Inventory Optimization

Nita H. Shah
Mandeep Mittal *Editors*

Soft Computing in Inventory Management

Springer

Inventory Optimization

Series Editors

Inventory management is a very tedious task faced by all the organizations in any sector of the economy. It makes decisions for policies, activities and procedures in order to make sure that the right amount of each item is held in stock at any time. Many industries suffer from indiscipline in ordering and production mismatch. Providing best policy to control such mismatch would be invaluable to them.

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Retailer's Optimal Ordering Policy Under Supplier Credits When Demand is Fuzzy and Cloud Fuzzy

Nita H. Shah and Milan B. Patel

Abstract This paper deals with retailer's optimal ordering inventory model under fuzzy and cloud fuzzy environment. In this study, crisp model is considered first and then by assuming demand rate as triangular fuzzy number and cloud triangular fuzzy number the model is formulated and solved. Extension of Yager's ranking index is utilized for defuzzification in cloud fuzzy model. The objective of the present work is to minimize the total inventory cost and to compare the results obtained by the existing crisp model. With the help of numerical examples for different cases under different environments, optimal solutions are compared and analysed by performing sensitivity analysis. For better visualization of results, graphical representation of solutions is given.

Keywords Inventory · Fuzzy demand · Cloud fuzzy demand · Deterioration · Delay in payments

1 Introduction

Among many of the factors affecting the performance of a business firm, management of inventory system is considered to be one of the most important aspects, as it directly affects the profit of the firm and the satisfaction of customers. From a small retailer shopkeeper to large industries always keep on applying new business tactics in order to attract new customers and to increase sales of their products. Out of these many business tactics, an idea implemented by many such suppliers is to provide a cash discount or grace period (i.e. trade credit period) to their customers in order to pay for the consignment. In such cases, it becomes indispensable for the retailer to make a balance between the situation of stock-out and the situation of overstocking. This study aims at modelling such phenomenon under uncertain demand rate and to provide retailers an optimal ordering policy when supplier offers some trade credit period.

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After the pioneering work of Harris (1913) of developing economic order quantity (EOQ) model, many researchers have devoted their efforts towards modelling the inventory system. Haley and Higgins (1973) introduced a trade credit policy of an inventory system having constant demand. Goyal (1985) firstly extended an EOQ model by allowing permissible delay in payments. For more literature in the field of trade credit policy under the crisp environment, one may refer to the works of Shah (1993), Aggarwal and Jaggi (1995), Liao et al. (2000), Chang and Teng (2004), Teng (2009), Shah and Cardenas-Barron (2015), Giri and Sharma (2016), and Mahata et al. (2020).

Since the decision-making process involves human thoughts and reasoning, it always has some imprecision in it. Fuzzy set theory introduced by Zadeh (1965) is the most powerful tool to express uncertainties. The above-mentioned inventory research papers were formulated by assuming the parameters to be crisp. However, to make these models more realistic and applicable, one needs to incorporate the fuzzy set theory. Park (1987) extended an EOQ model under fuzzy sense. Since then, many researchers have contributed significantly in developing inventory modelling under fuzzy sense. Following are some of the research papers related to this study.

Mahata and Goswami (2007) developed an EOQ model for deteriorating item by allowing delay in payments in fuzzy sense. The paper also generalizes the previous publications in this direction. Ouyang et al. (2010) worked on an optimal inventory policy by considering rate of interest earned, rate of interest charged and deterioration rate as triangular fuzzy number. Mahata and Mahata (2011) studied an inventory model for a retailer under two-level trade credit in fuzzy sense. Shah et al. (2012) established a fuzzy EOQ model by allowing demand rate, ordering cost and selling price as fuzzy quantities. They have used centre of gravity method for defuzzification. Jaggi et al. (2014) gave an inventory model in which they have used trapezoidal fuzzy numbers to represent uncertainty in some parameters. Bag and Chakraborty (2014) worked on a fuzzy inventory model with bi-level trade credit policy. Sujatha and Parvathi (2015) discussed an inventory model for variable deteriorating items with time-dependent Weibull demand rate by allowing shortages. Majumder et al. (2015) studied an economic production quantity (EPQ) model under partial trade credit by incorporating crisp as well as fuzzy demand rate. Das et al. (2015) developed an integrated inventory model for supplier and retailer with fuzzy credit period. Yadav et al. (2015) studied retailer's inventory model by discussing the effects of the inflation rate, deterioration rate and delay in payment on total profit of the inventory. Shukla and Suthar (2016) worked on fuzzy economic ordering policy to minimize total cost by taking into account the items having uncertain maximum lifetime. Huang et al. (2019) developed a vendor–buyer ordering policy of perishable items under crisp and fuzzy environment.

