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Jesús T. Pastor
Juan Aparicio
José L. Zofío

Benchmarking Economic Efficiency

Technical and Allocative Fundamentals

Springer

International Series in Operations Research & Management Science

Volume 315

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Jesús T. Pastor
Center of Operations Research (CIO)
Universidad Miguel Hernandez de Elche
Elche, Alicante, Spain

Juan Aparicio
Center of Operations Research (CIO)
Universidad Miguel Hernandez de Elche
Elche, Alicante, Spain

José L. Zofío
Department of Economics
Universidad Autónoma de Madrid
Madrid, Spain

ISSN 0884-8289 ISSN 2214-7934 (electronic)
International Series in Operations Research & Management Science
ISBN 978-3-030-84396-0 ISBN 978-3-030-84397-7 (eBook)
https://doi.org/10.1007/978-3-030-84397-7

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

To Alberto, Irene, Dieguito, and Blanca,
Diego, Alba, and Teseo, and Mercedes.

To Julia, Aitana, Hugo, and Marta

To Irene, Silvia, and Paula

Preface

This book is the result of the theoretical and empirical research that the authors have undertaken throughout their careers on the topic of economic and productive efficiency measurement. This has been a recursive research topic since the last quarter of the twentieth century, which started with Michael Farrell's seminal work on decomposing economic efficiency according to technical and allocative criteria. Our text offers a comprehensive account of the microeconomic foundations of the decomposition of cost, revenue, profitability, and profit efficiency using a wide range of technical efficiency models that result in distinct multiplicative and additive approaches.

The guiding framework is that of duality theory, relating the equivalent representation of firms' behavior either through the production technology or the aforementioned economic functions. This theoretical construction relates a (dual) representation of the economic behavior of the firm in terms of a supporting function with a (primal) characterization on the production technology. Under the assumption of producers behaving as price takers, their technologies could be equivalently described by dual cost, revenue, profitability, or profit functions. This simply states that thanks to duality theory we can mathematically recover the primal and dual representations of firms' behavior from each other, provided that some axioms or assumptions are satisfied, that is, regularity conditions, for example, convexity. As firms produce multiple outputs using multiple inputs, the primal representation of the technology relies on the technical efficiency concept, or, more generally, the mathematical notion of distance function. In a production context, duality has witnessed a revival in business economics as it represents the essential cornerstone in the benchmarking of firms through frontier analysis. The idea can be summarized in a simple way. Economic efficiency is defined as the gap between a maximum attainable economic goal and that which is actually achieved by a firm under evaluation (e.g., maximum profit versus observed profit), and this difference can be attributed to technical inefficiencies related to engineering shortcomings (in the quantity space) and allocative inefficiency related to market mismanagement practices (including the price space). Duality theory allows to decompose economic inefficiency in these two

mutually exclusive components and thereby identify the sources of a suboptimal economic behavior. In the benchmarking process, it provides guidance for understanding what is wrong within the firm when compared to its competitors. Moreover, the existence of many dual relationships between particular distance functions (input, output, generalized, directional, Hölder) and their supporting economic functions (cost, revenue, profit and profitability) offers the researcher the possibility of choosing which perspective of the firm is best suited for the analysis, depending on the specificities of the study at hand.

In this text, we are not only concerned about the theoretical underpinnings of economic efficiency, but we also pay attention to the practical side of efficiency measurement. With this aim in mind, we present, and introduce when necessary, the different formulations that allow implementing the different models through mathematical programming techniques known as data envelopment analysis (DEA). We present the optimization programs that solve both the classic and the new models. We illustrate the different models with straightforward examples and a real-life common dataset. This allows us to show that results vary depending on the specific model chosen by the analyst and that different answers to the benchmarking exercise may be obtained depending on the alternative characterizations of the economic goal and the technology. For example, whether the distance function can capture individual inefficiencies related to particular inputs or outputs (e.g., additive versus multiplicative models), how the reference efficient subset on the technological frontier is defined in terms of returns to scale, and the existence of strong or weak disposability. For each distance function definition and its associated economic efficiency DEA model, we discuss its relative pros and cons in terms of their economic interpretation, flexibility, and ability to capture all sources of (in)efficiency.

Moreover, the different examples and empirical applications are solved using a set of functions coded in the suitable and open environment represented by the Julia language. This set of functions is available to practitioners in the form of a self-contained package, allowing them to undertake research on their own without having to program the models by themselves. Practitioners can edit and change the specific functions, adapting the code according to their needs. The software is open source and is freely available at its dedicated site: www.benchmarkingeconomicefficiency.com. For most of the models it relies on linear optimization, but it also makes use of advanced computational methods of non-linear programming, including, among others, second-order cone programming (SOCP) and quadratic optimization methods linked to the use of special ordered sets (SOS). Using this toolbox and a collection of specific Jupiter notebooks associated with each chapter, we illustrate the practice of economic efficiency measurement following a step-by-step approach. This allows us to illustrate how different models lead to alternative decompositions of economic efficiency. Ultimately, our goal is to provide guidance on the best alternatives by taking into consideration the set of desirable properties that economic efficiency models should satisfy.

Most of the chapters draw from publications by the authors in field journals at the intersection of management science, economics, and operations research: *Omega-*

The International Journal of Management Science, *European Journal of Operational Research*, *Journal of Optimization Theory and Applications*, *Journal of Productivity Analysis*, *Economic Theory*, and *Journal of the Operational Research Society*, among others. In each of these chapters, we present a particular model of interest, including the details of the mathematical proofs and relevant examples that highlight its characteristics and how it compares to previous proposals in the literature. We are grateful to successive area editors that have promoted and managed the publications of relevant chapters in the field over the years, including Robert Dyson, Joe Zhu, William Greene, Robin Sickles, Knox Lovell, and many others. We have also benefited from regular conference meetings such as the *European Workshop on Efficiency and Productivity Analysis* and the *North American Productivity Workshop*, organized thanks to efforts of many members of its current partner society, the International Society for Efficiency and Productivity Analysis (ISEPA). We are grateful to participants at these conferences as well as to attendees at several workshops and seminars worldwide where we have presented the models included in the book.

