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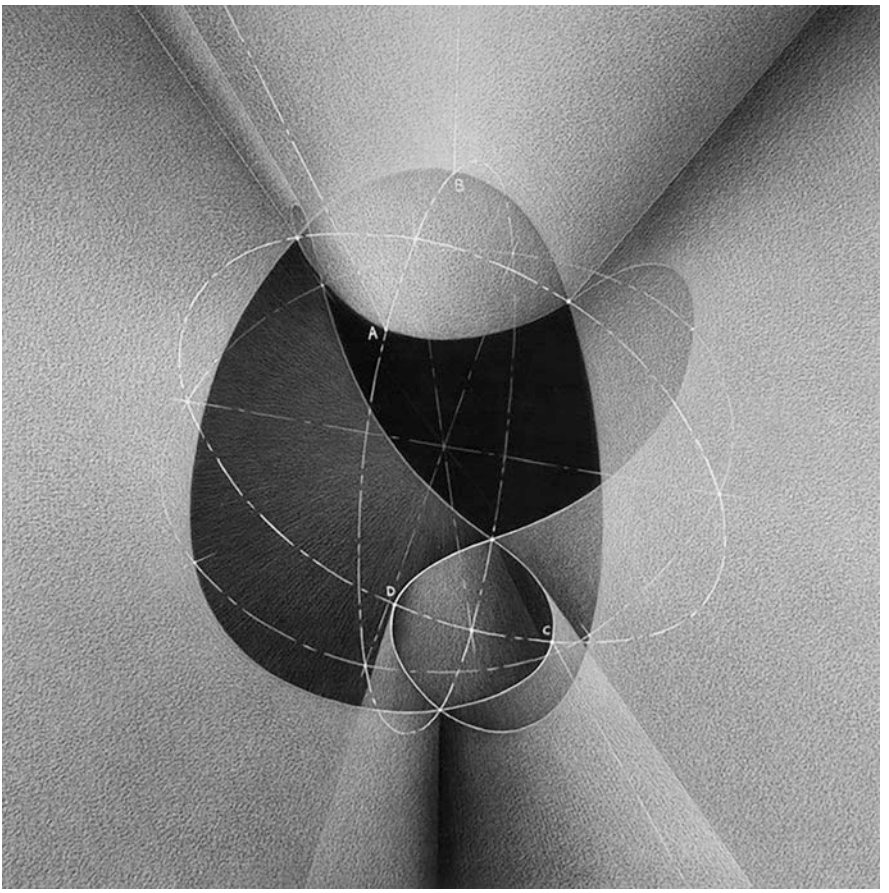

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Introduction

Specialists from various backgrounds, each recognised as references in their field or promising to become so, have been invited in January 2018 to reflect and exchange on a theme that was on the way to becoming a commonplace. This conference was guided by the firm intention of renewing this theme by reactivating all the dimensions that may have been convened in the past, but in scattered order, without avoiding salutary tensions and contradictions.

Such dispersion could not be avoided once again and is, no doubt, due to the nature of the theme itself. One of its poles is the *diagrammatic* and the other is the *question of space* as they intervene in the development of human knowledge in its broadest and most encompassing sense. By indexing these elements under the aegis of a questioning that crosses mathematics, physics, biology, philosophy, literature, and art, that of *form*, we have tried not to drown this heterogeneity under an even broader amphibology, but to turn it towards what resonates as a future or even a promise.

Indeed, diagrams play a fundamental role in the mathematical visualisation and philosophical analysis of forms in space. Some of the most interesting and profound recent developments in contemporary sciences, whether in topology, geometry, dynamic systems theory, quantum field theory, or string theory, have been made possible by the introduction of new types of diagrams, which, in addition to their essential role in the discovery of new classes of spaces and phenomena, have contributed to enriching and clarifying the meaning of the operations, structures, and properties which are at the heart of these spaces and phenomena. This multiplicity of uses covers a certain polysemy that should also be questioned. Diagrams, which are often related to images, drawings, figures, and models, implement a more imaginative and pictorial thinking of scientific and artistic practice, and combine *gesture*, *invention*, and *meaning*. They show that it is possible to elaborate a theory, a model, abstract or concrete, like a thought in movement, which originates in a space that is itself to be reinvented and unfolds in its own time. Developing a diagrammatic thinking thus amounts to trying to understand the dynamics of transformation and the processes of emergence of new properties and qualities of spaces and phenomena.

This volume wants to examine the importance today of diagrams of knots, links, braids, fields, interactions, strings, etc., in topology and geometry, in quantum physics and cosmology, but also in the theories of perception, in the plastic arts and in philosophy. To this end, we propose to study different cases of mathematical and physical theories in which diagrams play an important role, philosophical and phenomenological approaches to space, time, and perception, as well as artistic practices strongly inspired by diagrammatic thinking.

Topological Visualisation, or How to Apprehend the Invisible

The representation of space in topology require a process of "mathematical visualisation" (of idealisation or imagination), which calls upon a new type of intuition, more conceptual and at the same time more pictorial (diagrammatic), and resolutely distant from immediate sensations and empirical intuition. In topology, the figure, the drawing, the diagram, or the graph (in this context, we attribute to these words the same status) are no longer the image of something, of an external object that the image would be in charge of representing, but are themselves the object that represents a universe of relations and "hidden" properties absent from the image. The "semiotization" of the status of the image is even more developed there than in other sciences and has reached a very fine level.