The concept of fuzzy number utilized in the above-mentioned papers dealing with fuzzy inventory modelling assumes fuzziness to be constant forever which may not be the case in real scenario. Decision-maker can make better decision over time as he gains experiences from the previous consignments. Owing to this idea, the concept of cloud fuzzy number is introduced recently by De and Beg (2016) and applied by some researchers in order to make the inventory models more realistic. De and Mahata

(2016) introduced the concept of cloudy fuzzy number and formulated an inventory model with backorder. Berman et al. (2017) formulated a backordered inventory model with inflation under cloudy fuzzy environment. Karmakar et al. (2018) worked on extension of classical EOQ model under cloudy fuzzy demand rate. De and Mahata (2019a) studied cloudy fuzzy EOQ model for imperfect quality items. Further, De and Mahata (2019b) developed EOQ model under fuzzy monsoon demand. Maiti (2019) utilized the concept of cloudy fuzzy number and studied economic production lot-size model with fixed set-up cost with cloudy fuzzy demand rate.

The present study is an attempt to extend Chang and Teng (2004) model under fuzzy and cloud fuzzy demand rate. To study the model under fuzzy environment, demand rate is assumed to be triangular fuzzy number. For the extension of model under cloud fuzzy environment, cloud triangular fuzzy number is employed. For defuzzification in fuzzy environment, the researchers have used Yager's ranking index method (1981).

2 Notations and Assumptions

Following notations and assumptions are considered while formulating mathematical models.

2.1 Notations

h	holding cost (in \$/unit/year)
c	purchase cost (in \$/unit)
s	sales price (in \$/unit)
A	ordering cost (in \$/order)
θ	rate of deterioration $0 \le \theta < 1$
r	rate at which cash discount is given $0 < r < 1$
Ic	rate at which interest is charged (in %/year)
Ie	rate at which interest is earned (in %/year)
M_1	1st credit limit for retailer
M_2	2nd credit limit for retailer
T	cycle length (in year)
R	demand rate per year
$\overline{R}$	fuzzy demand rate per year
$\widetilde{R}$	cloud fuzzy demand rate per year
Q	order quantity

(continued)

(continued)

h	holding cost (in \$/unit/year)
$\overline{Q}$	fuzzy order quantity
$\widetilde{Q}$	cloud fuzzy order quantity
K	total inventory cost (in \$/year)
$\overline{K}$	total fuzzy inventory cost (in \$/year)
$\widetilde{K}$	total cloud fuzzy inventory cost (in \$/year)

2.2 Assumptions

(i) Rate of deterioration is considered to be constant. Further, no replenishment or repair of deteriorated items occurs during planning horizon.
(ii) Retailer has two choices for payment. Either pay at credit limit M_1 with discounted price $(1-r)c$ with $0 < r < 1$ or pay at credit limit M_2 without any discount. $(M_1 < M_2)$
(iii) Up to the credit period (i.e. M_1 or M_2), the amount generated by sales is deposited in an interest earning account. At the end of this period, retailer pays the amount generated in the account to supplier. If this amount is not sufficient to settle payment, the retailer starts paying off the remaining amount whenever he has money generated by sales.
(iv) Shortages are not permitted.
(v) Demand rate for crisp model is constant.
(vi) For fuzzy and cloud fuzzy model, demand is not precise and characterized by triangular fuzzy number $\overline{R} = (R_1, R_2, R_3)$ and cloud triangular fuzzy number $\widetilde{R} = \left(R_2\left(1 - \frac{\beta}{1+t}\right), R_2, R_2\left(1 + \frac{\gamma}{1+t}\right)\right)$, respectively.
(vii) Planning horizon is infinite.

