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Contents

Chapter 1
Introduction

Abstract A general introduction about the book 'Non-Gaussian Random Vibration Fatigue Analysis and Accelerated Test' is given, and the contents of chapters are summarized.

As a common problem in the engineering field, fatigue failure caused by vibration seriously endangers the safe and reliable operation of important equipment and structures. Vibration test is an important method for environmental adaptability, safety, reliability and life assessment of large equipment and structures in the fields of aviation, aerospace, shipbuilding, and mechanical engineering. How to ensure that the vibration test performed in the laboratory conforms to the actual service or transportation vibration environment of the equipment, and avoid under-testing and over-testing, has become an urgent problem to be solved.

In order to solve the above problem, we first need to understand the description of random vibration. The most common parameter used to describe random vibration is Power Spectral Density (PSD). However, PSD is not able to adequately portray all the characteristics of random vibration. For example, with the same PSD (also the same Root Mean Square), random vibration signals can have completely different probability density distribution, as shown in Fig. 1.1. Because the higher-order statistics over the second order are constantly zero for a Gaussian random process, only using PSD function or self-correlation function is enough to fully describe the characteristics of Gaussian random vibration. But for non-Gaussian (including super-Gaussian and sub-Gaussian) random vibration, in addition to PSD, higher-order statistics (above the second order) are necessary to describe its comprehensive characteristics, which will be discussed in detail in Chap. 2.

After understanding how to fully describe the characteristics of non-Gaussian random vibration, the next step is to simulate non-Gaussian random vibration. This is the key to the reproduction of the actual vibration environment experienced by the equipment, and it can also be used for structural dynamics and vibration fatigue simulation analysis. Because non-Gaussian random vibration also often appears in the form of non-stationary random vibration, in Chap. 3, besides discussing the simulation of non-Gaussian random vibration, the simulation of non-stationary random vibration is also discussed.

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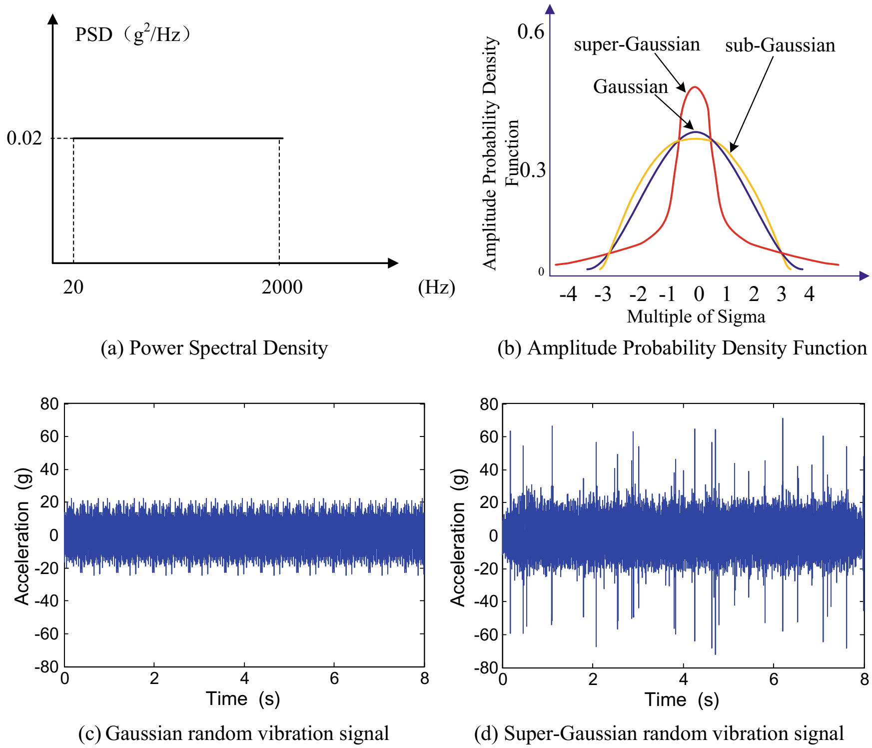

Fig. 1.1 Gaussian and non-Gaussian vibration signals with the same PSD

After realizing the simulation of non-Gaussian random vibration, using it for input excitation to act on the structure will produce a dynamic response, which is the root cause of the structure's vibration fatigue failure. Therefore, in order to study the influence of non-Gaussian random vibration on the reliability of the structure, we must first study the dynamic response of the structure under non-Gaussian random vibration. This is the main content of Chap. 4.