We are intellectually indebted to the innovators whose contributions have been a constant guidance and source of inspiration to our research. Besides Farrell's original contribution, we have also been influenced by the work of Gérard Debreu, Tjalling Koopmans, and Ronald Shephard in the field of economics, and Abraham Charnes and William Cooper in the area of operations research. Authors whose studies encouraged later developments by many other scholars are intellectually responsible for the thriving state of the discipline. Our book is also intended to continue the path marked by both classic and authoritative texts on production economics, duality theory, and economic efficiency. In the 1970s, these were represented by the second volume of the series Frontiers of Quantitative Economics, edited by Michael Intriligator and David Kendrick—particularly the chapter by Erwin Diewert, as well as Daniel McFadden's book, along with Melvin Fuss, *Production Economics: A Dual Approach to Theory and Applications*. In the 1980s, the *Measurement of Efficiency of Production* by Rolf Färe, Shawna Grosskopf, and C. A. Knox Lovell, who quite surprisingly made no reference to duality theory, and the more accessible text *Applied Production Analysis*: *A Dual Approach*, by Robert Chambers. In the 1990s, the concise *Multi-Output Production and Duality* by Rolf Färe and Daniel Primont ably summarized the state of the art on economic efficiency measurement based on duality theory, while Bert Balk followed suit focusing on index number theory with his *Industrial Price*, *Quantity, and Productivity Indices*. These contributions constitute a clear timeline in the discipline of economic efficiency measurement, which we intend to bring up to date with the latest research in the field. The references included in the bibliography bear witness of the exponential growth of interest that these methods have drawn among scholars, researchers, and practitioners in the field.

Our work has benefited from these and other outstanding personalities in the area, some of which we mention again now as coauthors to whom we owe stimulus and motivation. It has been a pleasure to collaborate with them throughout the years. In particular Bert Balk, William Cooper, Knox Lovell, Subhash Ray, and Joe Zhu. The

association with these authors has resulted in the edition of two relevant volumes, entitled *Advances in Efficiency and Productivity*, published in 2016 and 2020 by Springer Nature in its International Series in Operations Research & Management Science. Many ideas presented in the chapters of these books have a direct relation with our text, and we are grateful to the authors for their contributions.

On the Spanish side, it would not have been possible to complete the book without the constant support and understanding of our colleagues from the Center for Operations Research at Miguel Hernandez University of Elche (Alicante), and from the Economics Department at Universidad Autónoma de Madrid. Among former students, who are now outstanding colleagues, we would like to acknowledge Javier Barbero, currently economic analyst at the Directorate for Growth & Innovation in the European Commission-Joint Research Centre (JRC), for providing constant and invaluable support in the computational implementation of the models, as well as for compiling the set of functions in *Julia,* accompanying the book.

We thank Joe Zhu as area editor of Springer Nature's International Series in Operations Research & Management Science for his continuous support, patience, and understanding. We are also grateful to Matthew Amboy as the original senior editor for business and economics, as well as Maria David, the designated manager for this project. The authors thank the financial support from the Spanish Ministry of Science and Innovation and the State Research Agency under grant PID2019-105952GB-I00/ AEI / 10.13039/501100011033. J. L. Zofío also thanks the support from the Spanish Ministry of Science and Innovation and the State Research Agency under grant EIN2020-112260. Additionally, we acknowledge continuous support from the Spanish State Research Agency under several successive grants (MTM2013-43903-P, MTM2016-79765-P).

Last but not least, a few words of gratitude and encouragement to our readers interested in the measurement and decomposition of economic efficiency. At the time of writing the book, we intended it to be as accessible as possible for non-experts, so we included material covering the basics of the discipline. However, after the initial chapters devoted to this purpose, we delve into the most recent and advanced results offered in this active area of the efficiency and productivity literature. We also provide notebooks to solve the common examples aimed at facilitating the use of the accompanying software, so readers can undertake economic efficiency analysis on their own. In this respect, we thank you in anticipation for any comments or suggestions aimed at improving the text, the "hardware" of this project, as well as the software and learning material accompanying it.

Elche, Alicante, Spain — Jesús T. Pastor
Juan Aparicio
Madrid, Spain — José L. Zofío

Contents

1 Introduction 1
1.1 The Elements of Economic Efficiency Analysis: Markets, Management, and Production 1
1.2 Benchmarking Economic Performance: Multiplicative and Additive Approaches 4
1.3 Organization of the Book 11
1.4 Objectives of the Book 17

2 Conceptual Background: Firms' Objectives, Decision Variables, and Economic Efficiency 19
2.1 Introduction 19
2.2 The Technology: Input, Output, and Graph Technical Efficiency Measures 22
2.2.1 Input Technical (In)Efficiency Measures: Multiplicative and Additive Definitions 26
2.2.2 Output Technical (In)Efficiency Measures: Multiplicative and Additive Definitions 30
2.2.3 Graph Technical (In)Efficiency Measures: Multiplicative and Additive Definitions 33
2.2.4 Properties of Technical (In)Efficiency Measures: Multiplicative and Additive 39
2.3 Economic Behavior and Economic Efficiency 43
2.3.1 Cost Minimization and Cost (In)Efficiency 43
2.3.2 Revenue Maximization and Revenue (In)Efficiency 46
2.3.3 Profitability Maximization and Profitability Efficiency 48
2.3.4 Profit Maximization and Profit Inefficiency 50