3 Preliminary Concepts

3.1 Triangular Fuzzy Number (TFN)

A triangular fuzzy number (TFN) defined on the set of real numbers $\mathbb{R}$ can be expressed as $\overline{R} = (R_1, R_2, R_3)$. Its membership function can be defined as

$$f(\overline{R}) = \begin{cases} \frac{x-R_1}{R_2-R_1}, & R_1 \le x \le R_2 \\ \frac{x-R_3}{R_2-R_3}, & R_2 \le x \le R_3 \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

3.2 α- Cut of TFN

α- cut of TFN $\overline{R} = (R_1, R_2, R_3)$ is a crisp set $\alpha_{\overline{R}} = [L_\alpha, R_\alpha]$, where $L_\alpha = R_1 - \alpha(R_1 - R_2)$ is known as left α- cut and $R_\alpha = R_3 - \alpha(R_3 - R_2)$ is known as right α- cut $(0 \le \alpha \le 1)$.

3.3 Cloud Triangular Fuzzy Number (CTFN)

A triangular fuzzy number is known as cloud triangular fuzzy number (CTFN) if the set converge to a crisp number as time tends to infinity.

$$\widetilde{R} = \left(R_2\left(1 - \frac{\beta}{1+t}\right), R_2, R_2\left(1 + \frac{\gamma}{1+t}\right)\right) \tag{2}$$

where $\beta, \gamma \in (0, 1)$ and $t > 0$. From Eq. (2), it can be seen that as $t \to \infty$, $\widetilde{R} \to \{R_2\}$.

Membership function of CTFN can be defined as follows:

$$g(\widetilde{R}, t) = \begin{cases} \frac{x - R_2\left(1 - \frac{\beta}{1+t}\right)}{\frac{\beta R_2}{1+t}}, & R_2\left(1 - \frac{\beta}{1+t}\right) \le x \le R_2 \\ \frac{R_2\left(1 + \frac{\gamma}{1+t}\right) - x}{\frac{\gamma R_2}{1+t}}, & R_2 \le x \le R_2\left(1 + \frac{\gamma}{1+t}\right) \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

3.4 Left and Right α- cut of CTFN

Left and right α- cut of CTFN can be expressed as

$$L_{\alpha,t} = R_2\left(1 - \frac{\beta}{1+t}\right) + \frac{\alpha\beta}{1+t}R_2 \quad \& \quad R_{\alpha,t} = R_2\left(1 + \frac{\gamma}{1+t}\right) - \frac{\alpha\gamma}{1+t}R_2 \tag{4}$$

respectively.

3.5 Yager's Ranking Index Method (1981)

According to Yager's ranking index method, defuzzification for a TFN can be given by

$$YRI(\overline{R}) = \frac{1}{2}\int_{0}^{1} (L_\alpha + R_\alpha)\, d\alpha \tag{5}$$

where L_α and R_α are left and right α- cut of TFN, respectively. By substituting the value of L_α and R_α, Eq. (5) reduces to

$$YRI(\overline{R}) = \frac{1}{4}(R_1 + 2R_2 + R_3) \tag{6}$$

3.6 Yager's Ranking Index Method for CTFN

It is an extension of Yager's ranking index method for TFN given by De and Mahata (2017). By this method, defuzzification of CTFN can be given by

$$YRI(\widetilde{R}) = \frac{1}{2T}\int_{\alpha=0}^{\alpha=1}\int_{t=0}^{t=T} (L_{\alpha,t} + R_{\alpha,t})\, d\alpha\, dt \tag{7}$$

Substituting the value of left and right α- cut of CTFN from Eqs. (4) and (7) reduces to

$$YRI(\widetilde{R}) = R_2\left(1 - \frac{(\beta-\gamma)}{4}\frac{\log(1+T)}{T}\right) \tag{8}$$

4 Mathematical Modelling

As per the model given by Chang and Teng (2004), total inventory cost for different cases under crisp environment is as follows:

Case A: $T \geq M_1$

$$K_A(T) = \frac{A}{T} + \frac{R[h + c\theta(1-r)]}{\theta^2 T}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - \frac{hR}{\theta} - \frac{sIeR}{2T}M_1^2 + \frac{IcR}{2sT}\left[\frac{c(1-r)}{\theta}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - sM_1\left(1 + \frac{IeM_1}{2}\right)\right]^2 \tag{9}$$

Case B: $T < M_1$

$$K_B(T) = \frac{A}{T} + \frac{R[h + c\theta(1-r)]}{\theta^2 T}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - \frac{hR}{\theta} - sIeR\left(M_1 - \frac{T}{2}\right) \tag{10}$$

