Trends in Mathematics
Research Perspectives CRM Barcelona Vol.15

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Extended Abstracts GEOMVAP 2019

Geometry, Topology, Algebra, and Applications; Women in Geometry and Topology

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Trends in Mathematics

Research Perspectives CRM Barcelona

Volume 15

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ISSN 2297-0215 ISSN 2297-024X (electronic)
Trends in Mathematics
ISSN 2509-7407 ISSN 2509-7415 (electronic)
Research Perspectives CRM Barcelona
ISBN 978-3-030-84799-9 ISBN 978-3-030-84800-2 (eBook)
https://doi.org/10.1007/978-3-030-84800-2

Mathematics Subject Classification: 13A18, 13F30, 14C20, 14E15, 14E30, 15B51, 35F21, 53D17, 70H20, 70E60, 92D15

This book is published under the imprint Birkhäuser, www.birkhauser-science.com by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

The Geometry of Varieties and Applications Group (GEOMVAP) is a group of researchers with interests in a wide range of fields, which include algebraic, differential and symplectic geometries, algebraic topology, commutative algebra and their applications. The group is composed of researchers rooted or formed at the Universitat Politècnica de Catalunya (UPC).

The main objective of GEOMVAP is to take a deep dive into the study of geometric structures and their applications. The geometric structures that are considered are algebraic varieties, symplectic and differentiable manifolds and the applications are mainly focused in the fields of Phylogenetics, Robotics, Mathematical Physics, Control Theory, Dynamical Systems and Celestial Mechanics. In order to achieve that end, a broad range of tools are used (geometric, algebraic, topological, arithmetic, differential and computational), and in many cases techniques from different fields are combined.

The members of the group work in interdisciplinary teams and transversal research topics. GEOMVAP promotes, in particular, Responsible Research and Innovation (RRI) within the framework of Horizon 2020. Among the RRI initiatives, it strives for gender equality, public engagement, science communication and the visibility of women in Science and Society.

The extended abstracts in this volume stem from the contributions in two events organized by GEOMVAP during the year 2019.

On January 23 and 24 of 2019, the GEOMVAP group organized an offside meeting on the *Parador de Cardona*, a large ninth century castle perched on a steep hill overlooking the town and salt mine of Cardona. The goal of that meeting was to share the different objectives, strategies and advances of the different research topics of GEOMVAP. The event consisted of 22 talks from different members of the group: Ph.D. students, postdoctoral members and professors.

A list of the contributed talks is included below.

List of Talks

- Maria Alberich Carramiñana, *Action of Cremona maps on planar polynomial differential systems.*
- Patricio Almirón Cuadros, *On the Tjurina number of plane curve singularities.*
- Josep Àlvarez Montaner, *D-modules over direct summands.*
- Miguel Angel Barja, *Clasificación de variedades irregulars y el Teorema Fundamental del Cálculo.*
- Guillem Blanco, *Bernstein-Sato polynomials of plane curves.*
- Roisin Braddell, *Group symmetries of cosymplectic and b-symplectic manifolds.*
- Joaquim Brugués, *La construcció de l'homologia de Floer*.
- Robert Cardona, *Estructures geomètriques en hidrodinàmica.*
- Franco Coltraro, *Mechanics of inextensible surfaces.*
- Josep Elgueta, *Representacions categòriques.*
- Marina Garrote López, *Semi-algebraic conditions for phylogenetic varieties.*
- Xavier Gràcia, *Hamilton-Jacobi theory and geometric mechanics.*
- Juan Margalef, *De la mecánica clásica a la mecánica cuántica.*
- Anastasiia Matveeva, *Group valued moment maps and equivariant cohomologies.*
- Eva Miranda, *From Celestial Mechanics to Fluid Dynamics: contact structures with singularities, part I*.
- Miguel C. Muñoz Lecanda, *Sobre distribucions no integrables.*
- Cédric Oms, *From Celestial Mechanics to Fluid Dynamics: contact structures with singularities, part II*.
- Alessandro Oneto, *Looking for equations of mixtures of phylogenetic models.*
- Arnau Planas, *A* b^m*-symplectic KAM theorem.*
- Xavier Rivas Guijarro, *Singular Lagrangian field theories and k-cosymplectic geometry.*
- Jordi Roca-Lacostena, *On the embedding problem for evolutionary Markov matrices.*
- Narciso Román Roy, *Multisimplectic formulation of Lagrangian models in gravitation (GR).*

The workshop *Women in Geometry and Topology* was an endeavor organized by the GEOMVAP research group and financed under the AGAUR project 2017SGR932. It took place at the *Centre de Recerca Matemètica*, Barcelona, from Semptember 25 to 27, 2019.