2.3.5 Properties of Economic (In)Efficiency Measures: Multiplicative and Additive 52
2.4 Duality and the Decomposition of Economic Efficiency into Technical and Allocative Components: The Essential Properties of Allocative Efficiency 54
2.4.1 Decomposing Cost (In)Efficiency 55
2.4.2 Decomposing Revenue (In)Efficiency 62
2.4.3 Decomposing Profitability Efficiency 68
2.4.4 Decomposing Profit Inefficiency 76
2.4.5 An Essential Property for the Decomposition of Economic (In)Efficiency: Multiplicative and Additive 84
2.5 Data Envelopment Analysis Methods 87
2.5.1 The Production Technology 89
2.5.2 Calculating Technical (In)Efficiency Measures 94
2.5.3 Calculating and Decomposing Economic (In) Efficiency 98
2.6 Introducing a Free "Julia" Package to Calculate and Decompose Economic Efficiency Using DEA 99
2.6.1 Installing the Benchmarking Economic (In)Efficiency Julia Package 101
2.6.2 Examples and Empirical Data 102
2.6.3 Data Structures: Reading and Reporting Results 105
2.7 Summary and Conclusions 110

Part I Benchmarking Economic Efficiency: The Multiplicative Approach

3 Shephard's Input and Output Distance Functions: Cost and Revenue Efficiency Decompositions 115
3.1 Introduction 115
3.2 The Input and Output Correspondences: Shephard's Radial Distance Functions 117
3.2.1 Pareto-Koopmans Efficiency and Input and Output Disposability 121
3.2.2 Calculating Radial Technical Efficiency Using Data Envelopment Analysis 123
3.3 Economic Behavior and Cost and Revenue Efficiencies 130
3.3.1 Cost Minimization and Cost Efficiency 131
3.3.2 Revenue Maximization and Revenue Efficiency 132
3.3.3 Calculating Minimum Cost and Maximum Revenue Using Data Envelopment Analysis 133
3.4 Duality and the Decomposition of Economic Efficiency as the Product of Technical and Allocative Efficiencies 134
3.4.1 Decomposing Cost Efficiency 135

3.4.2 Decomposing Revenue Efficiency 141
3.4.3 Decomposing Cost and Revenue Efficiency Under Non-homothetic Technologies 148
3.5 Empirical Illustration of the Radial Cost and Revenue Efficiency Models 153
3.5.1 The Radial Cost Efficiency Model 154
3.5.2 The Radial Revenue Efficiency Model 159
3.5.3 An Application: Taiwanese Banking Industry 161
3.6 Summary and Conclusions 165

4 The Generalized Distance Function (GDF): Profitability Efficiency Decomposition 167
4.1 Introduction 167
4.2 The Generalized Distance Function: Productive, Technical, and Scale Efficiencies 170
4.2.1 Defining Productive, Technical, and Scale Efficiencies 174
4.2.2 Pareto-Koopmans Efficiency and Input and Output Disposability 176
4.2.3 Calculating the Generalized Distance Function Using Data Envelopment Analysis 178
4.3 Economic Behavior and Profitability Efficiency 186
4.3.1 Calculating Maximum Profitability Using Data Envelopment Analysis 189
4.4 Duality and the Decomposition of Profitability Efficiency as the Product of Technical, Scale, and Allocative Efficiencies 190
4.4.1 Duality Between the Technology Set and the Profitability Function 191
4.4.2 Duality Between the Generalized Distance Function and the Profitability Function 194
4.4.3 Calculating and Decomposing Profitability Efficiency 200
4.5 Empirical Illustration of the Profitability Efficiency Model . . . 201
4.5.1 An Application to the Taiwanese Banking Industry . . 206
4.6 Summary and Conclusions 210

Part II Benchmarking Economic Efficiency: The Additive Approach

5 The Russell Measures: Economic Inefficiency Decompositions . . . 215
5.1 Introduction 215
5.2 The Russell Graph Measure of Technical Efficiency and the Decomposition of Profit Inefficiency 217
5.3 The Russell Input Measure of Technical Efficiency and the Decomposition of Cost Inefficiency 224

5.4 The Russell Output Measure of Technical Efficiency and the Decomposition of Revenue Inefficiency 231
5.5 Empirical Illustration of the Russell Profit, Cost, and Revenue Inefficiency Models 233
5.5.1 The Russell Profit Inefficiency Model 233
5.5.2 The Russell Cost Inefficiency Model 236
5.5.3 The Russell Revenue Inefficiency Model 239
5.5.4 An Application to the Taiwanese Banking Industry 241
5.6 Summary and Conclusions 243

6 The Weighted Additive Distance Function (WADF): Economic Inefficiency Decompositions 245
6.1 Introduction 245
6.2 The Weighted Additive Distance Function and the Decomposition of Profit Inefficiency 248
6.3 The Input-Oriented WADF and the Decomposition of Cost Inefficiency 256
6.4 The Output-Oriented WADF and the Decomposition of Revenue Inefficiency 264
6.5 Empirical Illustration of the Weighted Additive Profit, Cost, and Revenue Inefficiency Models 266
6.5.1 The WADF Profit Inefficiency Model 268
6.5.2 The WADF Cost Inefficiency Model 270
6.5.3 The WADF Revenue Inefficiency Model 273
6.5.4 An Application to the Taiwanese Banking Industry 276
6.6 Summary and Conclusions 278

7 The Enhanced Russell Graph Measure (*ERG=SBM*): Economic Inefficiency Decompositions 279
7.1 Introduction 279
7.2 Formulation, Solution, and Properties of the Graph ERG=SBM 281
7.2.1 Formulating the *ERG=SBM* as a Linear Fractional Model 281
7.2.2 Solving the *ERG=SBM* 283
7.2.3 Basic Properties of the *ERG=SBM* 284
7.3 The Graph *ERG=SBM* and the Decomposition of Profit Inefficiency 286
7.4 The Input-Oriented *ERG=SBM* and the Decomposition of Cost Inefficiency 295
7.5 The Output-Oriented ERG=SBM and the Decomposition of Revenue Inefficiency 299
7.6 Empirical Illustration of the ERG=SBM Profit Inefficiency Model 304