Topology allows another approach to the study of objects which does not restrict itself to quantitative relations of size and visual aspects, but rather considers the form (i.e. the *image* of a deformation, either an embedding or an immersion) as a whole, as well as the spectrum of possible variations (continuous or discrete) of its configurations. It has changed profoundly our scientific thought and culture of the image; it is the domain *par excellence* of images (figures, drawings) in the new and particular sense we have just outlined. It is the most abstract (but, in another sense, the most concrete) part of mathematics, which is defined as the science of transformations of extended objects and more or less abstract spaces by continuous deformation, that is to say, without tearing or gluing. In this field of mathematics, the notions of distance and length no longer play any role, and the concept of homotopy, or more generally of homology proves to be powerful to account for the similarity and intrinsic relations between two objects, two figures, two surfaces, or, more generally, between two abstract spaces. The criterion of similarity by superimposition or by reproduction of sizes, and thus the very notion of visible resemblance, takes all meaning in topology. In no other field of mathematics is the distance from visible reality and the model of figuration as important as it is in topology, where a much subtler concept close to the intrinsic structure of objects plays a fundamental role, namely the qualitative concept (in the topological sense, i.e. where no metric relations intervene) of equivalence or homeomorphism. Thus, in its broad meaning, topology studies the properties of geometrical objects such as manifolds, spaces, knots, and braids that conserve their essential properties

when submitted to a certain kind of transformations called homeomorphisms and homotopies.

Feynman Diagrams: A New Way Forward in Theoretical Physics

The Feynman diagrams, and their relationship to knots and links diagrams, are a magnificent example of this point of view, and it is one of the most significant ways of thinking in twentieth century physics. It can be seen as a kind of grammar endowed with the power to create theoretical models and possible interactions between physical but immaterial particles. These theoretical models, symbolised by diagrams generated one after the other thanks to a method of invention, represent a process in the making. Indeed, it allows us to apprehend the meaning of a physical world (that of fields and subatomic particles) invisible and inaccessible to our perception, first through the dynamic construction of a space with a very large number n of parameters (where n is unknown), then by projecting this space onto a simple two-dimensional surface, but which nevertheless conceals hidden possibilities. A fundamental part of physical thought in the twentieth century is closely linked to the invention of the diagrammatic method, largely due to the genius of the physicist Richard Feynman, to whom we owe (together with Tomonaga, Schwinger, and Dyson) the creation in 1948 of Quantum Electrodynamics (QED). It is a quantum field theory that aims to reconcile electromagnetism with quantum mechanics. The concept of field designates a structure that makes it possible to account for the creation or annihilation of particles at any point "in space". Mathematically, it is an Abelian group with a $U(1)$ gauge group of symmetries. The gauge field that intervenes in the interaction between two charges represented by ½ integer spin fields is the electromagnetic field. Physically, this translates into the fact that the charged particles interact through the exchange of photons. Photons, which play a fundamental role in atoms, are like strings that bind electrons to the nucleus. Quantum electrodynamics was the first quantum field theory in which the difficulties in developing a purely quantum formalism allowing the creation and annihilation of particles were satisfactorily resolved, thanks to the so-called renormalisation method, designed to free itself from the infinite undesirable quantities encountered in quantum field theory.

Diagrammatics and Invariants in Knot and Braid Theory

It is a field of research that cuts across several fundamental areas of mathematics, physics, biology, and the philosophy of science. Knots and braids diagrams are among the most fascinating objects of current research at the crossroads of

algebraic and geometric topology and quantum field theories. Information about two-dimensional knotted surfaces (i.e. knots that allow for a planar projection) living in the three-dimensional space R^3 or in S^3 results from a structure called a *knot diagram*. For the mathematical study of knots, the fundamental question is whether two knots are equivalent. If they are not, then they can be distinguished, usually by means of numerical, geometrical, or algebraic invariants: one of these invariants is the Jones polynomial. Most of these invariants can be obtained from knots diagrams. If at least two knots are equivalent, then we can define an ambient isotopy that establishes an equivalence relationship between manifolds or subsets of R^n.

In its far-reaching meaning, a knot invariant is a function from the set of all knots to any other set such that the function does not change as the knot is changed (up to isotopy). In other words, a knot invariant always assigns the same value to equivalent knots (although different knots may have the same knot invariant). Standard knot invariants include the fundamental group of the knot complement, numerical knot invariants (such as Vassiliev invariants), polynomial invariants (knot polynomials such as the Alexander polynomial, Jones polynomial, Kauffman polynomial, Homfly-pt polynomial), and crossing, linking, or torsion numbers (i.e. numbers defined in terms of an invariant, and for the torsion number, generated by the finite cyclic covering space of a knot complement). Torsion invariants were introduced by Reidemeister in 1935, and were historically the first non-homotopy invariants for closed three-dimensional Piecewise Linear manifolds.

Starting in the eights the quest for knots' invariants has been one of the most significant lines of research in the field of three-manifold topology. This began with the Jones polynomial invariant for knots in S^3, which was used by Witten to show, with Chern–Simons theory, that the 2+1 dimensional quantum Yang–Mills theory is exactly soluble. There are combinatorial definitions of the Jones polynomial, basically skein relations that say how the invariant is related before and after changing a crossing in the diagram of a knot or link. These types of relations led to other combinatorial notions of knot invariants and sometimes to 3-manifold invariants. One of the most powerful seems to be the Khovanov homology of knots in S^3. These homology groups should be viewed as a categorification, a kind of enrichment, that is a process consisting in replacing sets point of view by category theory. Like the original Jones polynomial, these invariants are defined starting with a braid presentation of the knot. The invariants of knots and links in S^3 obtained from Chern–Simons theory (a topological three-dimensional quantum field theory) can be used to construct three-manifold invariants. This provides an important tool to study topological properties of three-manifolds.

In the 1990s, it has been uncovered a deep but unexpected relationship between conformal field theories and knot theory. Indeed, quantum field theory can be used to generate new knots polynomials and analytic expressions for them. Knot theory, in turn, is an important tool by which conformal field theories and statistical mechanics can be studied, giving us a topological meaning to quantum groups and to the Yang–Baxter relation.