The workshop Women in Geometry and Topology featured nine plenary talks by top female mathematicians and several contributed talks and poster presentations by speakers of any gender identity. Two of the plenary lectures were addressed to the general public, not only for mathematicians but also for anyone with curiosity. A panel open to the public was also organized in order to discuss the situation of women in mathematics, the gender gap and strategies to break the glass-ceiling inside and outside academia.

Below there is a list of plenary, contributed talks and poster presentations.

List of Plenary Talks

- Başak Gürel (University of Central Florida), *From Hamiltonian systems with infinitely many periodic orbits to pseudo-rotations via symplectic topology.*
- Kathryn Hess (École Polytechnique Fédérale de Lausanne, EPFL SV BMI UPHESS), *What does topology have to do with neuroscience?*
- Ann Lemahieu (Laboratoire de Mathématiques J. A. Dieudonné), *On the monodromy conjecture for nondegenerate hypersurface singularities.*
- Marta Macho-Stadler (Universidad del País Vasco-Euskal Herriko Unibertsitatea), *Sesgos de género en la Academia: cuando las matemáticas no funcionan.*
- Catherine Meusburger (FAU Erlangen-Nürnberg), *Ideal tetrahedra and their duals.*
- Emmy Murphy (Northwestern University), *The Koras-Russel cubic and Weinstein flexibility.*
- Rita Pardini (Università di Pisa), *Deformations of semi-smooth varieties.*
- M. Eugenia Rosado Mar a (Universidad Politécnica de Madrid), *Second-order Lagrangians admitting a first-order Hamiltonian formalism.*
- Lidia Stoppino (Università degli Studi di Pavia), *Clifford-Severi inequalities for varieties of maximal Albanese dimension.*
- Ulrike Tillman (University of Oxford), *Geometric groups via homotopy theory.*
- Carme Torras (Institut de Robòtica i Informàtica Industrial (CSIC-UPC)), *Cloth manipulation in assistive robotics: Research challenges, ethics and fiction.*

List of Contributed Talks

- Daria Alekseeva (National Research University Higher School of Economics), *Presentations of symplectic mapping class group of rational 4-manifolds.*
- Patricio Almirón (Universidad Complutense de Madrid), *Milnor and Tjurina, a 4/3 relation.*
- Guillem Blanco Fernandez (Universitat de Politècnica de Catalunya), *Yano's conjecture.*
- Melanie Bondorevsky (Universidad de Buenos Aires & IMAS-CONICET), *Topological degree and periodic orbits of semi-dynamical systems.*
- Robert Cardona (Universitat de Politècnica de Catalunya), *A contact topology approach to Euler flows universality.*
- Joana Cirici (Universitat de Barcelona), *Hodge theory of almost Kähler manifolds.*
- Franco Coltraro (Institut de Robòtica i Informàtica Industrial, CSIC-UPC), *Collisions and friction for inextensible cloth simulatio.*
- Aina Ferrà Marcús (Universitat de Barcelona), *Localizations of models of theories with arities.*
- Marina Garrote-López (Universitat de Politècnica de Catalunya), *Distance to the stochastic part of phylogenetic varieties.*

- Jordi Gaset (Universitat de Politècnica de Catalunya), *A contact geometry approach to symmetries in systems with dissipation.*
- Debora Gil (Computer Vision Center and Computer Science Department at Universitat Autonoma de Barcelona), *Topological Radiomics (TOPiomics): Early Detection of Genetic Abnormalities in Cancer Treatment Evolution.*
- Matias V. Moya Giusti (Université Paris-Est), *Dimension formulas for the cohomology of arithmetic groups.*
- Cédric Oms (Universitat de Politècnica de Catalunya), *Do overtwisted contact manifolds admit infinitely many periodic Reeb orbits?*
- Sinem Onaran (Hacettepe University), *Legendrian knots in contact 3-manifolds.*
- Maryam Samavaki (University of Eastern Finland), *On several classes of Ricci tensor.*
- Julia Semikina (University of Bonn), *G-theory of group rings for finite groups.*
- Paola Supino (Roma Tre University), *On complete intersections containing a linear subspace.*
- M. Pilar Vélez (Universidad Antonio de Nebrija), *Automated proving and discovery in elementary Geometry by means of algebraic geometry.*

