Inventory Optimization

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Decision Making in Inventory Management

Springer

Inventory Optimization

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Inventory management is a very tedious task faced by all the organizations in any sector of the economy. It makes decisions for policies, activities and procedures in order to make sure that the right amount of each item is held in stock at any time. Many industries suffer from indiscipline in ordering and production mismatch. Providing best policy to control such mismatch would be invaluable to them.

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Contents

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About the Editors

Prof. Dr. Nita H. Shah received her Ph.D. in Statistics from Gujarat University in 1994. Prof. Nita is HoD of Department of Mathematics in Gujarat University, India. She is post-doctoral visiting research fellow of University of New Brunswick, Canada. Prof. Nita's research interests include inventory modeling in supply chain, robotic modeling, Mathematical modeling of infectious diseases, image processing, Dynamical systems and its applications etc. She has completed 3-UGC sponsored projects. She has published 13 monograph, 5 textbooks, and 475+ peer-reviewed research papers. Five edited books are prepared for IGI-Global and Springer. Her papers are published in high impact Elsevier, Inderscience and Taylor and Francis Journals. By the Google scholar, the total number of citations is over 3268 and the maximum number of citations for a single paper is over 174. The H-index is 25 and i-10 index is 83 up to February 2021. She has guided 28 Ph. D. Students and 15 M.Phil. Students till now. Eight students are pursuing research for their Ph.D. degree and one post-doctoral fellow of D. S. Kothari-UGC. She has travelled in USA, Singapore, Canada, South Africa, Malaysia, and Indonesia for giving talks. She is Vice-President of Operational Research Society of India. She is council member of Indian Mathematical Society.

Mandeep Mittal started his career in the education industry in 2000 with Amity Group. Currently, he is working as Head and Associate Professor in the Department of Mathematics, Amity Institute of Applied Sciences, Amity University Uttar Pradesh, Noida. He earned his post-doctorate from Hanyang University, South Korea, 2016, Ph.D. (2012) from the University of Delhi, India, and postgraduation in Applied Mathematics from IIT Roorkee, India (2000). He has published more than 70 research papers in International Journals and International conferences. He authored one book with Narosa Publication on C language and edited five research books with IGI Global and Springer. He is a series editor of Inventory Optimization, Springer Singapore Pvt. Ltd. He has been awarded the Best Faculty Award by the Amity School of Engineering and Technology, New Delhi for the year 2016–2017. He guided four Ph.D. scholars, and 4 students working with him in the area of Inventory Control and Management. He also served as Dean of Students Activities at Amity School of Engineering and Technology, Delhi, for nine years, and worked as Head, Department of Mathematics in the same institute for one year. He is a member of editorial boards of Revista Investigacion Operacional, Journal of Control and Systems Engineering, and Journal of Advances in Management Sciences and Information Systems. He actively participated as a core member of organizing committees in the International conferences in India and outside India.

Leopoldo Eduardo Cárdenas-Barrón received the B.S. degree in industrial and systems engineering, the M.Sc. degree in manufacturing systems, the M.Sc. degree in industrial engineering, the Ph.D. in industrial engineering; all degrees from Tecnológico de Monterrey. In 1996, Leopoldo Eduardo Cárdenas-Barrón joined the Industrial and Systems Engineering Department at Tecnológico de Monterrey. At present, he is currently a Professor at Industrial and Systems Engineering Department of the School of Engineering and Sciences at Tecnológico de Monterrey, Campus Monterrey, México. He is a researcher of the research group in Optimization and Data Science. He is a member of the Mexican Research National System. He was the associate director of the Industrial and Systems Engineering programme from 1999 to 2005. Moreover, he was also the associate director of the Department of Industrial and Systems Engineering from 2005 to 2009. His research activities include inventory theory, optimization and supply chain. He has published 133 papers in international journals such as International Journal of Production Economics, Applied Mathematical Modelling, Computers and Industrial Engineering, Applied Mathematics and Computation, Mathematical and Computer Modelling, Expert Systems with Applications, OMEGA, European Journal of Operational Research, Computers and Operations Research, Applied Soft Computing, Transportation Research Part E: Logistics and Transportation Review, Computers and Mathematics with Applications, Mathematical Problems in Engineering, Production Planning and Control, Journal of the Operational Research Society, among others. He is a co-author of a book related to simulation (in Spanish).