Philosophical and Scientific Implications

This brings us to the epistemological and philosophical issues. (1) In general, and particularly in the case of knots s and Feynman diagrams in quantum electrodynamics, diagrams have not been used to simply "illustrate" something (objects and events), but rather as symbolic operations or operators of an algebraic and topological nature. Moreover, diagrams make it possible to show and know topological properties of the object, in particular concerning their possible transformations in space and their invariant characteristics. Once known, these topological properties can lead to the discovery of new algebraic invariants of the knot. (2) Diagrams are not limited to offering a picture of the world as it appears to us. A diagram is a powerful symbolic and conceptual construction, a key to reading and rewriting the processes of formation of the "real" world, a kind of open semio-dynamics of his becoming. From this point of view, the diagram translates the form of a way of thinking (of a conceptual strategy) that the physicist geometer is able to give to matter, to the physical world. As a set of plastic qualities of a thought in the process of forming itself as an effective model, as a form, the diagram tends to objectify itself in real processes. Diagrams have three functions at the same time: (a) to elucidate concepts by deploying their articulations within a possible form; (b) to introduce new concepts; (c) to create new properties of the objects that diagrams (graphs, trees) model. (3) Diagrammatic thinking has opened the way to new methods and techniques for visualising objects and phenomena inaccessible to ordinary perception, and also to experiments that can be carried out by even very sophisticated devices. This visualisation has additional explanatory power compared to other classical methods used in the mathematical and physical sciences. Through the creation of mental images of objects and equations, certain visualisation techniques, especially diagrams, are constituted, of which two levels can be distinguished: the first corresponds to models of real objects as they are imagined and not (directly) observed; the second concerns the elements that form the models themselves, i.e. the graphical, denotative and connotative aspects, which give to "see" what is not visible, by a creation of objects and new articulations of meaning.

Diagrammatics and Category Theory

The theory of knots and interlacing gives us to see the stitching of diagram and calculation, implied in each other as in a Feynman diagram. From this interaction between the image and its interpretation, a dialectical pulsation is created between seeing and enunciating in a series of relationships that the diagram imposes and exposes. Explored by Lacan in the meanders of the Borromean knot, the image of these interlaces serves as a support for thought in order to justify and develop the complicated relationships between the real, the symbolic, and the imaginary, and to

locate the place of modes of jouissance and meaning between the wedging points of the "bo-knot". It will therefore be a question of studying the theatre of operations of these diagrammatic entanglements, understanding their relationship to the virtual, and giving an account of their creative intuition. But also, to point out the question of indexing, present both in Châtelet's texts and in those of Charles Sanders Peirce, who liked to say that "algebra is nothing other than a kind of diagram".

It is around the algebraic expression of Jones's polynomial that the connections between the low-dimensional topology, of which the theory of knots and links is a part, and the mathematical theory of the categories of Saunders MacLane and Samuel Eilenberg are born. Developed from the middle of the twentieth century onwards, the latter gave rise to many links between various fields of mathematics. The diagram is one of its founding elements and modes of reasoning. It is at work, for example, in Charles Ehresmann's sketch theory and in the theory of locally free diagrams by René Guitart and Christian Lair. These theories deserve to be reinterpreted within the philosophical framework of Alain Badiou's recent writings as set out in his *Logique des mondes*, and of the texts of Gilles Deleuze and Michel Foucault, who saw in Bentham's panoptic a diagram of power, a "political device", organising, according to Deleuze, "a new type of reality". In this confrontation between science and philosophy, notions such as appearance, univocity, duality, and universality will find new dimensions that will extend the reflection on the diagram and the diagrammatic.

A French Singularity in Epistemological Field: Gilles Châtelet

Among the many authors who have devoted all or part of their reflections to this question, Gilles Châtelet is undoubtedly a leading figure to whom this colloquium will pay tribute. Indeed, the mathematical philosopher reminds us that the diagram is distinguished from the figure by the operation that makes it function, by a series of singular points that make it up or that emerge *ab initio* from its own tensions and its own virtualities. For Châtelet, the diagram is "the gestural unfolding of a space" that pushes calculation through "allusive stratagems" that the philosopher in *Les enjeux du mobile* never ceases to hunt down in Argand's texts, Maxwell's electromagnetism, and Grassmann's theories. No less suggestive is Châtelet's renewed look at the history of science and the philosophy of science, as well as at the way in which the phenomenology of space should be resourced with scientific practice. It never goes without a parallel, sometimes implicit and sometimes explicit, with the history of art and painting in particular. It is in this perspective that Châtelet's insistence, following Leibniz, on the "manner of the operation" must be understood. Against a purely operative conception of algebraic and geometric (understood as the provision of a figurative by means of a "simple abstraction" from sensitive space), he rightly insists on the fluidification of space in favour of a *spatium*, understood as the space of virtualities whose constitution "presupposes only a law of coordination of the internal spontaneity of monads".

The resignation that would be the acceptance of a space and a metric already won, attached to the position of a neutral observer, is refused in the name of the requirement of a conquest of space that is a "victory of the 'projective'". Whether it is the conquest of depth in pictorial perspective, the scale of speeds in Oresme's diagrams, Einstein's central intuition of the special relativity that underlies the understanding of Lorentz's contraction, all the experiences of thought or "intuitions" that preside over these "conquests" (or inventions), whether scientific, artistic, philosophical or simply "existential", proceed from a "decision of horizon", where the style of circumspection and the opening of the "thematic" field is decided; which is not surprising, since the horizon "allows us to venture into the turbulent space where science, art and philosophy come close without confusing science, art and philosophy". This decision or "pact" negotiates between two failures: that of geometrical intuition being frozen in "clichés", and that of units of meaning syntactically submitted to the administration of proof. Such pacts correspond to moments of institution and brackets of the "available", and allow for their rediscovery: "a flagrant example is the 'rediscovery' (in Hamilton's case) of imaginary numbers by the introduction of the plane"; "this new encounter of geometry and algebra (and therefore of the visible and calculable) will impose the work of Hamilton and Riemann (and many others), while having an impact on the very notion of the application of mathematics". Such pacts engage us in the wake of Kant, beyond the "classical pact that geometry and algebra had sealed, by tying the images of the first (the "figures") to the literality of the second (the sequences of formulae and calculations of magnitudes)", in this "underbelow" that transcendental aesthetics left in the shadows, without sending it back to an empirical psychology, be it a psychology of the creative imagination or of the mathematical imagination.