List of Poster Presentations

- Joaquim Brugués (Universitat de Politècnica de Catalunya), *Towards a Floer homology for singular symplectic manifolds.*
- Luciana Bonatto (University of Oxford), *Decoupling in Higher Dimensions.*
- Marta Mazzocco (Universitat de Birmingham), *Poisson Structures on Painlevé Monodromy Manifolds.*
- Pau Mir (Universitat de Politècnica de Catalunya), *Invariants in Semitoric Integrable Systems. Looking for a new interpretation.*
- Inasa Nakamura (Institute of Science and Engineering, Kanazawa University), *Branched covering surfaces in 4-space and simplifying numbers.*

We are very happy to attest that the atmosphere created by the participants of the workshop was very open and friendly, and we hope that it led to effective further collaborations.

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Contents

$\mathbb{Q}$-Hilbert Functions of Multiplier and Test Ideals

Josep Àlvarez Montaner and Luis Núñez-Betancourt

Abstract This is an extended abstract with some of the results that will appear in the forthcoming paper [3] in which we prove the rationality of the Poincaré series associated to multipliers and test ideals as long as we have discreteness and rationality of the corresponding jumping numbers and Skoda's theorem is available. In order to do so we extend the theory of Hilbert functions to the case of filtrations indexed over the rational numbers.

1 Introduction

Let A be a commutative Noetherian ring containing a field $\mathbb{K}$. Assume that A is either local or graded with maximal ideal $\mathfrak{m}$ and let $\mathfrak{a}$ be an $\mathfrak{m}$-primary ideal. Depending on the characteristic of the base field we may find two parallel sets of invariants associated to the pair $(A, \mathfrak{a}^c)$ where c is a real parameter. In characteristic zero we have the theory of *multiplier ideals* which play a prominent role in birational geometry and are defined using resolution of singularities (see [12] for more insight). In positive characteristic we may find the so-called *test ideals* which originated from the theory of tight closure [10, 11] and are defined using the Frobenius endomorphism. Despite its different origins, it is known that under some conditions on A, the reduction mod p of a multiplier ideal is the corresponding test ideal. Moreover, both theories share

JAM is partially supported by Generalitat de Catalunya 2017SGR-932 project and Spanish Ministerio de Economía y Competitividad MTM2015-69135-P. LNB is partially supported by CONACYT Grant 284598 and Cátedras Marcos Moshinsky.

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M. Alberich-Carramiñana et al. (eds.), *Extended Abstracts GEOMVAP 2019*, Trends in Mathematics 15,
https://doi.org/10.1007/978-3-030-84800-2_1

a lot of common properties which we summarize as saying that they form a filtration of $\mathfrak{m}$-primary ideals

$$\mathcal{J}: \quad A \supsetneq \mathcal{J}_{\alpha_1} \supsetneq \mathcal{J}_{\alpha_2} \supsetneq \cdots \supsetneq \mathcal{J}_{\alpha_i} \supsetneq \cdots$$

and the indices where there is an strict inequality is a discrete set of rational numbers. Following the ideas of [8] we define the *multiplicity* of $c \in \mathbb{R}_{>0}$ as $m(c) = \dim_{\mathbb{K}} (\mathcal{J}_{c-\varepsilon}/\mathcal{J}_c)$, for $\varepsilon > 0$ small enough, and the *Poincaré series* of $\mathcal{J}$ as

$$P_{\mathcal{J}}(T) = \sum_{c \in \mathbb{R}_{>0}} \dim_{\mathbb{K}} (\mathcal{J}_{c-\varepsilon}/\mathcal{J}_c) \; T^c.$$

The natural question is whether this is a rational function, in the sense that it belongs to the field of fractional functions $\mathbb{Q}(z)$ where the indeterminate z corresponds to a fractional power $T^{1/e}$ for a suitable $e \in \mathbb{N}_{>0}$. The only known results on this question were given in [9] and [1, 2] where the authors proved the rationality of the Poincaré series of multiplier ideals in rings of dimension two by giving an explicit formula for the multiplicities.

