective peak load-Similar valid inequalities can be formed for transmission lines. Moreover, storage devices are complicating components of GTEP problems as they are only available when they are charged. Because the investment cost of storage devices is usually lower than other technologies, the solution of the master problem may include too many storage devices at the first iteration. As a result, subproblems may be infeasible. To overcome this issue, some valid inequalities approximating storage constraints (charge and discharge processes) are useful in the master problem.

Given the significant number of variables and constraints, it may take several days to solve a practical problem even with very powerful machines. The feasible region can be reduced before the solution process by applying methods such as Merit Order or even simple economic analysis to only include power generation technologies which are likely to be a part of the optimal solution. In addition, decomposition methods such as Dantzig Wolfe decomposition and column generation methods have proved to be very efficient for optimisation problems with binary variables (Singh et al., [2009\)](#page--1-40). Nevertheless, they have not received much attention for reformulating and solving GTEP problems (Flores-Quiroz et al., [2016\)](#page--1-41).

A practical GTEP with fair number of details, which addresses concerns for independent system operators may not be solvable with a reasonable time. Key aspects of a power system such as frequency response control, power system inertia, various types of losses (particularly important for distributed generators) are usually ignored. As mentioned, gas network and storages are also mainly neglected in GTEP problems. Simulation-based optimisation is a practical way of addressing all important components, factors and constraints of GTEP and at the same time solving the resulting problem. The application of simulation-based optimisation to GTEP is an independent topic with many technical details. Here, we emphasise on its benefit for GTEP and outline some general steps. For more detail, we refer the reader to Rodgers et al. [\(2018\)](#page--1-42).

As it is illustrated in Fig. [2,](#page--1-43) we can start with a GTEP optimisation model including only key constraints. The GTEP optimisation model box may contain several subboxes in relation to modelling and solution methods (e.g., cutting planes and decomposition methods). In particular, with uncertainty assumptions, it is usually the case that the problem is decomposed into a master problem and several sub-problems. In the initial optimisation model, complicating components such as the gas network (i.e., constraints (17–21) and (23)) and N-k security criterion may be ignored. Once the optimisation model is solved, its solution can be used for a Monte-Carlo simulation model with more details (including the gas network and N-k security criterion) compared with the initial optimisation model. The aforementioned complicating components do not directly affect the objective function i.e., there is no term for these components in the objective function. It is worth noting that the focus here is to solve a GTEP in which the impact of the gas network is also considered, and we do not intend to optimise the gas network as well. Therefore, these components affect the feasibility of the problem. If the solution satisfies all requirements, then the optimal solution is generated. It is worth mentioning that the notion of optimal solution here may be challenged. Otherwise, constraints that enforce the violated requirements