The epistemology suggested by Châtelet is a tracking of these gestures and moments of disaffection with regard to clichés, misleading analogies, as well as the domesticated operative, which has become routine. However, this kind of parenthesis has the opposite side to a full and complete subjective involvement in an experience of thought and intropathy, of *Einfühlung*. This point is essential because it provides us with the link between this epistemological attitude induced by the explicit promotion of the diagram to the rank of formal object and writing and the aesthetic attitude instituted by explicitly diagrammatic practices in the plastic arts. Châtelet's study is exemplary in this respect with regard to the *diagrams of intentionality* presented on the occasion of the developments on Grassmann, whose schema and general tendency need only be retained. The reading of these diagrams is pushed by Châtelet to the level of a choreography, depicting the scene of intersubjectivity, resulting in a profound modification of the (classical) subject/object relationship. Intentionality is in turn disturbed in turn, the subject S being haunted by the object O, haunting or reflecting the impassable abyss between two I's.

Phenomenology of Space (and Time) and Diagrammatic Epistemology

In the wake of these reflections, we are invited to rethink the place and role of a phenomenology of space. By stripping away structures, we discover—underneath the constituted—the trace of these gestures and their rich potential. This is how the diagrams are introduced. It is still through the diagram that one can find and make one's own experiences of radical thought. This is the case with the thought experience of Galileo, the Einstein of General Relativity, or Archimedes. The diagram records and transmits these radical thought experiences. They are a trace of them. In this conception of science, understanding and scientific learning, the diagram plays a decisive role, because diagrammatic communication is based on "intropathy". The appropriation of these experiences therefore presupposes that we find the gesture, the embodied operativity; that we plunge back into the constituent variety of subjective ways of "doing", which appear, after the fact, in the eyes of a normalised science, like a halo or a contingent gangue. But it is first of all constitutive: inventio itself presupposes such a "putting oneself in the place of". What is sought in this way under the diagrams and the techniques of forcing intuition, of provoking an algebraic intuition likely to upset the trivialities of a logically domesticated calculable, are in fact the gestures of constitution and institution. Since the task of phenomenology is to "explicitly renew" these formations of meaning by reactivating them, it must stop naively making use of them to consider them in their formation and in their use. This supposes that, in the description and discourse held on this subject, another form of reading of these "legacies" is practised. This legacy presupposes that they are made available in the form of "writing systems" and a socially constituted and transmitted habitus capable of reading and using what is thus made available. This presupposes the acquisition of new linguistic and graphic "tools" to "describe" the provision itself.

A "transcendental" moment comes to lie at the heart of the institution or the renewal of these practices, each time the hypotheses on which the previous practices were based are awakened and shaken up. And, it is possible to consider as a sample of phenomenology the explicit reflection that accompanies these moments of invention. The attention to the foundations and underpinnings of constructive symbolic activity is phenomenological. Phenomenology digs into the "model cupboard", into the back-kitchen of science. In its exploration of the deepest constitutive levels of constituent subjectivity, it must strive to "re-seize" and "reactivate" the inaugural "gestures", which are not only those of a proto-geometer from an antiquity as mythical as it is remote, but also and first and foremost those of each "inventor". But it is one thing, it will be said, to elucidate the role of the diagrammatic in intellectual activity (categorical synthesis), it is another to use diagrams in the course of this elucidation itself, as Husserl does to study a level of synthesis that is precisely not intellectual, and that intervenes, it seems, in the lowest and most immediate levels of constitution, as is the case with everything that touches on "aesthetic" syntheses: those which are constitutive of our consciousness of space and time.

Towards a "Diagrammatic Critique of Aesthetics"

That this area is also the place of the technicality of reason, that is to say of mathematics and art, is what we should never cease to meditate on; but first and foremost, this deployment takes place through a communicability of gestures, that is to say of hands and manners, "all this talking with hands" "which should perhaps be better called talking with hands" and which should not lead us to conceal the "hands" or the "manners" of speaking, this dimension of the linguistic expression which Kant in the third critique designates as being that of gesture (and gesticulation) always joint, in the spoken word, to word and tone (to articulation and modulation). In the field of the arts, it will therefore be a question of questioning the diagram as a gesture, in the act of writing, photographing, painting, or composing music, but also in its own genesis, in what precedes thought, in the unthought: the diagram as a technical device for indexing, preconceived or not, random or deterministic. In Francis Bacon's interviews with David Sylvester, the act of painting presupposes that there exists on the canvas a set of figurative data, more or less virtual, more or less current, constituting an intermediate place where the play of forces can be exercised. According to these forces, new data will appear, disappear, fade, or stand out according to the artist's desire or will. From a chaotic, primitive, structural, or algorithmic primordial form, the artistic diagram will be born in a middle ground, in a pure becoming. Because etymologically, the word diagram in Greek is the deverbal noun of *diagraphein*, that is to say literally "through the writing", it is posed as a transverse axis to the artist's gesture. The function of a diagram is always to make something explicit, and according to Châtelet, to immobilise "a gesture in order to establish an operation of amplification and intuition". From Kandinsky to Paul Klee, from Pollock to Bacon, or for artists such as Ricardo Basbaum, Wacław Szpakowski, Daniel Sheets Dye, Mark Lombardi who work on lines, their arrangements and relational networks, or for artists such as Edward Tufte, who works on the notion of Feynman's diagram, the diagram does not have here the same meaning or the same status, or particularly for artists such as Jorge Eduardo Eielson and Robert Morris, who worked on different types of knots and their virtual artistic expressions. We will therefore attempt to analyse the aesthetic outline of the diagram, to problematise its links with the work and the artistic process, to situate diagrammatic creativity as a processual whole, that is to say, in short, we will attempt to sketch out a "diagrammatic critique of aesthetics".

The book is divided into eight parts devoted to various mathematical and philosophical themes. The first part of the book is on "Logic, Forms, and Diagrams". The purpose of the paper of L.H. Kauffman is to explore the idea of a sign, using G. Spencer-Brown's work "Laws of Form" as a pivot, a reference, and a place from which Kauffman makes excursions into simplicity and complexity. Julien Bernhard proposes a thorough inquiry on roles of diagrams in logic. Are they merely pedagogical tools, or are they effectively endowed with a demonstrative force and eventually a heuristic potential, at least in the discovery of inferential forms? The

paper of Franck Jedrzejewski is a rigorous tentative of giving an interpretation of knot and link diagrams and invariants in terms of theory of category.