Case C: $T \geq M_2$

$$K_C(T) = \frac{A}{T} + \frac{R(h + c\theta)}{\theta^2 T}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - \frac{hR}{\theta} - \frac{sIeR}{2T}M_2^2 + \frac{IcR}{2sT}\left[\frac{c}{\theta}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - sM_2\left(1 + \frac{IeM_2}{2}\right)\right]^2 \tag{11}$$

Case D: $T < M_2$

$$K_D(T) = \frac{A}{T} + \frac{R(h + c\theta)}{\theta^2 T}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - \frac{hR}{\theta} - sIeR\left(M_2 - \frac{T}{2}\right) \tag{12}$$

Order quantity can be expressed as

$$Q = \frac{R}{\theta}\left(e^{\theta T} - 1\right) \tag{13}$$

4.1 Formulation of Fuzzy Mathematical Model

In order to extend Chang and Teng (2004) model under fuzzy environment, demand is assumed to be triangular fuzzy number $\overline{R} = (R_1, R_2, R_3)$. Fuzzifying the expression given in Eq. (9), the problem under fuzzy environment for Case A reduces to

$$\text{Minimize } \overline{K_A}(T) = \frac{A}{T} + \frac{\overline{R}[h + c\theta(1-r)]}{\theta^2 T}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - \frac{h\overline{R}}{\theta} - \frac{sIe\bar{R}}{2T}M_1^2 + \frac{Ic\overline{R}}{2sT}\left[\frac{c(1-r)}{\theta}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - sM_1\left(1 + \frac{IeM_1}{2}\right)\right]^2 \tag{14}$$

$$\text{with respect to } \overline{Q} = \frac{\overline{R}}{\theta}\left(e^{\theta T} - 1\right) \tag{15}$$

With the help of Eq. (1), membership function for fuzzy objective function and fuzzy order quantity can be expressed as follows:

(i) Membership function for total fuzzy inventory cost:

$$f_1(K) = \begin{cases} \frac{K-K_1}{K_2-K_1}, K_1 \le K \le K_2 \\ \frac{K-K_3}{K_2-K_3}, K_2 \le K \le K_3 \\ 0, \quad \text{otherwise} \end{cases} \tag{16}$$

where K_i for $i = 1, 2, 3$ can be obtained by replacing R with R_i in fuzzy inventory cost function.

(ii) Membership function for fuzzy order quantity:

$$f_2(Q) = \begin{cases} \frac{Q-Q_1}{Q_2-Q_1}, Q_1 \le Q \le Q_2 \\ \frac{Q-Q_3}{Q_2-Q_3}, Q_2 \le Q \le Q_3 \\ 0, \quad \text{otherwise} \end{cases} \tag{17}$$

where $Q_i = \frac{R_i}{\theta}(e^{\theta T} - 1)$ for $i = 1, 2, 3$.

Other cases can be formulated under fuzzy environment similarly.

By applying Yager's ranking index method for TFN (see Sect. 1.3.5), defuzzified value of total fuzzy inventory cost and fuzzy order quantity for each case is obtained. Defuzzified value of total fuzzy inventory cost for Case A, Case B, Case C and Case D is

$$I(\overline{K_A}) = \frac{A}{T} + (R_1 + 2R_2 + R_3)\begin{Bmatrix} \frac{1}{4\theta^2}\left[(h + c\theta(1-r))\left(\theta + \frac{1}{2}\theta^2 T\right)\right] - \frac{h}{4\theta} - \frac{sIeM_1^2}{8T} \\ +\frac{Ic}{8sT}\left[c(1-r)\left(T + \frac{1}{2}\theta T^2\right) - sM_1\left(1 + \frac{IeM_1}{2}\right)\right]^2 \end{Bmatrix} \tag{18}$$

$$I(\overline{K_B}) = \frac{A}{T} + (R_1 + 2R_2 + R_3)\begin{Bmatrix} \frac{1}{4\theta^2}\left[(h + c\theta(1-r))\left(\theta + \frac{1}{2}\theta^2 T\right)\right] - \frac{h}{4\theta} \\ -\frac{sIe}{4}\left(M_1 - \frac{T}{2}\right) \end{Bmatrix} \tag{19}$$