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Chapter 1
Upper-Lower Bounds for the Profit of an Inventory System Under Price-Stock Life Time Dependent Demand

Nita H. Shah, Ekta Patel, and Kavita Rabari

Abstract Price is the most important factor influencing demand rate based on marketing and economic theory. Along with price, stock display is also a major factor, as displayed stocks may induce customers to purchase more due to its visibility. Moreover, the demand for perishable products depends on its freshness. However, relatively little devotion has been paid to the influence of expiration dates despite the fact that they are an important factor in consumers' purchase decisions. As a result, we develop an inventory model for perishable products in which demand explicitly in a multivariate function of price, displayed stocks, and expiration dates. We then formulate the model by determining the optimal selling price to maximize the total profit by using classical optimization method with the necessary condition given by Kuhn-Tucker. Furthermore, we discuss the optimal decisions under two scenarios: upper bound of profit and lower bound of profit by taking holding cost as a function of upper and lower bound respectively. Finally, a numerical example is demonstrated along with sensitivity analysis to describe the impact of inventory parameters on the optimal decisions.

Keywords Perishable products · Price · Stock and life time dependent demand · Expiration date · Lot-sizing and classical optimization method

MSC 90B05

1.1 Introduction

Inventory management for enterprises is continuously facing challenges associated with the development, quality, design, and manufacturing of new products.

Thus the demand for new products comes and goes at a faster pace. Recently, it is observed that customers are becoming more alert and cognizant about their health as their standard of living gets better than earlier, so the demand for products with a long life cycle has drastically increased in recent years. Only an increasing

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number of products are becoming subject to loss of utility, evaporation, degradation, and devaluation because of the launch of new technology or the substitutions like fashion and seasonal goods, electronic equipment, and so on. Even products like durable furniture, high technology goods, medicines, vitamins, and cosmetics are becoming victims of perishability, so managing such perishable inventory can be very challenging. To be competitive in today's grocery industry, there is a big task for getting the right product to the right place at the right time in the right condition. Determining price and order quantity jointly is recognized for perishable products as an essential way to intensify profitability and maintain competition in the market.

It is observed that the age of perishable products has a negative impact on the demand because of the loss of consumer's confidence in the product quality. Hence, in today's market, the freshness of the product has a major effect on demand. Moreover, the expiration date is one of the major concerns to assess the freshness of a product and could significantly affect its demand. Consequently, perishable products have become more and more significant fonts of revenue in the grocery industry. Fujiwara and Perera (1993) proposed an EOQ model for perishable products in which product devalues over time by considering exponential distribution. Sarker et al. (1997) developed an inventory model for perishable products by taking the negative effect of age of the on-hand socks into consideration. Later, Hsu et al. (2006) established an inventory model by considering expiration dates for deteriorating items. Bai and Kendall (2008) studied optimal shelf space allocation for perishable products in which demand is considered to be a function of displayed stock level and freshness condition. Then, Avinadav et al. (2014) explored an inventory model for perishable products by measuring product freshness until the expiration date. Dobson et al. (2017) studied an EOQ model for perishable products with age-dependent demand in which the lower and upper bound of the cycle length and profit are analyzed. After that Chen et al. (2016) studied an inventory model for perishable products in which demand is close to zero when it approaches its expiration date. Further, Feng et al. (2017) extended Chen et al. (2016) model by adding a pricing strategy.

In practice, the demand for fresh products is influenced by the stock level, as an increase in the displayed stock level attracts more customers to purchase more. Various types of inventory models have been derived to quantify this phenomenon in studying the optimal inventory policies. Baker and Urban (1988) proposed an inventory model in which the demand rate is a polynomial function form depending on the displayed stock level. Thereafter, the first EOQ model was proposed by Urban (1992) with non-zero ending inventory with displayed stock-dependent demand. Then Urban and Baker (1997) derived an EOQ model in which demand is a deterministic and multivariate function of price, time, and level of inventory. Teng and Chang (2005) extended Urban and Baker (1997) model by scrutinizing the effect of trade credit financing along with stock level. Dye and Ouyang (2005) investigated an inventory model for perishable products under stock and price-dependent demand by considering partial backlogging. Soni and Shah (2008) formulated an inventory model in which demand is partially constant and partially dependent on stock. One step ahead, Chang et al. (2010) scrutinized an optimal replenishment