Chapter 5 mainly studies how to obtain accurate and reliable calculation results of fatigue life after obtaining the non-Gaussian random stress response. First, the influence of sampling frequency on the calculation accuracy of fatigue damage is studied, and a stress signal reconstruction method based on Shannon's formula is proposed. Then, based on the rain-flow counting method, the calculation methods of the fatigue life of the structure under the action of narrowband and wideband non-Gaussian random stress are, respectively, proposed. Finally, the applicability and accuracy of the calculation method of non-Gaussian fatigue life are verified through specific example analysis.

Chapter 6 refers to fatigue reliability evaluation of structural components under random loadings. The randomness of fatigue damage is treated in two aspects. The first one is the uncertainty quantification from the external random loading. The second one is the uncertainty quantification of the fatigue property of the structural component. The former is characterized by Gaussian distribution derived from the rainflow cycle distribution, medium stress-life (S–N) curve, and the linear damage accumulation rule. The latter is described with the probabilistic stress-life (P–S–N) curve based on log-normal distribution. The proposed method has colligated these two aspects together to evaluate the expectation and confidence interval of fatigue reliability.

In Chap. 7, a novel non-Gaussian random vibration accelerated test methodology was proposed, which can significantly reduce the test time and the sample size. First, fatigue life prediction models of Gaussian and non-Gaussian random vibration were proposed based on random vibration and fatigue theory. Meanwhile a detailed solving method was also presented for determining the unknown parameters in the models. Second, a non-Gaussian random vibration accelerated test system was designed. Third, several groups of random vibration fatigue tests were designed and conducted with the aim of investigating effects of both Gaussian and non-Gaussian random excitation on the vibration fatigue. Finally, an application case for the fatigue life prediction of electronic product structures verified the effectiveness of the above non-Gaussian random vibration accelerated test method.

Chapter 2
Statistic Analysis of Non-Gaussian Random Load

Abstract Statistic analysis of non-Gaussian random load underpins the calculation and reliability analysis of the fatigue life for products under non-Gaussian loads. Skewness and kurtosis only are insufficient to express the non-Gaussian statistical properties. In view of this, chapter sets up the mathematical model of probability density function (PDF) for the symmetrical and oblique non-Gaussian random load amplitude based on the Gaussian Mixture Model (GMM). Higher-order statistics are sub-stituted into the math model to determine the parameters and obtain the analytical expression of non-Gaussian PDF. Validity of this method is verified by simulation and measured signals. The analytical expression of PDF contributes to a precise definition of the non-Gaussian random load, and underpins the calculation and reliability analysis of the fatigue life as well as the testing program.

2.1 Common Non-Gaussian Statistical Parameters

In theory, statistics fully describing the non-Gaussian properties in the random process include higher moment $m_n(\tau_1, \ldots, \tau_{n-1})$ or high-order cumulants $c_n(\tau_1, \ldots, \tau_{n-1})$ [4] $(n > 2)$. $m_n(\tau_1, \ldots, \tau_{n-1})$ and $c_n(\tau_1, \ldots, \tau_{n-1})$ are multivariate functions of the time interval $\{\tau_i\}$, which has complicated computation with calculation errors. Higher-order statistics is rarely applied in the quantitative analysis of non-Gaussian random load due to its complexity. For the purpose of convenience, static higher-order statistics, as a result of setting the time interval τ_i of $m_n(\tau_1, \ldots, \tau_{n-1})$ or $c_n(\tau_1, \ldots, \tau_{n-1})$ as 0, is used to express the non-Gaussian properties. The static higher-order statistics of the random process, in essence, amounts to the higher-order statistics of the random variable. Normalized 3-order and 4-order static matrix are called skewness γ_3 and kurtosis γ_4, respectively.

Skewness of the Gaussian random process $\gamma_3 = 0$, and kurtosis $\gamma_4 = 3$; skewness of symmetrical non-Gaussian random process $\gamma_3 = 0$, and kurtosis $\gamma_4 > 3$ (Strictly, $\gamma_4 \neq 3$, but the non-Gaussian random load in practice is usually $\gamma_4 > 3$ [1]). Skewness and kurtosis of the gaussian random process $X(t)$ can be estimated by the time series $x(t)$ of the sample:

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$$\hat{\gamma}_3 = \frac{\frac{1}{T}\left[\int_0^T x^3(t)\mathrm{d}t\right]}{\hat{\sigma}_X^3} = \frac{\hat{m}_3(0,0)}{\hat{\sigma}_X^3}; \qquad \hat{\gamma}_4 = \frac{\frac{1}{T}\left[\int_0^T x^4(t)\mathrm{d}t\right]}{\hat{\sigma}_X^4} = \frac{\hat{m}_4(0,0,0)}{\hat{\sigma}_X^4} \tag{2.1}$$

Of which, T is time span of the sample. Skewness and kurtosis only are insufficient to characterize the probability statistics of the non-Gaussian random process, therefore, higher-order statistics is required for the analytical expression of PDF of the non-Gaussian amplitude.