7.6.1 An Application to the Taiwanese Banking Industry . . 307
7.7 Summary and Conclusions 309

8 The Directional Distance Function (DDF): Economic Inefficiency Decompositions 311
8.1 Introduction 311
8.2 The Directional Distance Functions: Orientation, Calculation, and Properties 314
8.2.1 Graph, Input, and Output Directional Distance Functions 314
8.2.2 Calculating the Directional Distance Functions Using Data Envelopment Analysis 317
8.2.3 Characterizing the Technical Inefficiency of Firms Through the DDF 318
8.2.4 Properties of the Directional Distance Function 320
8.3 Exogenous and Endogenous Directional Vectors, DVs 322
8.3.1 The Family of Exogenous DVs 322
8.3.2 The Family of Endogenous DVs 326
8.4 The Graph DDFs and the Decomposition of Profit Inefficiency 327
8.5 The Input-Oriented DDFs and the Decomposition of Cost Inefficiency 331
8.6 The Output-Oriented DDFs and the Decomposition of Revenue Inefficiency 335
8.7 A Price-Based Method for Comparing DDF Inefficiencies based on the Normalization of the DVs 338
8.7.1 A Procedure for Normalizing the DVs 339
8.8 Empirical Illustration of the DDF Profit, Cost, and Revenue Inefficiency Models 341
8.8.1 The Graph DDF Profit Inefficiency Model 343
8.8.2 The Input-Oriented DDF Cost Inefficiency Model . . . 346
8.8.3 The Output-Oriented Revenue Inefficiency Model . . 348
8.8.4 An Application to the Taiwanese Banking Industry . . 351
8.9 Summary and Conclusions 353

9 The Hölder Distance Functions: Economic Inefficiency Decompositions 355
9.1 Introduction 355
9.2 The Weakly and Strongly Efficient Graph Hölder Distance Functions 357
9.3 The Hölder Distance Functions and the Decomposition of Profit Inefficiency 363
9.3.1 Decomposing Profit Inefficiency Based on the Weakly Efficient Hölder Distance Functions 363

9.3.2 Decomposing Profit Inefficiency Based on the Strongly Efficient Hölder Distance Functions 367
9.4 The Input-Oriented Hölder Distance Functions and the Decomposition of Cost Inefficiency 370
9.5 The Output-Oriented Hölder Distance Functions and the Decomposition of Revenue Inefficiency 377
9.6 Empirical Illustration of the Hölder Profit, Cost, and Revenue Inefficiency Models 384
9.6.1 The ℓ_2 Hölder Profit Inefficiency Model 386
9.6.2 The ℓ_1 Hölder Cost Inefficiency Model 389
9.6.3 The ℓ_∞ Hölder Revenue Inefficiency Model 391
9.6.4 An Application to the Taiwanese Banking Industry 394
9.7 Summary and Conclusions 396

10 The Loss Distance Function: Economic Inefficiency Decompositions 399
10.1 Introduction 399
10.2 The Graph Loss Distance Function and the Decomposition of Profit Inefficiency 402
10.3 The Input-Oriented Loss Distance Function and the Decomposition of Cost Inefficiency 407
10.4 The Output-Oriented Loss Distance Function and the Decomposition of Revenue Inefficiency 411
10.5 Summary and Conclusions 413

11 The Modified Directional Distance Function (MDDF): Economic Inefficiency Decompositions 415
11.1 Introduction 415
11.2 The Modified Directional Distance Function 416
11.3 Duality and the Decomposition of the Lost Profit on Outlay 419
11.4 Empirical Illustration of the Modified DDF Profit Inefficiency Model 425
11.4.1 An Application to the Taiwanese Banking Industry 428
11.5 Summary and Conclusions 431

12 The Reverse Directional Distance Function (RDDF): Economic Inefficiency Decompositions 433
12.1 Introduction 433
12.2 The Reverse Directional Distance Function $RDDF\left(EM^S, F_J, \widehat{F}_J\right)$ Associated with the Efficiency Measure Triad $\left(EM^S, F_J, \widehat{F}_J\right)$ 436

12.3 Improving the Profit Inefficiency Decomposition of the Graph Efficiency Measure $\left(EM^S(G), F_J, \widehat{F}_J\right)$ Resorting to Its *RDDF* 448
12.4 Improving the Cost Inefficiency Decomposition of $\left(EM^S(I), F_J, \widehat{F}_J\right)$ Resorting to Its $RDDF\left(EM^S(I), F_J, \widehat{F}_J\right)$ . . 456
12.5 Improving the Revenue Inefficiency Decomposition of $\left(EM^S(O), F_J, \widehat{F}_J\right)$ Resorting to Its $RDDF\left(EM^S(O), F_J, \widehat{F}_J\right)$ 460
12.6 Introducing the Bidirectional Distance Functions $\left(BDF, F_J, \widehat{F}_J\right)$: Deriving for Each $\left(BDF^S, F_J, \widehat{F}_J\right)$ Its Reverse Directional Distance Function $RDDF\left(BDF^S, F_J, \widehat{F}_J\right)$ 463
12.6.1 The Bidirectional Distance Function $\left(BDF, F_J, \widehat{F}_J\right)$ 464
12.6.2 Defining the RDDF Associated with a Bidirectional Distance Function $\left(BDF^S, F_J, \widehat{F}_J\right)$ 466
12.7 Empirical Illustration of the RDDF Profit, Cost, and Revenue Inefficiency Models 468
12.7.1 The RDDF (ERG = SBM) Profit Inefficiency Model 470
12.7.2 The RDDF (Russell) Cost Inefficiency Model 473
12.7.3 The RDDF (Russell) Revenue Inefficiency Model . . . 477
12.7.4 An Application to the Taiwanese Banking Industry 480
12.8 Summary and Conclusions 482