Table 1.1 Literature survey is exhibited in Table 1.1

Authors	Demand pattern		Deterioration	Expiration date	Variable holding cost
	Price sensitive	Stock dependent			
Agi and Soni (2020)	✓	✓	Instantaneous	✘	✘
Avinadav et al. (2014)	✓	✘	✘	✘	✘
Bai and Kendall (2008)	✘	✓	Instantaneous	✘	✘
Baker and Urban (1988)	✘	✓	✘	✘	✘
Chang et al. (2010)	✘	✓	Non-instantaneous	✘	✘
Chen et al. (2016)	✘	✓	Instantaneous	✓	✘
Cohen (1977)	✓	✘	Instantaneous	✓	✘
Dobson et al. (2017)	✘	✘	Instantaneous and non-instantaneous	✘	✓
Dye (2007)	✓	✘	Instantaneous	✘	✘
Dye and Ouyang (2005)	✓	✓	Instantaneous	✘	✘
Feng et al. (2017)	✓	✓	Instantaneous	✓	✘
Fujiwara and Perera (1993)	✘	✘	Instantaneous	✘	✘
Hsu et al. (2006)	✓	✘	Instantaneous	✓	✘
Maihami and Abadi (2012)	✓	✘	Non-instantaneous	✘	✘
Mishra and Tripathy (2012)	✘	✘	Instantaneous	✘	✘
Mishra (2013)	✘	✘	Instantaneous	✘	✓
Papachristos and Skouri (2003)	✓	✘	Instantaneous	✘	✘
Sarker et al. (1997)	✘	✓	Instantaneous	✘	✘
Soni and Shah (2008)	✘	✓	✘	✘	✘
Teng and Chang (2005)	✓	✓	Instantaneous	✘	✘

(continued)

Table 1.1 (continued)

Authors	Demand pattern		Deterioration	Expiration date	Variable holding cost
	Price sensitive	Stock dependent			
Urban (1992)	✘	✓	✘	✘	✘
Urban and Baker (1997)	✓	✓	✘	✘	✘
Wee (1999)	✓	✘	Instantaneous	✘	✘
Wu et al. (2016)	✘	✓	Instantaneous	✓	✘
Chen et al. (2020)	✘	✘	Instantaneous	✘	✘
Amiri et al. (2020)	✘	✘	Instantaneous	✘	✘
Proposed model	✓	✓	Instantaneous	✓	✓

policy by taking stock-dependent demand for non-instantaneous perishable products. Mishra and Tripathy (2012) exploring inventory policy for time-dependent Weibull deterioration, in this study shortages are allowed and partially backlogged. After that Mishra (2013) proposed optimal inventory policies for instantaneous perishable items with the controllable deterioration rate in which demand and holding cost are time-dependent. Wu et al. (2016) established an inventory model for fresh produce in which demand is a time-varying function of its freshness, displayed volume, and expiration date.

Selling price is also a major concern to create a repeated purchasing environment in today's competitive market scenario. In this context, Cohen (1977) proposed an inventory model for ordering and pricing decisions by considering deterministic price-dependent demand. After that Wee (1999) established an inventory model for joint pricing and order quantity decision with selling price dependent demand and partial backlogging of unsatisfied demand. Papachristos and Skouri (2003) extended the work of Wee (1999) by taking demand as continuous, convex, and decreasing in selling price. Dye (2007) addressed an inventory problem with decreasing price demand in which marginal revenue is increased. In this study, demand is not affected by product age or its freshness. More recently, Maihami and Abadi (2012) investigated an inventory model in which demand is to be a function of age and price. More recently, Agi and Soni (2020) present a deterministic model for perishable items with age, stock and price-dependent demand rate. Feng et al. (2017) scrutinized pricing and lot sizing policy for perishable items in which demand is a multivariate function of price, freshness, and displayed stocks. Chen et al. (2020) proposed an inventory model for perishable products with two self-life. Amiri et al. (2020) studied an inventory model for perishable products in a two-echelon supply chain.

The Remainder of the article is structured as follows: Sect. 1.2 defines notations and assumptions. Section 1.3 formulates the mathematical model. Section 1.4 provides numerical results. Sensitivity analysis is carried out in Sect. 1.5. Section 1.6 concludes the proposed model with future research directions.

1.2 Notations and Assumptions

1.2.1 Notations

These are the notations that are used throughout the article (Table 1.2).