The approach that we are going to give is completely algebraic and will provide an unified proof of the rationality of the Poincaré series for both the multiplier and the test ideals in any dimension. To such purpose we will develop a theory of Hilbert functions indexed over $\mathbb{Q}$.

2 $\mathbb{Q}$-Good Filtrations

Let A be a commutative Noetherian ring. Assume that A is either local or graded with maximal ideal $\mathfrak{m}$ and let $\mathfrak{a}$ be an $\mathfrak{m}$-primary ideal. The theory of *good* $\mathfrak{a}$*-filtrations* gives an approach to the study of Hilbert functions that covers most of the classical results in an unified way. We start recalling briefly this notion but we refer to the monograph [13] and the references therein for more insight.

Let M be a finitely generated A-module such that $\lambda\,(M/\mathfrak{a}M) < \infty$, where $\lambda(\cdot)$ denotes the length as A-module. A *good* $\mathfrak{a}$*-filtration* on M is a decreasing filtration

$$\mathcal{M}: \quad M = M_0 \supseteq M_1 \supseteq \cdots$$

by A-submodules of M such that $M_{j+1} = \mathfrak{a}M_j$ for $j >> 0$ large enough. Under these premises we may consider the *Hilbert* and the *Hilbert–Samuel function* of M with respect to the filtration $\mathcal{M}$ defined as

$$H_{\mathcal{M}}(j) := \lambda\left(M_j/M_{j+1}\right) \quad \text{and} \quad H^1_{\mathcal{M}}(j) := \lambda\left(M/M_j\right)$$

respectively. Moreover, we consider the *Hilbert* and the *Hilbert–Samuel series*

$$HS_{\mathcal{M}}(T) := \sum_{j\geq 0} \lambda\left(M_j/M_{j+1}\right) T^j \quad \text{and} \quad HS^1_{\mathcal{M}}(T) := \sum_{j\geq 0} \lambda\left(M/M_j\right) T^j.$$

Notice that we have $HS_{\mathcal{M}}(T) = (1-T)HS^1_{\mathcal{M}}(T)$. As a consequence of the Hilbert–Serre theorem we can express them as rational functions

$$HS_{\mathcal{M}}(T) = (1-T)HS^1_{\mathcal{M}}(T) = (1-T)\frac{h_{\mathcal{M}}(T)}{(1-T)^{d+1}},$$

where $h_{\mathcal{M}}(T) \in \mathbb{Z}[T]$ satisfies $h_{\mathcal{M}}(1) \neq 0$ and d is the Krull dimension of M. The polynomial $h_{\mathcal{M}}(T)$ is the *h-polynomial* of $\mathcal{M}$.

The aim of this section is to extend the notion of good $\mathfrak{a}$-filtrations by allowing filtrations indexed over $\mathbb{Q}$ and thus mimicking properties satisfied by filtrations given by multiplier and test ideals.

Definition 1 Let M be a finitely generated A-module such that $\lambda(M/\mathfrak{a}M) < \infty$. A $\mathbb{Q}$-good $\mathfrak{a}$-filtration is a decreasing filtration $\mathcal{M} := \{M_\alpha\}_{\alpha\geq 0}$ of submodules of $M_0 = M$, indexed by a discrete set of positive rational numbers such that $M_{\alpha+1} = \mathfrak{a}M_\alpha$ for all $\alpha > j$ with $j >> 0$ large enough.

Indeed, we may think of $\mathcal{M}$ as a filtration of submodules M_c indexed over the real numbers for which there exist an increasing sequence of rational numbers $0 < \alpha_1 < \alpha_2 < \cdots$ such that $M_{\alpha_i} = M_c \supsetneq M_{\alpha_{i+1}}$ for any $c \in [\alpha_i, \alpha_{i+1})$. In particular we have a discrete filtration of submodules

$$\mathcal{M}: \quad M \supsetneq M_{\alpha_1} \supsetneq M_{\alpha_2} \supsetneq \cdots \supsetneq M_{\alpha_i} \supsetneq \cdots$$

and we say that the α_i are the *jumping numbers* of $\mathcal{M}$. A crucial observation is that, once we fix an index $c \in \mathbb{R}$, the filtration

$$\mathcal{M}_c: \quad M_c \supseteq M_{c+1} \supseteq M_{c+2} \supseteq \cdots$$

is a good $\mathfrak{a}$-filtration.