$$I(\overline{K_C}) = \frac{A}{T} + (R_1 + 2R_2 + R_3)\begin{Bmatrix} \frac{1}{4\theta^2}\left[(h + c\theta)\left(\theta + \frac{1}{2}\theta^2 T\right)\right] - \frac{h}{4\theta} - \frac{sIeM_2^2}{8T} \\ +\frac{Ic}{8sT}\left[c\left(T + \frac{1}{2}\theta T^2\right) - sM_2\left(1 + \frac{IeM_2}{2}\right)\right]^2 \end{Bmatrix} \tag{20}$$

$$I(\overline{K_D}) = \frac{A}{T} + (R_1 + 2R_2 + R_3)\left\{\frac{1}{4\theta^2}\left[(h + c\theta)\left(\theta + \frac{1}{2}\theta^2 T\right)\right] - \frac{h}{4\theta} - \frac{sIe}{4}\left(M_2 - \frac{T}{2}\right)\right\} \tag{21}$$

respectively.

Defuzzified value of fuzzy order quantity can be represented as

$$I(\overline{Q}) = \frac{1}{4\theta}(R_1 + 2R_2 + R_3)\left(e^{\theta T} - 1\right) \tag{22}$$

4.2 Formulation of Cloud Fuzzy Mathematical Model

Fuzzifying the expression given in Eq. (9), the problem under cloud fuzzy environment is given by

$$\text{Minimize } \widetilde{K}_A(T) = \frac{A}{T} + \frac{\widetilde{R}[h + c\theta(1-r)]}{\theta^2 T}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - \frac{h\widetilde{R}}{\theta} - \frac{sIe\widetilde{R}}{2T}M_1^2 + \frac{Ic\widetilde{R}}{2sT}\left[\frac{c(1-r)}{\theta}\left(\theta T + \frac{\theta^2 T^2}{2}\right) - sM_1\left(1 + \frac{IeM_1}{2}\right)\right]^2 \tag{23}$$

$$\text{with respect } to \quad \widetilde{Q} = \frac{\widetilde{R}}{\theta}\left(e^{\theta T} - 1\right) \tag{24}$$

With the help of Eq. (3), membership function for cloud fuzzy objective function and cloud fuzzy order quantity can be expressed as follows:

(i) Membership function for total cloud fuzzy inventory cost:

$$g_1(K, T) = \begin{cases} \frac{K-K_1}{K_2-K_1}, \; K_1 \le K \le K_2 \\ \frac{K-K_3}{K_2-K_3}, \; K_2 \le K \le K_3 \\ 0, \quad \text{otherwise} \end{cases} \tag{25}$$

where K_1, K_2, K_3 can be obtained by replacing $\widetilde{R}$ with $R_2\left(1 - \frac{\beta}{1+t}\right)$, R_2 & $R_2\left(1 + \frac{\gamma}{1+t}\right)$,respectively, in Eq. 23.

(ii) Membership function for cloud fuzzy order quantity:

$$g_2(Q, T) = \begin{cases} \frac{Q-Q_1}{Q_2-Q_1}, \; Q_1 \le Q \le Q_2 \\ \frac{Q-Q_3}{Q_2-Q_3}, \; Q_2 \le Q \le Q_3 \\ 0, \quad \text{otherwise} \end{cases} \tag{26}$$

where $Q_1 = \frac{1}{\theta}R_2\left(1 - \frac{\beta}{1+T}\right)\left(e^{\theta T} - 1\right)$, $Q_2 = \frac{R_2}{\theta}\left(e^{\theta T} - 1\right)$ & $Q_3 = \frac{1}{\theta}R_2\left(1 + \frac{\gamma}{1+T}\right)\left(e^{\theta T} - 1\right)$

Defuzzified value of cloud fuzzy total inventory cost and cloud fuzzy order quantity as per extension of Yager's ranking index method can be derived using the following equation.

$$I\left(\widetilde{K}\right) = \frac{1}{T}\int_{t=0}^{t=T} \frac{1}{4}(K_1 + 2K_2 + K_3)\, dt \tag{27}$$

where the value of $\frac{1}{4}(K_1 + 2K_2 + K_3)$ for different cases is as follows:

For Case A:

$$\frac{1}{4}(K_1+2K_2+K_3)=\frac{A}{T}+\left(4-\frac{(\beta-\gamma)}{1+T}\right)\left\{\begin{array}{c}\frac{1}{4\theta^2}\left[R_2(h+c\theta(1-r))\left(\theta+\frac{1}{2}\theta^2T\right)\right]\\+\frac{IcR_2}{8sT}\left[c(1-r)\left(T+\frac{1}{2}\theta T^2\right)-sM_1\left(1+\frac{IeM_1}{2}\right)\right]^2\\-\frac{hR_2}{4\theta}-\frac{sIeR_2M_1^2}{8T}\end{array}\right\} \tag{28}$$

For Case B:

$$\frac{1}{4}(K_1+2K_2+K_3)=\frac{A}{T}+\left(4-\frac{(\beta-\gamma)}{1+T}\right)\left\{\begin{array}{c}\frac{R_2}{4\theta^2}\left[(h+c\theta(1-r))\left(\theta+\frac{1}{2}\theta^2T\right)\right]-\frac{hR_2}{4\theta}\\-\frac{sIeR_2}{4}\left(M_1-\frac{T}{2}\right)\end{array}\right\} \tag{29}$$

For Case C:

$$\frac{1}{4}(K_1+2K_2+K_3)=\frac{A}{T}+\left(4-\frac{(\beta-\gamma)}{1+T}\right)\left\{\begin{array}{c}\frac{R_2}{4\theta^2}\left[(h+c\theta)\left(\theta+\frac{1}{2}\theta^2T\right)\right]-\frac{hR_2}{4\theta}-\frac{sIeR_2M_2^2}{8T}\\+\frac{R_2Ic}{8sT}\left[c\left(T+\frac{1}{2}\theta T^2\right)-sM_2\left(1+\frac{IeM_2}{2}\right)\right]^2\end{array}\right\} \tag{30}$$

For Case D:

$$\frac{1}{4}(K_1+2K_2+K_3)=\frac{A}{T}+\left(4-\frac{(\beta-\gamma)}{T}\right)\left\{\begin{array}{c}\frac{R_2}{4\theta^2}\left[(h+c\theta)\left(\theta+\frac{1}{2}\theta^2T\right)\right]-\frac{hR_2}{4\theta}\\-\frac{sR_2Ie}{4}\left(M_2-\frac{T}{2}\right)\end{array}\right\} \tag{31}$$

Defuzzified value of cloud fuzzy order quantity by using extension of Yager's ranking index method can be expressed by

$$I(\widetilde{Q})=\frac{R_c}{\theta}\left(e^{\theta T}-1\right)\text{ where, }R_c=R_2\left(1-\frac{(\beta-\gamma)\log(1+T)}{4T}\right) \tag{32}$$

5 Numerical Analysis and Proof of Convexity

Along with the proof of convexity in all four cases under fuzzy and cloud fuzzy environment, this section consists of numerical examples in all four cases in order to compare the results obtained in Chang and Teng model with the fuzzy and cloud fuzzy model derived in the present study.

Example 1: (for Case A) Let us consider the value of various parameters for crisp, fuzzy and cloud fuzzy environment as follows: $R = 1000$ units/year, $h = \$\,3$/unit/year, $A = \$\,10$/order, $Ic = 9\%$/year, $Ie = 3\%$/year, $c = \$\,20$/unit,

$s = \$30$/unit, $r = 0.02$, $\theta = 0.03$, $M_1 = 30/365$ years. For fuzzy model, consider $R_1 = 950$, $R_2 = 1000$, $R_3 = 1040$, and for cloud fuzzy model, let $\beta = 0.20$, $\gamma = 0.14$.

Example 2: (for Case B) Consider $R = 1000$ units/year, $h = \$3$/unit/year, $A = \$10$/order, $Ie = 3\%$/year, $c = \$20$/unit, $s = \$30$/unit, $r = 0.02$, $\theta = 0.03$, $M_1 = 30/365$ years. For fuzzy model, consider $R_1 = 950, R_2 = 1000$, $R_3 = 1040$, and for cloud fuzzy model, let $\beta = 0.95$, $\gamma = 0.10$.

Example 3: (for Case C) Consider $R = 1000$ units/year, $h = \$4$/unit/year, $A = \$25$/order, $Ic = 9\%$/year, $Ie = 6\%$/year, $c = \$30$/unit, $s = \$45$/unit, $r = 0.02$, $\theta = 0.03$, $M_1 = 20/365$ years, $M_2 = 30/365$ years. For fuzzy model, consider $R_1 = 950$, $R_2 = 1000$, $R_3 = 1040$, and for cloud fuzzy model, let $\beta = 0.14$, $\gamma = 0.15$.