The second part of the book is on "Geometrical Spaces and Topological Knots, Old and New". The nice paper of Marco Andreatta is an historical presentation of some ideas and concrete constructions of geometrical surfaces. He briefly considers some recent results in higher dimensional algebraic geometric, which can be summarised in few diagrams. He also points out that some of these ideas and diagrams could be directly connected to biology and life sciences. Alessandro Verra shows that geometry undergone revolutionary shifts of paradigm during all the last century. He addresses the issue of the difference between historical and contemporary geography of algebraic geometry, trying to bring some evidence and concrete examples from the experience of a person working in this field. A major attention is payed to the history of classical and modern rationality problems, for some famous examples of algebraic varieties. In its paper, Luciano Boi shows that knot theory has extensive interactions, not only with different branches of mathematics but also with various and fundamental areas of physics. Knots and links are deeply related to the geometry of 3-manifolds and low-dimensional topology, quantum field theory, and fluid mechanics. The paper surveys some current topics in the mathematical theory of knots and some of their more striking ramifications in physics and biology. This article aims at stressing the importance of considering diagrams in the study of topological and geometrical objects and the key role of knot theory for the understanding of the structure of space and space-time.

The third part of the book is on "Diagrams, Graphs, and Representation". Carlo Petronio explains how planar diagrams, equipped with suited depictions, can be used for describing topological objects of dimension 1, 2, 3, and 4. In that paper, Patrick Popescu-Pampu presents some problems which led to the introduction of special kinds of graphs as tools for studying singular points of algebraic surfaces. He explains how such graphs were first described using words, and how several classification problems made it necessary to draw them, leading to the elaboration of a special kind of calculus with graphs. This non-technical paper is intended to be readable both by mathematicians and philosophers or historians of mathematics.

The fourth part of the book deals with "Diagrams, Physical Forces, and Paths Integrals". In the paper of Sergio Albeverio, Feynman path integrals are first presented for the case of non-relativistic quantum mechanics, both in physical and mathematical terms. Then the case of scalar relativistic and Euclidean quantum fields is discussed. The methods of (constructive) perturbation theory and renormalisation theory in relation to Feynman path integrals are briefly considered, in particular mentioning the visual help provided by Feynman diagrams. The paper ends with mentioning some open problems and presenting some philosophical remarks and reflections on the description of natural phenomena, in particular those of fundamental physics, in mathematical terms. Jean-Jacques Szczeciniarz paper contains some remarks on Penrose diagrams. He explains what a conformal diagram is by expounding the theoretical and geometrical elements which constitute it: complex geometry in several variables, projective geometry, conformal geometry.

Finally, he briefly presents the cosmological and philosophical significance of the synthesis realised by Roger Penrose.

The theme of the fifth part is "Phenomenology in and of Mathematical Diagrams". Frédéric Patras paper is a tentative of giving representations of elementary geometric and combinatorial objects, with particular emphasis on the so-called non-crossing set partitions, in a Husserlian perspective. The paper of Arturo Romero first presents the inner relation between the phenomenological concept of intentionality and space in a general mathematical sense. Then briefly characterises the use of the geometrical concept of manifold (*Mannigfaltigkeit*) in Husserl's work. Next, he presents some examples of the use of the concept in Husserl's analyses of space, time, and intersubjectivity, pointing out some difficulties in his endeavour. Finally, he offers some points of coincidence between phenomenology and category theory suggesting that the latter can work as a formal frame for ontology in the former. The thesis of the paper is that intentionality operates in different levels as a morphism, functor, and natural transformation. Carlos Lobo's paper is a study of the way in which diagrams of time have functioned in Husserlian phenomenology in the most fertile period on the subject, i.e. between the lessons of 1905 and the Manuscripts from 1917-1918. While doing so, he offers a phenomenological clarification on the role of diagrams in science as well as in phenomenology.

The sixth part is on "Diagrams, Gestures, and Subjectivity". The paper of Hye Young Kim presents a topological analysis of space-time consciousness. The paper attempts to explore a possibility to visualise the structure of time-consciousness in a knot shape. By applying Louis Kauffman's knot-logic, the consistency of subjective consciousness, the plurality of now's, and the necessary relationship between subjective and intersubjective consciousness are represented in topological space. Philippe Roy paper is on gestures, diagrams, and subjectivity. According to the author, the subjectivity can be thought through the new articulations of the dualities "Individual/Society" or "Individual unity/Multiplicity of becoming", rather than by the category of substance. The paper of Filipe Varela presents some grounds to further understand what imagination is, what it does, how it possibly works, and where it may be located in our nervous systems. Such an approach follows the imperative to correlate the precious insights coming from philosophy, with those of the sciences that nowadays study our perceptual and cognitive apparatus from a physiological and bio-chemical perspective. Such correlation is vital to a comprehensive understanding of this elusive thing, imagination. The paper of Fabien Ferri is on the diagrammatic language beyond the phenomenological difference. In other words, following the ideas proposed by Bruno Bachimont, the author discusses the possibility of overcoming the traditional difference between scientific language or knowledge whose main function is to calculate, and phenomenological language or knowledge, which can be expressed by natural language and where "saying stands for having a meaning".

The seventh part is about "Diagrams, from Mathematics to Aesthetics". The paper of Charles Alunni shows how *Ars diagrammaticae* discusses the significance of the art of diagram and its philosophical implications through the analysis of some examples took from modern and contemporary mathematical physics. Alunni draws

on the concepts and operations at work in contemporary mathematics to question the classical philosophical distinctions between image, figure, and diagram. The paper of Jakub Zdebik shows how, despite its abstract and non-figurative character, Gilles Deleuze's diagram has stimulated a deep methodological renewal in aesthetics of visual arts. Amélie de Beauffort's paper focuses on the idea that drawing allow for a plastic foundation of thought, which is important for creativity and also for living in the world. This viewpoint is motivated by a graphic practice which rests on a plastic morphology of the manipulation of knots and gestures that make and unmake the drawing. Farah Khelil paper deals with the emergence of a reasoning owing to the diagram in its artistic practice and in particular in a recent work entitled *Point d'étape*. She clarifies the singularity of the diagram with respect to the relation of art with philosophy and its relationship with the reality. She also enquires the painting and its relationship to space.