Part III New Approaches to Decompose Economic Efficiency

13 A Unifying Framework for Decomposing Economic Inefficiency: The General Direct Approach and the Reverse Approaches 487
13.1 Introduction 487
13.2 Decomposing Profit Inefficiency 490
13.2.1 The Traditional Approach Based on a Specific Graph Efficiency Measure 490
13.2.2 The General Direct Approach Based on a Specific Graph Measure 492
13.2.3 Profit Inefficiency Decompositions Based on a Traditional Approach and on the General Direct Approach: A Comparison, a Numerical Example, and Some Properties of the Direct Approach 502

13.2.4 The Exceptional Case of the Directional Graph Distance Function . . . 511
13.2.5 The Graph Reverse Approaches for Profit Decompositions . . . 513
13.3 Decomposing Cost Inefficiency . . . 532
13.3.1 The Traditional Approaches Based on Input-Oriented Efficiency Measures . . . 532
13.3.2 The General Direct Approach Based on a Specific Input-Oriented Efficiency Measure . . . 534
13.3.3 Cost Inefficiency Decomposition Based on the Traditional and on the General Direct Approach: A Comparison, a Numerical Example, and Some Properties of the Direct Approach . . . 538
13.3.4 The Exceptional Case of the Directional Input Distance Function . . . 545
13.3.5 The Input-Oriented Reverse Approaches for Cost Decompositions . . . 547
13.4 Decomposing Revenue Inefficiency . . . 562
13.4.1 The Traditional Approaches Based on Output-Oriented Efficiency Measures . . . 562
13.4.2 The General Direct Approach Based on Output-Oriented Efficiency Measures . . . 564
13.4.3 Revenue Inefficiency Decomposition Based on the Traditional and on the General Direct Approach: A Comparison, a Numerical Example, and Some Properties of the Direct Approach . . . 566
13.4.4 The Exceptional Case of the Directional Output Distance Function . . . 574
13.4.5 The Output-Oriented Reverse Approaches for Revenue Decompositions . . . 575
13.5 Empirical Illustration of the General Direct Approach to Decompose Economic Inefficiency . . . 589
13.5.1 The General Direct Approach (ERG = SBM) to Decompose Profit Inefficiency . . . 591
13.5.2 The General Direct Approach (Russell) to Decompose Cost Inefficiency . . . 594
13.5.3 The General Direct Approach (Russell) to Decompose Revenue Inefficiency . . . 596
13.5.4 An Application: Taiwanese Banking Industry . . . 599
13.6 Summary and Conclusions . . . 600

14 A Final Overview: Economic Efficiency Models and Properties . . . 605
14.1 Introduction . . . 605
14.2 Multiplicative or Additive Decompositions of Economic (In)efficiency . . . 607

14.3 On the Choice of Economic (In)efficiency Models 608
14.4 Decompositions of Economic (In)efficiency of the Taiwanese Banking Industry 609
14.5 Properties of the Economic Efficiency Models 613

Bibliography 619

Author Index 631

Subject Index 635

List of Figures

Fig. 1.1 (a–b). Profitability (**a**), profit (**b**), and technology 8
Fig. 1.2 Duality and economic (in)efficiency: multiplicative and additive approaches 10

Fig. 2.1 (**a**–**b**) Technology sets 24
Fig. 2.2 (**a**–**b**) Input-oriented technical (in)efficiency 27
Fig. 2.3 (**a**–**b**) Output-oriented technical (in)efficiency 31
Fig. 2.4 (**a**–**b**) Graph (hyperbolic)-oriented technical (in)efficiency 35
Fig. 2.5 (**a**–**b**) Cost minimization, revenue maximization, and economic (in)efficiency 45
Fig. 2.6 (**a**–**b**) Profitability maximization, profit maximization, and economic (in)efficiency 49
Fig. 2.7 (**a**–**b**) Duality between the input technical efficiency measure and the cost function 56
Fig. 2.8 (**a**–**b**) Duality between the output technical efficiency and the revenue function 63
Fig. 2.9 (**a**–**b**) Duality between the graph technical efficiency and the profitability function 70
Fig. 2.10 (**a**–**b**) Duality between slack-based technical inefficiency and the profit function 77
Fig. 2.11 (**a**–**b**) DEA approximation of the production technology and technical efficiency 92
Fig. 2.12 The Julia REPL 101
Fig. 2.13 Jupyter Notebook for the cost model using BEE for Julia 107
Fig. 2.14 Jupyter Notebook for the profitability model using BEE for Julia 108

Fig. 3.1 (**a**, **b**) Input and output distance functions and technical efficiency 120
Fig. 3.2 (**a**, **b**) DEA approximation of the input and output production sets 124

Fig. 3.3 (**a**, **b**) Cost minimization, revenue maximization, and economic efficiency 132
Fig. 3.4 (**a**, **b**) Duality between the input distance function and the cost function 136
Fig. 3.5 (**a**, **b**) Duality between the output distance function and the revenue function 143
Fig. 3.6 (**a**, **b**) Cost and revenue efficiency under non-homotheticity 151
Fig. 3.7 Example of the radial cost efficiency model using BEE for Julia 155
Fig. 3.8 Example of the radial revenue efficiency model using BEE for Julia 155

Fig. 4.1 (**a**–**b**) Technology sets, GDF, and technical and scale efficiencies 172
Fig. 4.2 DEA approximation or the technology set 180
Fig. 4.3 Profitability maximization and profitability efficiency 187
Fig. 4.4 (**a**–**b**) Duality between the technology and the profitability function 192
Fig. 4.5 Example of the GDF profitability efficiency model using BEE for Julia 202

Fig. 5.1 Example of the Russell profit inefficiency model using BEE for Julia 235
Fig. 5.2 Example of the Russell cost inefficiency model using BEE for Julia 237
Fig. 5.3 Example of the Russell revenue inefficiency model using BEE for Julia 240