Example 4: (for Case D) Consider $R = 1000$ units/year, $h = \$6$/unit/year, $A = \$10$/order, $Ie = 6\%$/year, $c = \$20$/unit, $s = \$30$/unit, $r = 0.02$, $\theta = 0.03$, $M_2 = 30/365$ years. For fuzzy model, consider $R_1 = 950$, $R_2 = 1000$, $R_3 = 1040$, and for cloud fuzzy model, let $\beta = 0.18\gamma = 0.14$.

Using the method explained in Sects. 1.4.1 and 1.4.2 for fuzzy and cloud fuzzy model, respectively, the values of decision variables are obtained under different environments for Example 1, Example 2, Example 3 and Example 4 and results are shown in Table 1. Also, the comparison between all cases under a different environment is graphically represented in Fig. 1.

Proof of convexity of total inventory cost function for fuzzy and cloud fuzzy model for Case A, Case B, Case C and Case D is shown in Figs. 2, 3, 4, 5, respectively.

Table 1 Optimal solutions for all cases under different environments

Case	Environment	Cycle time T (in year)	Order quantity Q	Total cost K (in \$)
A ($T \geq M_1$)	Crisp	0.08241	82.51	19,845.49
	Fuzzy	0.08247	82.37	19,796.18
	Cloud fuzzy	0.19573	195.14	19,751.01
B ($T < M_1$)	Crisp	0.0667	66.82	19,825.62
	Fuzzy	0.0668	66.73	19,776.43
	Cloud fuzzy	0.0820	74.91	15,961.41
C ($T \geq M_2$)	Crisp	0.0940	94.15	30,407.54
	Fuzzy	0.0941	94.42	30,332.19
	Cloud fuzzy	0.5328	537.17	30,322.53
D ($T < M_2$)	Crisp	0.0487	48.83	20,261.93
	Fuzzy	0.0488	48.76	20,211.79
	Cloud fuzzy	0.0812	80.96	20,129.51

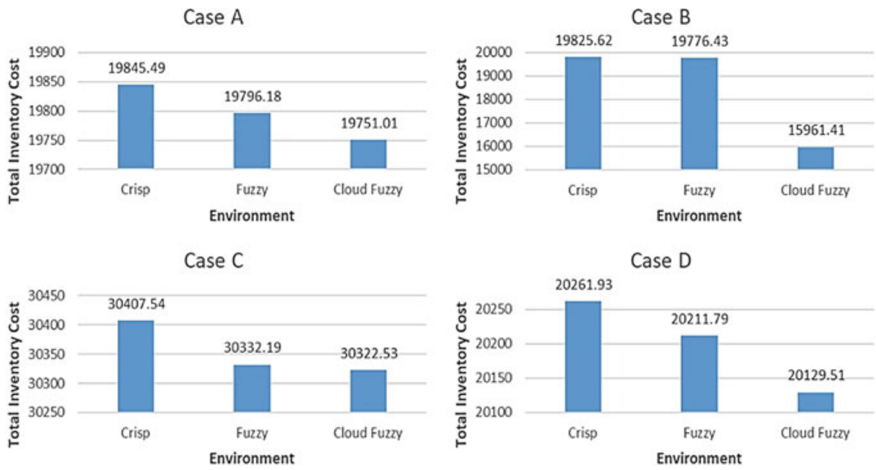

Fig. 1 Total inventory cost of all cases under different environments

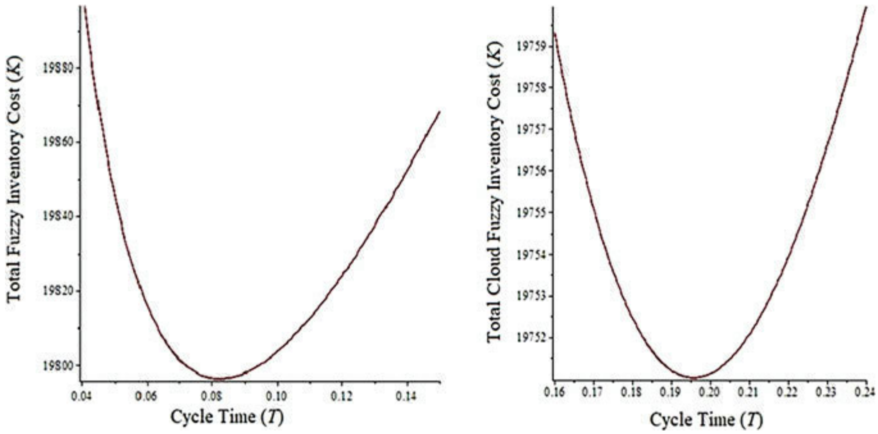

Fig. 2 Convexity of objective function of Case A under fuzzy and cloud fuzzy environment