1.2.2 Assumptions

Proposed inventory model is constructed on the following assumptions.

- Fresh produce has been affected by many factors such as temperature, humidity, refrigeration, time in stock among others. It seems impossible to obtain an explicit freshness of the product. However, it is well-known that fresh produce has its

Table 1.2 Notations

α	Scale demand, $\alpha > 0$
β	Mark up, $\beta > 0$
p	Selling price per unit (dollars/unit)
c	Purchase cost per unit per dollar, $p > c$
Q	The order quantity
η	Price elasticity, $\eta > 1$
A	Ordering cost per order (dollars/order)
h	Holding cost per unit per unit time in dollars
h_l	Holding cost for lower bound per unit per unit time in dollars
h_u	Holding cost for upper bound per unit per unit time in dollars
m	Expiration date (in months)
T	Cycle time (in months)
$I(t)$	The inventory level at time $t \in [0, T]$
TP	Total profit in dollars
TP_l	Total profit for lower bound in dollars
TP_u	Total profit for upper bound in dollars

expiration date. To make the problem easy and tractable, we may assume the maximum lifetime $f(t) = \frac{m-t}{m}, 0 < t \le m$.

- The demand rate $R(p, I(t))$ is assumed to be a function of price, stock, and life time which is given by $R(p, I(t)) = (\alpha + \beta I(t))p^{-\eta}\frac{(m-t)}{m}$, where α is scale demand ($\alpha > 0$), $\beta > 0$ is mark-up, p is a selling price per unit, $\eta > 1$ denotes price elasticity mark-up and m is a life time of the product.
- The inventory cycle is lower than the maximum life time of the product.
- Holding cost for lower bound is defined as $h_l = \frac{p}{m} + \frac{h}{4}$ and for upper bound holding cost is $h_u = \frac{p}{m} + \frac{h}{2}$ where p is a selling price per unit m is a maximum life time of the product and h is a constant holding cost.
- Shortages are not allowed.
- The time horizon is infinite.

1.3 Mathematical Model

In Sect. 1.3, an inventory model is developed where the product loses its freshness with time. Initially, at time $t = 0$, the order quantity is Q, that reduced due to the effect of demand which depends upon price, stock, and life time of the product and reaches zero at time $t = T$.

The differential equation governing the inventory level at time t during the interval $[0, T]$ is given by

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = -(\alpha + \beta I(t))p^{-\eta}\left(\frac{m-t}{m}\right), \quad 0 \le t \le T \tag{1.1}$$

With the boundary condition $I(T) = 0$. Solving the differential equation in (1.1), we express the inventory level as follows

$$I(t) = -\frac{\alpha}{\beta} + \frac{\alpha \mathrm{e}^{\frac{-1}{2}\frac{\beta p^{-\eta}t(2m-t)}{m}}}{\beta \mathrm{e}^{\frac{-1}{2}\frac{\beta p^{-\eta}T(2m-T)}{m}}}, \quad 0 \le t \le T \tag{1.2}$$

Thus the order quantity could be expressed as follows:

$$Q = \frac{1}{2}\frac{T\alpha\left(p^{-2\eta}T^3\beta - 4p^{-2\eta}T^2\beta m + 4p^{-2\eta}T\beta m^2 + 2p^{-\eta}mT - 4p^{-\eta}m^2\right)}{m^2} \tag{1.3}$$

Based on the above, the profit function through the cycle consists of the following terms:

Ordering cost per cycle

$$OC = A \tag{1.4}$$

Purchase cost is given by

$$\begin{aligned} PC &= cQ \\ &= \frac{1}{2}\frac{cT\alpha\left(p^{-2\eta}T^3\beta - 4p^{-2\eta}T^2\beta m + 4p^{-2\eta}T\beta m^2 + 2p^{-\eta}mT - 4p^{-\eta}m^2\right)}{m^2} \end{aligned} \tag{1.5}$$