Fig. 6.1 Illustration of the weighted additive distance function (WADF) 253
Fig. 6.2 Example of the WADF profit inefficiency model using BEE for Julia 268
Fig. 6.3 Example of the WADF cost inefficiency model using BEE for Julia 271
Fig. 6.4 Example of the WADF revenue inefficiency model using BEE for Julia 274

Fig. 7.1 Example of the *ERG*=*SBM* profit inefficiency decomposition, $(p, w) = (2, 1)$ 291
Fig. 7.2 Example of the *ERG*=*SBM* (*I*) cost inefficiency decomposition, $w = (2, 1)$ 297
Fig. 7.3 Example of the $ERG = SBM(O)$ revenue inefficiency decomposition, $p = (1, 2)$ 302
Fig. 7.4 Example of the *ERG*=*SBM* profit inefficiency model using BEE for Julia 306

Fig. 8.1 Strongly efficient projections and *DVs* 324

Fig. 8.2 Profit inefficiency decomposition based on the *graph DDF* 330
Fig. 8.3 Cost inefficiency decomposition based on the *input-oriented DDF* 334
Fig. 8.4 Revenue inefficiency decomposition based on an *output-oriented DDF* 338
Fig. 8.5 Example of the *graph DDF* profit inefficiency model using BEE for Julia 344
Fig. 8.6 Example of the input-oriented DDF cost inefficiency model using BEE for Julia 347
Fig. 8.7 Example of the output-oriented DDF revenue inefficiency model using BEE for Julia 350

Fig. 9.1 An example of the weakly efficient Hölder distance functions for $h = \infty$ 358
Fig. 9.2 Example of the ℓ_2 Hölder profit inefficiency model using BEE for Julia 387
Fig. 9.3 Example of the ℓ_1 weakly Hölder cost inefficiency model using BEE for Julia 390
Fig. 9.4 Example of the ℓ_∞ Hölder revenue inefficiency model using BEE for Julia 393

Fig. 11.1 Example of the MDDF profit inefficiency model using BEE for Julia 426

Fig. 12.1 Example of the *ERG* = *SBM* profit inefficiency decomposition, $(p, w) = (2, 1)$ 449
Fig. 12.2 Example of the *RDDF (ERG* = *SBM)* profit inefficiency model using BEE for Julia 472
Fig. 12.3 Example of the *RDDF* (Russell) cost inefficiency model using BEE for Julia 475
Fig. 12.4 Example of the *RDDF* (Russell) revenue inefficiency model using BEE for Julia 478

Fig. 13.1 Example of the general direct approach for profit inefficiency decomposition, $(p, w) = (2, 1)$ 498
Fig. 13.2 Example of the traditional and general approaches for decomposing profit inefficiency, $(p, w) = (6, 1)$ 506
Fig. 13.3 Example of the traditional and general approaches to decompose cost inefficiency, $w = (2, 1)$ 540
Fig. 13.4 Example of the traditional and general approaches to decompose revenue inefficiency, $p = (1, 2)$ 570
Fig. 13.5 Example of the GDA (ERG = SBM) profit inefficiency model using BEE for Julia 592
Fig. 13.6 Example of the GDA (Russell) cost inefficiency model using BEE for Julia 595
Fig. 13.7 Example of the GDA (Russell) revenue inefficiency model using BEE for Julia 598

List of Tables

Table 2.1 Example data illustrating the economic (in)efficiency models 103
Table 2.2 Descriptive statistics. Taiwanese banks, 2010 104
Table 2.3 Illustration of the cost efficiency model using BEE for Julia 106
Table 2.4 Illustration of the profitability model using BEE for Julia 107
Table 2.5 Illustration of the radial efficiency measure model using BEE for Julia 109
Table 2.6 Illustration of the hyperbolic efficiency measure model using BEE for Julia 109
Table 2.7 Information on the reference peers of the cost efficiency model using BEE for Julia 110

Table 3.1 Example data illustrating the cost and revenue efficiency models 154
Table 3.2 Implementation of the radial cost efficiency model using BEE for Julia 156
Table 3.3 Calculating the radial input efficiency measure using BEE for Julia 157
Table 3.4 Reference peers of the radial input efficiency measure model using BEE for Julia 157
Table 3.5 Implementation of the radial revenue efficiency model using BEE for Julia 159
Table 3.6 Calculating the radial output efficiency measure using BEE for Julia 160
Table 3.7 Reference peers of the radial output efficiency measure model using BEE for Julia 161
Table 3.8 Decomposition of cost and revenue efficiency based on Shephard's radial distance functions 163
Table 3.9 Input and output slacks in the cost and revenue efficiency models 164

Table 4.1 Example data illustrating the profitability efficiency model 201
Table 4.2 Implementation of the profitability efficiency model using BEE for Julia 203
Table 4.3 Calculating the generalized distance function with BEE for Julia 204
Table 4.4 Reference peers of the GDF efficiency measure using BEE for Julia 205
Table 4.5 Decomposition of profitability efficiency based on the generalized distance function 207
Table 4.6 Input and output slacks in the profitability efficiency model 209

Table 5.1 Example data illustrating the economic inefficiency models 233
Table 5.2 Implementation of the Russell profit inefficiency model using BEE for Julia 234
Table 5.3 Implementation of the Russell graph efficiency measure using BEE for Julia 235
Table 5.4 Reference peers of the Russell graph efficiency measure using BEE for Julia 236
Table 5.5 Implementation of the Russell cost inefficiency model using BEE for Julia 237
Table 5.6 Implementation of the Russell input efficiency measure using BEE for Julia 238
Table 5.7 Reference peers of the Russell input efficiency measure using BEE for Julia 238
Table 5.8 Implementation of the Russell revenue inefficiency model using BEE for Julia 239
Table 5.9 Implementation of the Russell output efficiency measure using BEE for Julia 240
Table 5.10 Reference peers of the Russell output efficiency measure using BEE for Julia 241
Table 5.11 Decomposition of profit inefficiency based on Russell inefficiency measure 242