The eighth part of the book is on "Poetics and Politics of Diagrams". Catherine Paoletti focuses on the tentative by Gilles Châtelet, who stressed that the diagram is at the same time tool, object, and locus of thinking, which give rise to new spatial configurations, therefore to potential movements in space and also to new dynamics in the writing. The paper of Tatiana Roque is on diagrams of the possibility, by establishing a link between the phase space (a mathematical object) and the political subject. The function of the diagram is, as stressed by Deleuze, a goal which introduces new effective possibilities in science, as well in social sciences; in other words, it is an operative concept which can change reality. In its paper, Noëlle Batt, after briefly recalling the evolution of the concept of diagram in the philosophical work of Gilles Deleuze, first present the hypothesis that literary and more particularly poetic writing has a diagrammatic dimension. The hypothesis is sustained by a number of resonances between, on the one hand, the features and morpho-dynamic processes which are prominent in the definition of diagram proposed by different disciplines, and, on the other hand, those which characterise the specific reorganisation of signs and infra-semantic elements of the language which constitute the material of the literary text.

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Logic, Forms and Diagrams

The Semiotics of Laws of Form

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Abstract The purpose of this essay is to explore the idea of a sign, using G. Spencer-Brown's work "Laws of Form" as a pivot, a reference and a place from which to make excursions into both simplicity and complexity. In order to handle the simplicity of the issues involved in thinking about distinction, Spencer-Brown's introduction of a language that has only one sign is an instrument of great delicacy.

Keywords Semiotic · Diagram · Logic · Proof · Knots

Introduction

The purpose of this essay is to explore the idea of a sign, using G. Spencer-Brown's work "Laws of Form" (Spencer-Brown 1969) as a pivot, a reference and a place from which to make excursions into both simplicity and complexity. In order to handle the simplicity of the issues involved in thinking about distinction, Spencer-Brown's introduction of a language that has only one sign is an instrument of great delicacy.

The Spencer-Brown mark ⌝ is a sign that can represent any sign, and so begins semiotics in both universal and particular modes. The mark is seen to make a distinction in the space in which it is written, and so can be seen, through this distinction, to refer to itself. In the language of Charles Sanders Peirce, the mark is its own representamen and it is also its own interpretant. The sign that the mark produces for somebody is, in its form, the mark itself. By starting with the idea of distinction we find, in the mark, the first sign and the beginning of all possible signs.

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Right, that's how long it takes, not a day less,—Qfwfq said,—once, as I went past, I drew a sign at a point in space, just so I could find it again two hundred million years later, when we went by the next time around. What sort of sign? It's hard to explain because if I say sign to you, you immediately think of a something that can be distinguished from a something else, but nothing could be distinguished from anything there; you immediately think of a sign made with some implement or with your hands, and then when you take the implement or your hands away, the sign remains, but in those days there were no implements or even hands, or teeth, or noses, all things that came along afterwards, a long time afterwards. As to the form a sign should have, you say it's no problem because, whatever form it may be given, a sign only has to serve as a sign, that is, be different or else the same as other signs: here again it's easy for you young ones to talk, but in that period I didn't have any examples to follow, I couldn't say I'll make it the same or I'll make it different, there were no things to copy, nobody knew what a line was, straight or curved, or even a dot, or a protuberance or a cavity. I conceived the idea of making a sign, that's true enough, or rather, I conceived the idea of considering a sign a something that I felt like making, so when, at that point in space and not in another, I made something, meaning to make a sign, it turned out that I really had made a sign, after all.

In other words, considering it was the first sign ever made in the universe, or at least in the circuit of the Milky Way, I must admit it came out very well. Visible? What a question! Who had eyes to see with in those days? Nothing had ever been seen by anything, the question never even arose. Recognizable, yes, beyond any possibility of error: because all the other points in space were the same, indistinguishable, and instead, this one had the sign on it.

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I thought about it day and night; in fact, I couldn't think about anything else; actually, this was the first opportunity I had had to think something; or I should say: to think something had never been possible, first because there were no things to think about, and second because signs to think of them by were lacking, but from the moment there was that sign, it was possible for someone thinking to think of a sign, and therefore that one, in the sense that the sign was the thing you could think about and also the sign of the thing thought, namely, itself.

So the situation was this: the sign served to mark a place but at the same time it meant that in that place there was a sign (something far more important because there were plenty of places but there was only one sign) and also at the same time that sign was mine, the sign of me, because it was the only sign I had ever made and I was the only one who had ever made signs. It was like a name, the name of that point, and also my name that I had signed on that spot; in short, it was the only name available for everything that required a name.

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In the universe now there was no longer a container and a thing contained, but only a general thickness of signs superimposed and coagulated, occupying the whole volume of space; it was constantly being dotted, minutely, a network of lines and scratches and reliefs and engravings; the universe was scrawled over on all sides, along all its dimensions. There was no longer any way to establish a point of reference: the Galaxy went on turning but I could no longer count the revolutions, any point could be the point of departure, any sign heaped up with the others could be mine, but discovering it would have served no purpose, because it was clear that, independent of signs, space didn't exist and perhaps had never existed. [A Sign in Space, Cosmicomix by Italo Calvino. Copyright © 1965 by Giulio Einaudi Editore, S.p.A. English translation copyright © 1968 by Harcourt Brace & Company and Jonathan Cape Limited]

Finding Distinction

We begin by discussing (the idea of) distinction. If one looks for the definition of distinction in any dictionary the result is circular. Distinction is defined as a difference. Difference is defined as a form of distinction. The meaning of distinction as an indication of outstanding value is also an instance of special difference. Fields of study are founded in the use and examination of certain basic distinctions.

Mathematics is constructed set theoretically by using the concept of a collection. A collection is a distinction of membership. For example the set of prime numbers connotes the distinction between composite and prime among the positive integers. At the level of sets themselves, the empty set, denoted by brackets containing nothing {}, is a distinction between void and an empty container. The very sign for the empty set consists of two brackets (left and right) that together can be interpreted as a container for something that is placed between them. In the case of the empty set, nothing is placed between the brackets. The brackets themselves are shaped as cusps.