Holding cost during the time interval $[0,\ T]$ is given by

$$\begin{aligned} \text{HC} &= h\int_0^T I(T)\text{d}t \\ &= h\left(\begin{array}{l} \frac{1}{40}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^5}{m^2} - \frac{1}{8}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^4}{m} \\ +\frac{1}{3}\frac{1}{\beta}\left(\alpha\left(\frac{1}{2}\frac{\beta p^{-\eta}}{m} + \frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2\left(-(2m-T)T + m^2\right)}{m^2}\right)T^3\right) \\ +\frac{1}{2}\alpha\frac{\left(-\beta p^{-\eta} + \frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)T}{m^2}\right)T^2}{\beta} - \frac{\alpha T}{\beta} \\ +\frac{1}{\beta}\left(\alpha\left(1 - \frac{\beta p^{-\eta}(2m-T)T}{m} + \frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)^2T^2}{m^2}\right)T\right) \end{array}\right) \end{aligned} \tag{1.6}$$

Sales Revenue

$$\text{SR} = pQ = \frac{pT\alpha\left(p^{-2\eta}T^3\beta - 4p^{-2\eta}T^2\beta m + 4p^{-2\eta}T\beta m^2 + 2p^{-\eta}mT - 4p^{-\eta}m^2\right)}{2m^2} \tag{1.7}$$

So, from Eqs. (1.4)–(1.7) the total profit can be calculated by following equation

$$\text{TP} = \frac{1}{T}(\text{SR} - \text{PC} - \text{HC} - \text{OA})$$

$$\mathrm{TP}=\frac{1}{T}\left(\begin{array}{l}\frac{1}{2}\frac{1}{m^2}\left(pT\alpha\left(p^{-2\eta}T^3\beta-4p^{-2\eta}T^2\beta m+4p^{-2\eta}T\beta m^2+2p^{-\eta}Tm-4p^{-\eta}m^2\right)\right)\\-\frac{1}{2}\frac{1}{m^2}\left(cT\alpha\left(p^{-2\eta}T^3\beta-4p^{-2\eta}T^2\beta m+2p^{-\eta}Tm-4p^{-\eta}m^2\right)\right)-A\\-h\left(\begin{array}{l}\frac{1}{40}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^5}{m^2}-\frac{1}{8}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^4}{m}\\+\frac{1}{3}\frac{1}{\beta}\left(\alpha\left(\frac{1}{2}\frac{\beta p^{-\eta}}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2\left(-(2m-T)T+m^2\right)}{m^2}\right)T^3\right)\\+\frac{1}{2}\alpha\frac{\left(-\beta p^{-\eta}+\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)T}{m^2}\right)T^2}{\beta}-\frac{\alpha T}{\beta}\\+\frac{1}{\beta}\left(\alpha\left(1-\frac{\beta p^{-\eta}(2m-T)T}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)^2T^2}{m^2}\right)T\right)\end{array}\right)\end{array}\right) \tag{1.8}$$

Instead of dealing with constant holding cost, the model defines the holding in terms of expiration date and selling price, based on the holding cost, proposed inventory system can be classified into the following two categories:

- **Lower bound**

Holding cost for lower bound during the interval $[0, T]$ is given by Chen et al. (2016)

$$\mathrm{HCl}=\left(\frac{p}{m}+\frac{h}{4}\right)\int\limits_0^T I(t)\mathrm{d}t$$

$$\mathrm{HCl}=\left(\frac{p}{m}+\frac{h}{4}\right)\left(\begin{array}{l}\frac{1}{40}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^5}{m^2}-\frac{1}{8}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^4}{m}\\+\frac{1}{3}\frac{1}{\beta}\left(\alpha\left(\frac{1}{2}\frac{\beta p^{-\eta}}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2\left(-(2m-T)T+m^2\right)}{m^2}\right)T^3\right)\\+\frac{1}{2}\alpha\frac{\left(-\beta p^{-\eta}+\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)T}{m^2}\right)T^2}{\beta}-\frac{\alpha T}{\beta}\\+\frac{1}{\beta}\left(\alpha\left(1-\frac{\beta p^{-\eta}(2m-T)T}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)^2T^2}{m^2}\right)T\right)\end{array}\right) \tag{1.9}$$