2.2 Symmetrical Non-Gaussian Probability Density Function

2.2.1 Gaussian Mixed Model

Middleton [5] proposed GMM for the probability distribution of the amplitude of the noise signal with multi-source superimposition in the communication system, expressed as:

$$f_{\mathrm{NG}}(x) = \sum_{i=0}^{N} \alpha_i f_i(x) \tag{2.2}$$

Of which, $f_{\mathrm{NG}}(x)$ is the non-Gaussian PDF, $f_i(x)$ is the PDF of the ith Gaussian component, N is dimension, α_i is weight of the ith Gaussian component, $0 \le \alpha_i \le 1$, $\sum \alpha_i = 1$. Generally, 2D or 3D GMM can obtain precise non-Gaussian PDF. Here, we adopt the 2D GMM.

$$f_{\mathrm{NG}}(x) = \alpha f_1(x|\sigma_1) + (1-\alpha) f_2(x|\sigma_2) \tag{2.3}$$

2.2.2 Estimation of Parameters

Since the PDF of the Gaussian component is determined by the standard deviation σ, the 2D GMM of the zero-mean non-Gaussian process $X(t)$ can be expressed as:

$$f_{\mathrm{NG}}(x) = \alpha \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{x^2}{2\sigma_1^2}\right) + (1-\alpha) \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{x^2}{2\sigma_2^2}\right) \tag{2.4}$$

Of which, σ_1 and σ_2 are standard deviation of the Gaussian component 1 and 2, respectively; α and 1–α are weight of the Gaussian component 1 and 2, respectively. Three unknowns in Eq. (2.4) are σ_1, σ_2, and α. For the zero-mean process $X(t)$, the

2nd, 4th, and 6th central moment can be estimated via the time series $x(t)$ of the sample:

$$\begin{cases} \hat{m}_2 = \frac{1}{T}\int_0^T x^2(t)\mathrm{d}t \\ \hat{m}_4 = \frac{1}{T}\int_0^T x^4(t)\mathrm{d}t \\ \hat{m}_6 = \frac{1}{T}\int_0^T x^6(t)\mathrm{d}t \end{cases} \tag{2.5}$$

With sufficient time span T, estimated value $\hat{m}_n$ can well approach the truth-value m_n [2]. With 2D GMM (Eq. 2.4), we obtain the the following:

$$\begin{cases} m_2 = \alpha m_2^{(1)} + (1-\alpha)m_2^{(2)} \\ m_4 = \alpha m_4^{(1)} + (1-\alpha)m_4^{(2)} \\ m_6 = \alpha m_6^{(1)} + (1-\alpha)m_6^{(2)} \end{cases} \tag{2.6}$$

Of which, $m_2^{(1)}$, $m_4^{(1)}$, and $m_6^{(1)}$ are the 2nd, 4th, and 6th moment of the Gaussian component 1, respectively; $m_2^{(2)}$, $m_4^{(2)}$, and $m_6^{(2)}$ are the 2nd, 4th, and 6th moment of the Gaussian component 2, respectively. 2-order matrix is quadratic mean. For the zero-mean gaussian random process, there is the relationship among the moments:

$$\begin{cases} m_2^{(1)} = \sigma_1^2; \quad m_2^{(2)} = \sigma_2^2 \\ m_4^{(1)} = 3\sigma_1^4; \quad m_4^{(2)} = 3\sigma_2^4 \\ m_6^{(1)} = 15\sigma_1^6; \quad m_6^{(2)} = 15\sigma_2^6 \end{cases} \tag{2.7}$$

Equation (2.7) is substituted into (2.6), then:

$$\begin{cases} m_2 = \alpha\sigma_1^2 + (1-\alpha)\sigma_2^2 \\ m_4 = 3\alpha\sigma_1^4 + 3(1-\alpha)\sigma_2^4 \\ m_6 = 15\alpha\sigma_1^6 + 15(1-\alpha)\sigma_2^6 \end{cases} \tag{2.8}$$