Table 6.1 Example data illustrating the economic efficiency models 267
Table 6.2 Implementation of the WADF profit inefficiency model using BEE for Julia 269
Table 6.3 Implementation of the WADF graph inefficiency measure using BEE for Julia 269
Table 6.4 Reference peers of the WADF graph inefficiency measure using BEE for Julia 270
Table 6.5 Implementation of the WADF cost inefficiency model using BEE for Julia 271

Table 6.6 Implementation of the WADF input inefficiency measure using BEE for Julia 272
Table 6.7 Reference peers of the WADF cost inefficiency measure using BEE for Julia 272
Table 6.8 Implementation of the WADF revenue inefficiency model using BEE for Julia 274
Table 6.9 Implementation of the WADF output inefficiency measure using BEE for Julia 275
Table 6.10 Reference peers of the WADF output inefficiency measure using BEE for Julia 275
Table 6.11 Decomposition of profit inefficiency based on the weighted additive distance function (WADF) 277

Table 7.1 Results based on the decomposition of Aparicio et al. (2017a, b, c), $(p, w) = (2,1)$ 295
Table 7.2 Results based on the input-oriented *ERG=SBM* (*I*), $w = (2, 1)$ 298
Table 7.3 Results based on the output-oriented *ERG=SBM* (*O*), $p = (1, 2)$ 302
Table 7.4 Example data illustrating the economic inefficiency models 304
Table 7.5 Implementation of the *ERG=SBM(G)* profit inefficiency model using BEE for Julia 305
Table 7.6 Implementation of the *ERG=SBM* graph inefficiency measure using BEE for Julia 306
Table 7.7 Reference peers of the *ERG=SBM* graph inefficiency measure using BEE for Julia 307
Table 7.8 Decomposition of profit inefficiency based on the *ERG=SBM(G)* efficiency measure 308

Table 8.1 Example data illustrating the economic inefficiency models 341
Table 8.2 Implementation of the *DDF* profit inefficiency model using BEE for Julia 343
Table 8.3 Implementation of the graph DDF inefficiency measure using BEE for Julia 344
Table 8.4 Reference peers of the graph DDF inefficiency measure using BEE for Julia 345
Table 8.5 Implementation of the *DDF* cost inefficiency model using BEE for Julia 346
Table 8.6 Implementation of the input DDF technical inefficiency measure using BEE for Julia 348
Table 8.7 Reference peers of the input DDF technical inefficiency measure using BEE for Julia 348

Table 8.8 Implementation of the DDF revenue inefficiency model using BEE for Julia 349
Table 8.9 Implementation of the output DDF inefficiency measure using BEE for Julia 350
Table 8.10 Reference peers of the output DDF inefficiency measure using BEE for Julia 351
Table 8.11 Decomposition of profit inefficiency based on the proportional directional distance function. 352

Table 9.1 Example data illustrating the economic inefficiency models 384
Table 9.2 Directional vectors and weights corresponding to Hölder norms 385
Table 9.3 Implementation of the ℓ_2 Hölder profit inefficiency model using BEE for Julia 386
Table 9.4 Implementation of the ℓ_2 Hölder graph inefficiency measure using BEE for Julia 387
Table 9.5 Reference peers of the ℓ_2 Hölder graph inefficiency measure using BEE for Julia 388
Table 9.6 Implementation of the ℓ_1 Hölder cost inefficiency model using BEE for Julia 389
Table 9.7 Implementation of the ℓ_1 Hölder input inefficiency measure using BEE for Julia 390
Table 9.8 Reference peers of the ℓ_1 Hölder input inefficiency measure using BEE for Julia 391
Table 9.9 Implementation of the ℓ_∞ Hölder revenue inefficiency model using BEE for Julia 392
Table 9.10 Implementation of the ℓ_∞ Hölder output inefficiency measure using BEE for Julia 393
Table 9.11 Reference peers of the ℓ_∞ Hölder output inefficiency measure using BEE for Julia 394
Table 9.12 Decomposition of profit inefficiency based on the ℓ_∞ Hölder distance function 395

Table 10.1 Different normalization sets and their corresponding DEA technical inefficiency measures 404
Table 10.2 Different values for k and their corresponding DEA technical inefficiency measures 407
Table 10.3 Different normalization sets and their corresponding input-oriented DEA technical efficiency measures 409
Table 10.4 Different values for k and their corresponding DEA input-oriented technical inefficiency measures 410
Table 10.5 Different normalization sets and their corresponding output-oriented DEA technical efficiency measures 412

Table 10.6 Different values for k and their corresponding DEA output-oriented technical efficiency measures 413
Table 11.1 Example data illustrating the profit inefficiency model 425
Table 11.2 Implementation of the modified DDF profit inefficiency model using BEE for Julia 426
Table 11.3 Implementation of the modified DDF graph inefficiency measure using BEE for Julia 427
Table 11.4 Implementation of the modified DDF graph efficiency measure using BEE for Julia 427
Table 11.5 Decomposition of profit inefficiency based on the Modified Directional Distance Function (MDDF) 429
Table 12.1 The *RDDF* associated with the ERG = SBM 445
Table 12.2 Profit inefficiency decompositions for $ERG = SBM$, $(p, w) = (2, 1)$ 446
Table 12.3 Profit inefficiency decompositions for the *RDDF*, $(p, w) = (2, 1)$ (associated with Table 12.2) 447
Table 12.4 New results for firms F, G, and H of Table 12.3 with new projections 452
Table 12.5 Profit inefficiency decompositions for $ERG = SBM$ based on the best projections, $(p, w) = (2, 1)$. (Reproduction of Table 7.1) 454
Table 12.6 Profit inefficiency decompositions for the *RDDF* based on the best projections,$(p, w) = (2, 1)$ (associated with Table 12.5) 455
Table 12.7 Results based on the $ERG = SBM(I)$, $w = (2, 1)$. (Reproduction of Table 7.2) 457
Table 12.8 Results based on the associated $RDDF(I)$, $w = (2, 1)$. (Reproduction of Table 8.3) 459
Table 12.9 Results based on the $ERG = SBM(O)$, $p = (1, 2)$ 461
Table 12.10 Results based on the associated $RDDF(O)$, $p = (1, 2)$ 462
Table 12.11 Example data illustrating the economic inefficiency models 469
Table 12.12 Implementation of the *RDDF (ERG = SBM)* profit inefficiency model using BEE for Julia 471
Table 12.13 Implementation of the *RDDF (ERG = SBM)* graph inefficiency measure using BEE for Julia 472
Table 12.14 Reference peers of the *RDDF (ERG = SBM)* graph inefficiency measure using BEE for Julia 473
Table 12.15 Implementation of the *RDDF* (Russell) cost inefficiency model using BEE for Julia 474
Table 12.16 Implementation of the *RDDF* (Russell) input inefficiency measure using BEE for Julia 475