6 Sensitivity Analysis

To figure out the most critical inventory parameters in fuzzy and cloud fuzzy model, sensitivity analysis is performed by changing one inventory parameter by −20%, −10%, 10% and 20% while keeping other parameters unchanged.

Sensitivity analysis for Case A as shown in Figs. 6 and 7 reveals that holding cost and order cost are highly sensitive parameters under both environments. It can also be observed from the graphs that increase in selling price also increases the total inventory cost significantly, while the period of cash discount has negligible effect on the total inventory cost. Increase of deterioration rate increases the total cost. Further,

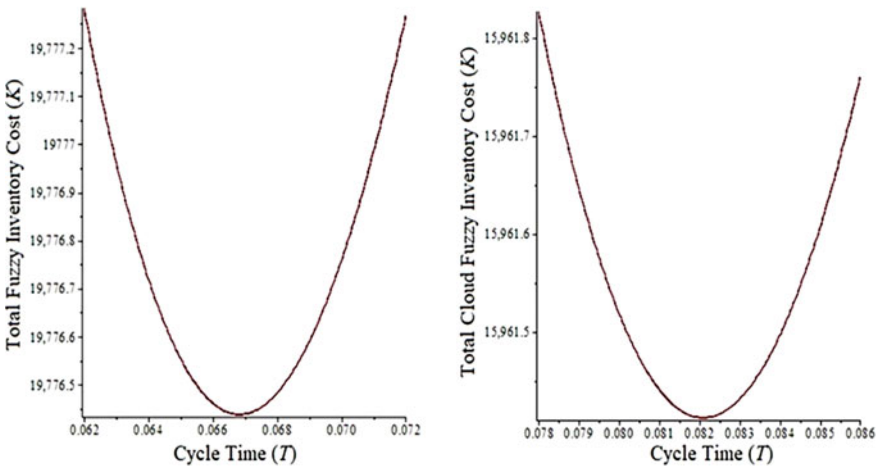

Fig. 3 Convexity of objective function of Case B under fuzzy and cloud fuzzy environment

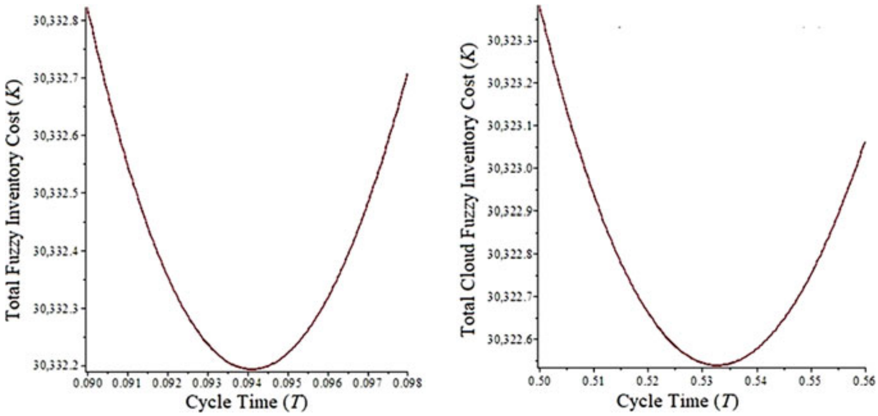

Fig. 4 Convexity of objective function of Case C under fuzzy and cloud fuzzy environment

increase in interest earned results in to lower inventory cost. Under both fuzzy and cloud fuzzy environments, increase in interest charged increases the total inventory cost, which suggests retailer to make the payment as early as possible in order to minimize the total inventory cost. The sensitivity analysis also concludes that cloud fuzzy parameters β and γ are highly sensitive to the inventory cost. Further, it can be clearly concluded from the graph that the behaviour of all the inventory parameters is same under both the environments.