Definition 2 Let $\mathcal{M} := \{M_c\}_{c\geq 0}$ be a $\mathbb{Q}$-good $\mathfrak{a}$-filtration. We define the multiplicity of $c \in \mathbb{R}_{>0}$ as

$$m(c) := \lambda\left(M_{c-\varepsilon}/M_c\right)$$

for $\varepsilon > 0$ small enough. Clearly, c is a jumping number if and only if $m(c) > 0$.

Definition 3 Let $\mathcal{M} := \{M_c\}_{c\geq 0}$ be a $\mathbb{Q}$-good $\mathfrak{a}$-filtration. We define the Poincaré series of $\mathcal{M}$ as

$$P_{\mathcal{M}}(T) = \sum_{c\in\mathbb{R}_{>0}} m(c) \quad T^c.$$

The question that we want to address is whether the Poincaré series is rational in the sense that it belongs to the field of fractional functions $\mathbb{Q}(T^{1/e})$ where $e \in \mathbb{N}_{>0}$ is the least common multiple of the denominators of all the jumping numbers.

Proposition 4 *Let $\mathcal{M} := \{M_c\}_{c\geq 0}$ be a $\mathbb{Q}$-good $\mathfrak{a}$-filtration. Given $c \in \mathbb{R}_{>0}$ we have that*

$$\sum_{j\geq 0} m(c+j)T^j$$

is a rational function in $\mathbb{Q}(T^{1/e})$.

Theorem 5 *Let $\mathcal{M} := \{M_c\}_{c\geq 0}$ be a $\mathbb{Q}$-good $\mathfrak{a}$-filtration. Then, the Poincaré series $P_{\mathcal{M}}(T)$ is rational. Indeed we have*

$$P_{\mathcal{M}}(T) = \sum_{c\in(0,1]} \left(\frac{m(c)}{1-T} + \frac{h_{\mathcal{M}_c}(T) - h_{\mathcal{M}_{c-\varepsilon}}(T)}{(1-T)^{d+1}} \right) T^c$$

where $h_{\mathcal{M}_c}(T)$ is the h-polynomial associated to $\mathcal{M}_c$ and $d = \dim A$.

3 Poincaré Series of Multiplier and Test Ideals

In this Section we specialize the results obtained above to the case of multiplier and test ideals.

3.1 Multiplier Ideals

Let $(A, \mathfrak{m})$ be a normal local ring essentially of finite type over an algebraically closed field $\mathbb{K}$ of characteristic zero and $\mathfrak{a} \subseteq A$ an $\mathfrak{m}$-primary ideal. Under these general assumptions we ensure the existence of canonical divisors K_X on $X = \operatorname{Spec} A$ which are not necessarily $\mathbb{Q}$-Cartier. Then we may find some effective boundary divisor Δ such that $K_X + \Delta$ is $\mathbb{Q}$-Cartier with index m large enough. Now, given a *log-resolution* $\pi : X' \to X$ of the triple $(X, \Delta, \mathfrak{a})$ we pick a canonical divisor $K_{X'}$ in X' such that $\pi^* K_{X'} = K_X$ and let F be an effective divisor such that $\mathfrak{a} \cdot \mathcal{O}_{X'} = \mathcal{O}_{X'}(-F)$.

The *multiplier ideal* associated to the triple $(X, \Delta, \mathfrak{a}^c)$ for some real number $c \in \mathbb{R}_{>0}$ is defined as

$$\mathcal{J}(X, \Delta, \mathfrak{a}^c) = \pi_* \mathcal{O}_{X'} \left(\left\lceil K_{X'} - \frac{1}{m}\pi^*(m(K_X + \Delta)) - cF \right\rceil \right).$$

This construction allowed de Fernex and Hacon [7] to define the multiplier ideal $\mathcal{J}(\mathfrak{a}^c)$ associated to $\mathfrak{a}$ and c as the unique maximal element of the set of multiplier ideals $\mathcal{J}(X, \Delta, \mathfrak{a}^c)$ where Δ varies among all the effective divisors such that $K_X + \Delta$ is $\mathbb{Q}$-Cartier. The key point proved in [7] is the existence of such a divisor Δ that realizes the multiplier ideal as $\mathcal{J}(\mathfrak{a}^c) = \mathcal{J}(X, \Delta, \mathfrak{a}^c)$.