Each cusp can be seen as a process of bifurcation that gives rise to the distinction between the branches of the cusp. The two cusps (brackets) are mirror imaged with respect to one another, and it is this symmetry across an imaginary mirror between them that gives us the possibility to see them together as one container. The brackets are two and yet they are one (via the mirror symmetry).

At this point (in the encounter with the empty set) we reach a semantic divide between the mode of speaking of mathematicians trained in logical formalism and a wider analysis of language that I refer to as semiotic. In speaking of semiotics I am relying for its root meanings as expressed by Charles Sanders Peirce:

> [Semiotics is a] quasi-necessary, or formal doctrine of signs ... which abstracts what must be the characters of all signs used by an intelligence capable of learning by experience, ... and which is philosophical logic pursued in terms of signs and sign processes. [Peirce, C. S., *Collected Papers of Charles Sanders Peirce*, vol. 2, paragraph 227]

> A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign. That sign which it creates I call the interpretant of the first sign. The sign stands for something, its object not in all respects, but in reference to a sort of idea which I have sometimes called the ground of the representation. [Peirce—Vol. 2, p. 228]

Peirce goes on to say

> The object of representation can be nothing but a representation of which the first representation is the interpretant. But an endless series of representations, each representing the one behind it, may be conceived to have an absolute object as its limit. The meaning of a representation can be nothing but a representation. In fact, it is nothing but the representation

> itself conceived as stripped of irrelevant clothing. But this clothing never can be completely stripped off; it is only changed for something more diaphanous. So there is an infinite regression here. Finally, the interpretant is nothing but another representation to which the torch of truth is handled along; and as representation, it has its interpretant again. Lo, another infinite series. [Peirce—Vol. 1, p. 339]

Peirce concentrates on the structure of signs and that for him signs either are, or stand for, certain distinctions. To begin with signs is to begin with something apparently definite and yet, as soon as the discussion begins, we find there are only signs (see above) "A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign." Thus what is in the mind of another person is also a sign, albeit a sign that is understood internally by that person. One can look and look for substance that may underlie the sign but the search always leads to more signs. In this expansion of signs related to signs, signs describing signs, signs and interpretant signs, the self becomes yet another sign standing in relation to all the signs that work at the nexus that the person represents. The sign of the self becomes a limit of all the signs that are the life of that self. The distinction of a person is her sign of distinction, her sign of self.

Spencer-Brown in his book "Laws of Form" (here to be abbreviated LOF), makes a new semiotic start, beginning with the idea of distinction. Signs arise as we shall see, but Spencer-Brown begins with the pronouncement:

> We take as given the idea of distinction and the idea of indication, and that we cannot make an indication without drawing a distinction. We take, therefore, the form of distinction for the form. (LOF, page 1).

It is as this point that Peirce and Spencer-Brown come into contact. For in Chapter 12 in the last sentence of Laws of Form, Spencer-Brown writes "We see now that the first distinction, the mark, and the observer are not only interchangeable, but, in the form, identical." Here the mark is the first made sign or indication of a first distinction. The observer can be identified with the interpretant in so much as the interpretant (see the quote from Peirce above) is an equivalent sign created in the mind of somebody, and must for its existence partake of the being of that somebody. At this nexus Spencer-Brown indicates the essential identity of sign, representamen and interpretant. The three coalesce into the form that is the form of distinction.

The form of distinction becomes, in Spencer-Brown, a background for the entire play of signs that is the context of Peircean semiotics. We take the form of distinction for the form. And in this saying "the form" becomes a noun as elusive as it seems to be concrete, just as is the nature of the sign in Peirce. The form of a distinction drawn as a circle in the plane is geometrical form, the circle. But the form of distinction, the form of the idea of distinction, what is this form?

The echo from Peirce is clear as a bell. The form of distinction calls up a sign in the mind of some person. It is an amalgamation or superposition of all the signs for distinction in the history of that mind, that observer or all observers. We come forth in the complexity of experience to a sharp idea of the distinct. We can give instructions for the performance of an act of distinction, while simultaneously

Definition

Distinction is perfect continence.

That is to say, a distinction is drawn by arranging a boundary with separate sides so that a point on one side cannot reach the other side without crossing the boundary. For example, in a plane space a circle draws a distinction.

Once a distinction is drawn, the spaces, states, or contents on each side of the boundary, being distinct, can be indicated.

There can be no distinction without motive, and there can be no motive unless contents are seen to differ in value.

Fig. 1 Definition

understanding that it is a creative act, not bound by any given set of rules or regulations.

The next lines of Laws of Form are shown in Fig. 1. We give these quotes by direct photocopy to show the layout in the text of Laws of Form. After some thought the reader may come to realize that this paragraph is an amalgam of words that all stand for aspects of distinction: *definition, continence, boundary, separate, sides, point, draw a distinction, spaces, states, contents, side of the boundary, being distinct, indicated, motive, differ in value*. The paragraph is not a definition in the mathematical sense of definition: something in terms of previously defined things. There is no possibility to define distinction in terms of previous somethings that are not distinctions. The only possibility is to define distinction in terms of itself. We take the form of distinction for the form.

The paragraph is nevertheless readable. How did this happen? How could readability arise from circularity? The answer is in the injunctive power of language. This same paragraph contains the phrases: "arranging a boundary so that a point on one side cannot reach the other side without crossing the boundary", "a circle draws a distinction", "a distinction is drawn", "spaces, states, or contents ... can be indicated", "contents are seen to differ in value". At once the paragraph is an amalgam of synonyms for distinction and it is a catalog of injunctions to arrange a boundary, to draw a circle, to indicate, to see the difference in value. We are invited to take these steps and so enter into a contract of exploring the concept and practice of distinction.

Let us not forget that we have followed already the injunction of the first line: "We take as given the idea of distinction and the idea of indication and that we cannot make an indication without drawing a distinction." It is already given that there is a something called indication that entails the making of a distinction. And implicitly it is called up that a distinction could occur without any indication. We cannot make an indication without drawing a distinction. Can we have a distinction without making an indication? We are falling down the rabbit hole.