Table 12.17 Reference peers of the *RDDF* (Russell) input inefficiency measure using BEE for Julia 476
Table 12.18 Implementation of the *RDDF* (Russell) revenue inefficiency model using BEE for Julia 477
Table 12.19 Implementation of *RDDF* (Russell) output inefficiency measure using BEE for Julia 479
Table 12.20 Reference peers of the *RDDF* (Russell) output inefficiency measure using BEE for Julia 479
Table 12.21 Decomposition of profit inefficiency based on the reverse directional distance function, $RDDF\left(BDF, F_J, \widehat{F}_J\right)$ 481
Table 13.1 Results based on the traditional best decomposition of Aparicio et al. (2017a), $(p,w) = (6,1)$ 507
Table 13.2 Results based on the general direct decomposition, $(p,w) = (6,1)$ 509
Table 13.3 Results based on *the SR approach*, $(p, w) = (6,1)$ 522
Table 13.4 Results based on *the FR approach*, $(p, w) = (6,1)$ 531
Table 13.5 Results based on the traditional decomposition of Aparicio et al. (2015a) (Chap. 5), $w = (2, 1)$ 541
Table 13.6 Results based on the general direct decomposition, $w = (2, 1)$ 543
Table 13.7 Results based on the standard reverse approach cost decompositions, $w = (2,1)$ 553
Table 13.8 Results based on *the FR cost approach*, $w = (2,1)$ 561
Table 13.9 Results based on the traditional decomposition of Aparicio et al. (2015a),$p = (1, 2)$ 572
Table 13.10 Results based on the general direct approach, $p = (1, 2)$ 573
Table 13.11 Results based on the standard reverse approach, $p = (1, 2)$ 580
Table 13.12 Results based on the *output flexible reverse approach*, $p = (1, 2)$ 588
Table 13.13 Example data illustrating the economic efficiency models 590
Table 13.14 Implementation of the GDA (ERG = SBM) profit inefficiency model using BEE for Julia 591
Table 13.15 Implementation of the GDA (ERG = SBM) inefficiency using BEE for Julia 593
Table 13.16 Implementation of the GDA (Russell) cost inefficiency model using BEE for Julia 594
Table 13.17 Implementation of the GDA (Russell) input inefficiency using BEE for Julia 596
Table 13.18 Implementation of the GDA (Russell) revenue inefficiency model using BEE for Julia 597
Table 13.19 Implementation of the GDA (Russell) output inefficiency using BEE for Julia 599

Table 13.20 Normalized general direct approach decomposition of profit inefficiency based on the *ERG* = *SGM* (*G*) 601
Table 13.21 General direct approach decomposition of profit inefficiency based on the ERG = SGM (G) (monetary values) 602
Table 14.1 Multiplicative decompositions of cost, revenue, and profitability efficiency of Taiwanese banks 610
Table 14.2 Additive decomposition of profit inefficiency of Taiwanese banks 611
Table 14.3 Properties of economic efficiency decompositions: technical, economic, and allocative 615

About the Authors

Jesús T. Pastor is a professor of statistics and operations research at the Universidad Miguel Hernandez of Elche, Spain. He earned an MBA and a PhD in mathematical sciences from Valencia University, Spain. He has been visiting researcher at the Universities of Georgia (USA), Toronto (Canada), Queensland (Australia), and Warwick (UK). Prof. Pastor's research fields include location science, banking, and, for the last 25 years, efficiency analysis. He has served on the editorial review or advisory board of more than 20 international journals. He has been guest editor of one issue of European Journal of Operational Research (*EJOR*) and of two issues of Journal of Productivity Analysis (*JPA*). He has authored or co-authored 9 books in various fields of mathematics and has published over 150 research papers. His research has appeared in a wide variety of refereed top economic and operations research journals. In 2013, the Spanish Scientists Association accredited Prof. Pastor as one of the three distinguished scientists of the year. Lastly, in 2020, he was appointed by his University as Professor Emeritus.

Juan Aparicio is a professor in the Department of Statistics, Mathematics and Information Technology at the Miguel Hernandez University of Elche (UMH), Spain, and the head of the Center of Operations Research. He has been co-chair (with Prof. Knox Lovell) of the Santander Chair on Efficiency and Productivity. His research interest includes efficiency and productivity analysis combined with machine learning and data science. He has published and co-edited several books focusing on performance evaluation and benchmarking using data envelopment analysis, as well as publishing approximately 100 scientific articles in different international journals. He is associate editor of *Omega—The International Journal of Management Science*, *Journal of Productivity Analysis*, *Mathematics*, and *Advances in Operations Research*.

José Luis Zofío (www.joselzofio.net) is a professor of economics at the Universidad Autónoma de Madrid, and former chair of the Department of Economics. He is also visiting professor at Erasmus University and Wageningen University and Research, and visiting fellow to the Erasmus Research Institute of Management