The model is analyzed the lower bound for holding cost that defines the lower range of profit function which can be calculated from Eqs. (1.4), (1.5), (1.7) and (1.9), is given by the following equation

$$\mathrm{TPl}=\frac{1}{T}(\mathrm{SR}-\mathrm{PC}-\mathrm{HCl}-\mathrm{OA})$$

$$\mathrm{TPl}=\frac{1}{T}\left(\begin{array}{l}\frac{1}{2}\frac{1}{m^2}\left(pT\alpha\left(p^{-2\eta}T^3\beta-4p^{-2\eta}T^2\beta m+4p^{-2\eta}T\beta m^2+2p^{-\eta}Tm-4p^{-\eta}m^2\right)\right)\\ -\frac{1}{2}\frac{1}{m^2}\left(cT\alpha\left(p^{-2\eta}T^3\beta-4p^{-2\eta}T^2\beta m+2p^{-\eta}Tm-4p^{-\eta}m^2\right)\right)-A\\ -\left(\frac{p}{m}+\frac{h}{4}\right)\left(\begin{array}{l}\frac{1}{40}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^5}{m^2}-\frac{1}{8}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^4}{m}\\ +\frac{1}{3}\frac{1}{\beta}\left(\alpha\left(\frac{1}{2}\frac{\beta p^{-\eta}}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2\left(-(2m-T)T+m^2\right)}{m^2}\right)T^3\right)\\ +\frac{1}{2}\alpha\frac{\left(-\beta p^{-\eta}+\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)T}{m^2}\right)T^2}{\beta}-\frac{\alpha T}{\beta}\\ +\frac{1}{\beta}\left(\alpha\left(1-\frac{\beta p^{-\eta}(2m-T)T}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)^2T^2}{m^2}\right)T\right)\end{array}\right)\end{array}\right) \tag{1.10}$$

- **Upper bound**

Holding cost for upper bound during the interval $[0, T]$ is given by is Chen et al. (2016)

$$\mathrm{HCu}=\left(\frac{p}{m}+\frac{h}{2}\right)\int_0^T I(t)\mathrm{d}t$$

$$\mathrm{HCu}=\left(\frac{p}{m}+\frac{h}{2}\right)\left(\begin{array}{l}\frac{1}{40}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^5}{m^2}-\frac{1}{8}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^4}{m}\\ +\frac{1}{3}\frac{1}{\beta}\left(\alpha\left(\frac{1}{2}\frac{\beta p^{-\eta}}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2\left(-(2m-T)T+m^2\right)}{m^2}\right)T^3\right)\\ +\frac{1}{2}\alpha\frac{\left(-\beta p^{-\eta}+\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)T}{m^2}\right)T^2}{\beta}-\frac{\alpha T}{\beta}\\ +\frac{1}{\beta}\left(\alpha\left(1-\frac{\beta p^{-\eta}(2m-T)T}{m}+\frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)^2T^2}{m^2}\right)T\right)\end{array}\right) \tag{1.11}$$

One can replace the holding cost taken in traditional model by the holding cost given in Eq. (1.11). To achieve upper bound of the total profit which is calculated from Eqs. (1.4), (1.5), (1.7) and (1.11):

$$\text{TPu} = \frac{1}{T}(\text{SR} - \text{PC} - \text{HCu} - \text{OA})$$

$$\text{TPu} = \frac{1}{T}\left(\begin{array}{l} \frac{1}{2}\frac{1}{m^2}\left(pT\alpha\left(p^{-2\eta}T^3\beta - 4p^{-2\eta}T^2\beta m + 4p^{-2\eta}T\beta m^2 + 2p^{-\eta}Tm - 4p^{-\eta}m^2\right)\right) \\ -\frac{1}{2}\frac{1}{m^2}\left(cT\alpha\left(p^{-2\eta}T^3\beta - 4p^{-2\eta}T^2\beta m + 2p^{-\eta}Tm - 4p^{-\eta}m^2\right)\right) - A \\ -\left(\frac{p}{m} + \frac{h}{2}\right)\left(\begin{array}{l} \frac{1}{40}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^5}{m^2} - \frac{1}{8}\frac{\alpha\beta\left(p^{-\eta}\right)^2T^4}{m} \\ +\frac{1}{3}\frac{1}{\beta}\left(\alpha\left(\frac{1}{2}\frac{\beta p^{-\eta}}{m} + \frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2\left(-(2m-T)T + m^2\right)}{m^2}\right)T^3\right) \\ +\frac{1}{2}\alpha\frac{\left(-\beta p^{-\eta} + \frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)T}{m^2}\right)T^2}{\beta} - \frac{\alpha T}{\beta} \\ +\frac{1}{\beta}\left(\alpha\left(1 - \frac{\beta p^{-\eta}(2m-T)T}{m} + \frac{1}{2}\frac{\beta^2\left(p^{-\eta}\right)^2(2m-T)^2T^2}{m^2}\right)T\right) \end{array}\right) \end{array}\right) \tag{1.12}$$

1.3.1 Optimal Solution

The model uses classical optimization method to maximize the total profit

Step 1: Differentiate all the three profit functions derived in Eqs. (1.8), (1.10) and (1.12) with respect to inventory parameters T and p partially.