If a content is of value, a name can be taken to indicate this value.

Thus the calling of the name can be identified with the value of the content.

Axiom 1. The law of calling

The value of a call made again is the value of the call.

That is to say, if a name is called and then is called again, the value indicated by the two calls taken together is the value indicated by one of them.

That is to say, for any name, to recall is to call.

Fig. 2 Calling

But here, we have to look and see. In most circumstances, to draw is to indicate. A notion of privacy is another form of distinction. Can I hide distinctions within the boundary of my privacy distinction? Then I can pretend that there are distinctions that do not have indications. What a tangled web we weave in order to believe. In order to make a distinction without an indication, we are entangled in a web of new distinctions. The very act of drawing is a form of indication, and it must be concealed? Must we search for distinctions that are made without drawing a distinction? I sit before an emptiness. The emptiness is distinct for me. It is empty and I am empty before it. It is possible to have less action not more. And in the limit of acting gently in emptiness, or not at all, there seems to be the possibility of distinction without indication.

Figure 2 illustrates the next few lines of Laws of Form. Take the first sentence shown in Fig. 2: "If a content is of value, then a name can be taken to indicate this value." Already we have faced the multiplicity of names for a distinction and that making an indication is a special act that cannot happen without the making of a distinction. Nevertheless, it comes as a shock that suddenly a *name* can be called forth. A name can be taken to indicate a value. A distinction can be performed that allows the performance of a distinction. We begin to realize that in this condensed place where there is only creation of distinction, boundary or the crossing of the boundary, the only distinction is at first the distinction between nothing (the unmarked) and the act of creation, and then arises a distinction between name and act. If a state is of value then a name can be taken to indicate this value. If a distinction is a distinction then a distinction of distinctioning can be distinctioned to distinction this distinction. We are down the rabbit hole again. One side makes you smaller. One side makes you larger. Choose a door and pass through it. The act and the name are not different. The indication of a distinction, the crossing of the boundary of the distinction and the distinction "itself" are in the form identical.

We come to the creation of a name and find that this is the same as the creation of a distinction. They are one and the same. And yet a name can be separated from the distinction to which it refers. The name can be taken to be a new distinction that refers to the first distinction. Indeed we can imagine that the original distinction (for example a circle drawn in the plane) is seen (in all quietness) to stand for, to indicate, itself. But in the act of recognizing this possibility that "it" could stand for "itself" we have made a distinction between "it" and "itself". We have allowed a condensation by making the possibility of a separation. The name and the sign are born in that process. The name, the sign, is Peirce's representamen, a sign residing in the mind of somebody. And we conclude that both the sign, the name and the original distinction all reside in the mind of somebody. At the point of condensation, the mind is the sign and the sign is the mind. No mind, no distinction. No distinction, no mind. We take the form as the form of distinction. Form is emptiness. Emptiness is form.

In his "A Note on the Mathematical Approach" Spencer-Brown writes "The act is itself already remembered . . . as our first attempt to distinguish different things in a world where, in the first place, the boundaries can be drawn anywhere we please. At this stage the universe cannot be distinguished from how we act upon it, and the world may seem like shifting sand beneath our feet." The act of naming ("If a content is of value, then a name can be taken to indicate this value.") is the key step toward a world of apparent distinctions. It is by naming a distinction that we call it into being. In Fig. 2 we see the first of the Laws of Form, the "Law of Calling: The value of a call made again is the value of the call." It is enough to indicate the name once. For any name, to recall is to call.

In Fig. 3 we find the "Law of Crossing: The value of a crossing made again is not the value of the crossing.". At this point a distinction is made between crossing (the boundary of a distinction) and calling the name of a distinction. For "The value of a call made again is the value of the call." Crossing and calling appear to be given as terms in a similar level of speech, and yet they are declared to be different. We understand that the crossing of the boundary can be the act of naming the distinction. I cross into "riding" when I cross the boundary of balance and actually ride the bicycle. I name riding by actually engaging in the act of riding. If I cease to ride,

Axiom 2. The law of crossing

The value of a crossing made again is not the value of the crossing.

That is to say, if it is intended to cross a boundary and then it is intended to cross it again, the value indicated by the two intentions taken together is the value indicated by none of them.

That is to say, for any boundary, to recross is not to cross.

Fig. 3 Crossing

Construction

Draw a distinction.

Content

Call it the first distinction.

Call the space in which it is drawn the space severed or cloven by the distinction.

Call the parts of the space shaped by the severance or cleft the sides of the distinction or, alternatively, the spaces, states, or contents distinguished by the distinction.

Intent

Let any mark, token, or sign be taken in any way with or with regard to the distinction as a signal.

Call the use of any signal its intent.

Fig. 4 Construction, content and intent

then the value of riding ceases. The distinction of riding is no longer present. And yet it can still be named.

At this point, we have come to the end of Spencer-Brown's discussion of Laws of Form that makes no explicit use of a sign of distinction. The word "sign" has not yet occurred for Spencer-Brown. The first use of the word sign is in the next chapter of the book entitled "Forms Taken Out of the Form" and is shown in Fig. 4.

In this development. The injunctive mode has taken priority. The text tells its reader to "Draw a distinction." and to "Call it the first distinction.". This should sweep away any notion that first distinction is an absolute concept.

The first distinction is the one that is under discussion. The form is the form of the first distinction. And so the form of the first chapter has shifted from the universal to the particular, and the form of distinction is the form of that first distinction. The form is inherent in any act of distinction. Still speaking of Fig. 4, we find that at the point of intent "Let any mark, token, or sign be taken in any way with or with regard to the distinction as a signal. Call the use of any signal its intent.". Here is the entry of the word sign into Spencer-Brown's consideration of distinction and form.

Now we listen again to Peirce. "A sign, or representamen, is something which stands to somebody for something in some respect or capacity." Indeed Spencer-Brown's mark, token or sign is a sign in the Peircean sense. This sign is taken as a signal (in the condensation of Laws of Form, a signal is yet a sign) with regard to or