Step 2: Solve the equations for T and p**.**

Step 3: Allocate the values to all the inventory parameters except decision variables.

Step 4: Substitute in all the profit functions.

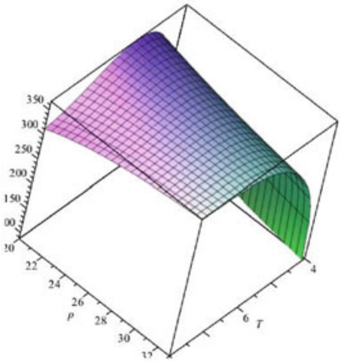
1.4 Numerical Validation

This section validates the proposed model with a numerical example and managerial insights are also given.

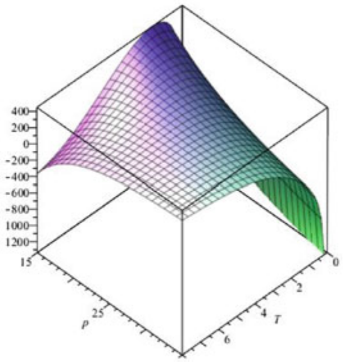
$$A = \$140, \alpha = 650, \beta = 3.5, \eta = 1.002,$$
$$m = 8\text{ months}, h = \$5/\text{unit}, c = \$20/\text{unit}$$

In such condition the solution: cycle time $T = 6.85$ months, selling price $p =$ \$27.65 / unit and total profit is $TP = \$290.59$.

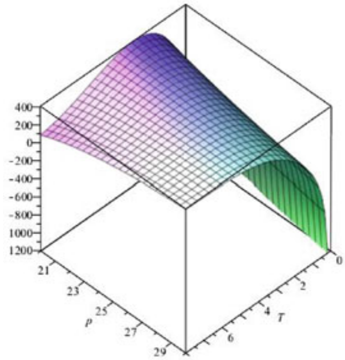
Graphical representation in all the three cases: lower bound of total profit, total profit, and upper bound of total profit are validated in maple 18 as shown below:



(a) Concavity of total profit w.r.to cycle time and selling price



(b) Concavity of lower bound of total profit w.r.to cycle time and selling price



(c) Concavity of upper bound of total profit w.r.to cycle time and selling price

Fig. 1.1 Concavity of total profit

Figure 1.1 show the concavity of total profit with respect to cycle time and selling price. Figure 1.1a is of total profit with constant holding cost. Lower bound of total profit is presented in Fig. 1.1b. Upper bound of profit is displayed in Fig. 1.1c.

1.5 Sensitivity Analysis

Based on the result, we performed the sensitivity analysis by changing the value of one parameter at a time by a factor of negative and positive of 10 and 20%. Effects of such changes in each parameter on the optimal solutions are studied. Based on the holding cost this analysis is categorized into two different cases:

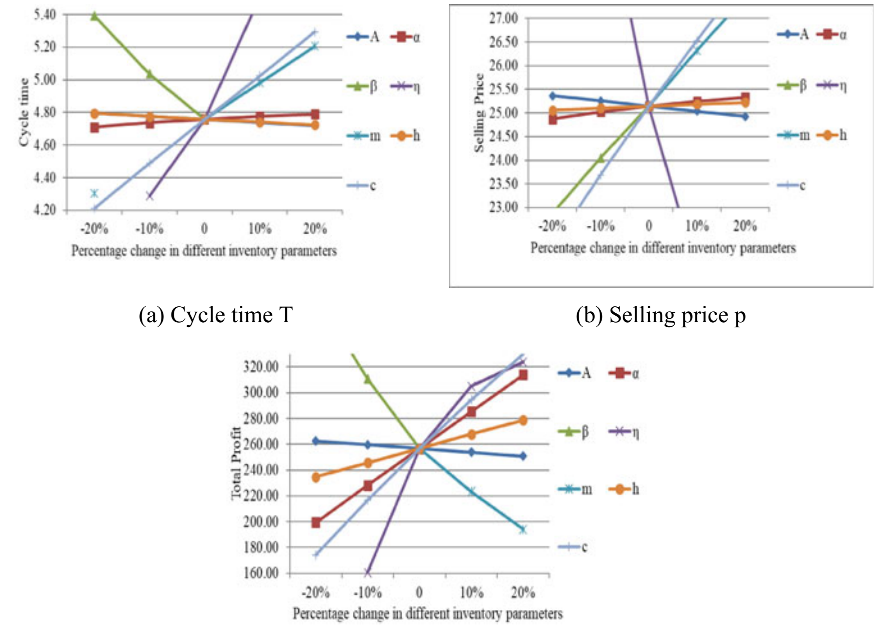
Case: 1 Lower bound

The variation in cycle time, selling price, and total profit are presented in Fig. 1.2a–c respectively. It is observed from Fig. 1.2a that cycle time is more sensitive to purchase cost c, expiration date m, price elasticity η and mark-up β. As purchase cost c, expiration date m and price elasticity η increases, cycle time will increase. Inventory parameters scale demand α, ordering cost A and holding cost h have reasonable effects on cycle time.

Figure 1.2b, c show that with a rise in scale demand, mark-up and purchase cost, selling price as well as total profit will increase. So it is advisable for a profitable business. Expiration dates have a huge impact on the model. If the duration of the expiration date is short, a business faces financial loss in terms of the reduced profit function. Inventory parameters ordering cost A and price elasticity η plays a negative impact on profitability. An increase in ordering cost and price elasticity reduces the total profit. Hence, an increase is not preferable.

Case: 2 Upper bound

Figure 1.3a–c shows the change in cycle time, selling price, and total profit with respect to other inventory parameters. Figure 1.3a shows inventory parameters scale demand α, ordering cost A, and holding cost h have a significant effect on cycle time. An increase in purchase cost c, price elasticity η and expiration date m increases cycle

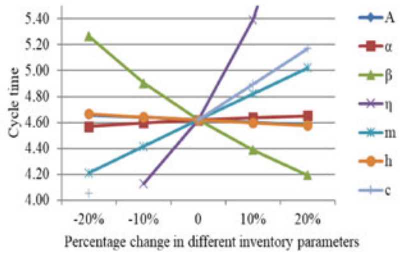


(a) Cycle time T

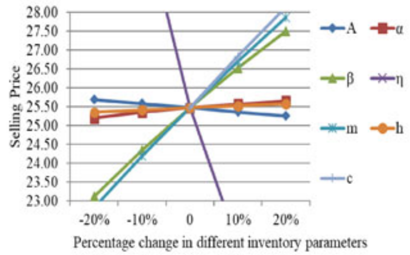
(b) Selling price p

(c) Total profit

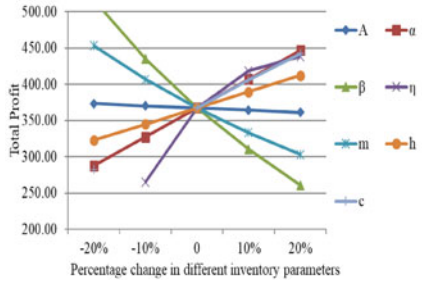
Fig. 1.2 Sensitivity analysis of lower bound



(a) Cycle time T



(b) Selling price p



(c) Total profit

Fig. 1.3 Sensitivity analysis of upper bound

time. On the other hand, cycle time decreases with an increase of mark up β. From Fig. 1.3b, it can be shown that purchase cost c, expiration date m and mark-up β have a positive impact on selling price whereas price elasticity η has a reversible effect on selling price. The total profit gets increased with the rise of scale demand α, ordering cost A, holding cost h, purchase cost c and price elasticity η. Profit will decrease for expiration date m and mark up β. The rest of the parameters have a reasonable effect on total profit depicted in Fig. 1.3c.

Sensitivity analysis of cycle time, selling price, and total profit with lower and upper bound is exposed in the following figures.

From Fig. 1.4a, it is observed that total profit is not affected by increasing ordering cost in both the cases: lower bound and upper bound. In Fig. 1.4b, there is no significant change in lower bound of profit due to a change in holding cost. Figure 1.4c, d represent the effect of change in scale demand and mark-up on total profit. In both scenarios, the total profit increase with an increase in scale demand while total profit gets decreases with increases in inventory parameter mark-up.