The truth-value is replaced by the estimated results of high-order moment of the non-Gaussian random process given by Eq. (2.5), then:

$$\begin{cases} \hat{m}_2 = \alpha\sigma_1^2 + (1-\alpha)\sigma_2^2 \\ \hat{m}_4 = 3\alpha\sigma_1^4 + 3(1-\alpha)\sigma_2^4 \\ \hat{m}_6 = 15\alpha\sigma_1^6 + 15(1-\alpha)\sigma_2^6 \end{cases} \tag{2.9}$$

Ternary system of nonlinear equations defined by Eq. (2.9) can be determined through the calculation software (Matlab symbolic operation). The determined parameters α, σ_1, and σ_2 are substituted into Eq. (2.4) to obtain PDF of the symmetrical non-Gaussian amplitude of GMM. It can be seen that the PDF of the symmetrical non-Gaussian amplitude set by Eqs. (2.4) and (2.9) considers the 6th moment of the non-Gaussian random process, unlike the single application of kurtosis, which improves the description accuracy of non-Gaussian statistical property.

2.3 Asymmetrical Non-Gaussian Probability Density Function

2.3.1 Gaussian Mixed Model

Uniform expression of GMM is shown by Eq. (2.2), but the math model of asymmetrical non-Gaussian PDF differs from the symmetrical non-Gaussian PDF. Take the 2D GMM as example, the Gaussian component $f_1(x)$ and $f_2(x)$ of the symmetrical non-Gaussian random load are zero-mean Gaussian PDF while the Gaussian component $f_1(x)$ and $f_2(x)$ of the asymmetrical non-Gaussian random process have different means μ and standard deviations σ.

$$f_{\mathrm{NG}}(x) = \alpha f_1(x|\mu_1, \sigma_1) + (1-\alpha) f_2(x|\mu_2, \sigma_2) \tag{2.10}$$

In this way, the odd higher moment of the asymmetrical non-Gaussian random process cannot be 0.

2.3.2 Estimation of Parameters

We first presume the analysis based on zero-mean, because the effects of means on the probability distribution can be solved by translation. By Eq. (2.10), the amplitude PDF of the asymmetrical non-Gaussian random process can be expanded as:

$$f_{\mathrm{NG}}(x) = \alpha \frac{1}{\sqrt{2\pi}\sigma_1} \exp\left[-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right] + (1-\alpha)\frac{1}{\sqrt{2\pi}\sigma_2} \exp\left[-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right] \tag{2.11}$$

Of which, α, $\mu_1$andσ_1 are weight, mean and standard deviation of the Gaussian component 1, respectively; $1-\alpha$, $\mu_{2,}$ and σ_2 are weight, mean, and standard deviation of the Gaussian component 2. The means μ_1 and μ_2 are introduced for GMM to fit the asymmetrical non-Gaussian PDF curve. Equation (2.11) has five unknowns: μ_1, $\mu_{2,}$ $\sigma_{1,}$ $\sigma_{2,}$ and α. As for the zero-mean, the central moment of the non-Gaussian process amounts to the origin moment, hereinafter uniformly referred to as moment; 1st moment is 0.

$$\begin{aligned} m_1^{(\mathrm{NG})} &= \int_{-\infty}^{\infty} x f_{\mathrm{NG}}(x)\mathrm{d}x = \alpha \int_{-\infty}^{\infty} x f_1(x)\mathrm{d}x + (1-\alpha)\int_{-\infty}^{\infty} x f_2(x)\mathrm{d}x \\ &= \alpha\mu_1 + (1-\alpha)\mu_2 = 0 \end{aligned} \tag{2.12}$$

The 2nd moment of the non-Gaussian process equals to quadratic mean.

$$\begin{aligned} m_2^{(\mathrm{NG})} &= \int_{-\infty}^{\infty} x^2 f_{\mathrm{NG}}(x)\mathrm{d}x = \alpha \int_{-\infty}^{\infty} x^2 f_1(x)\mathrm{d}x + (1-\alpha)\int_{-\infty}^{\infty} x^2 f_2(x)\mathrm{d}x \\ &= \alpha\Psi_2^{(1)}(x) + (1-\alpha)\Psi_2^{(2)}(x) \end{aligned} \tag{2.13}$$

Of which, $\Psi_2^{(1)}(x)$ and $\Psi_2^{(2)}(x)$ are quadratic mean of the Gaussian component 1 and 2, respectively, also the function of quadratic mean and variance:

$$\begin{cases} \Psi_2^{(1)}(x) = \mu_1^2 + \sigma_1^2 \\ \Psi_2^{(2)}(x) = \mu_2^2 + \sigma_2^2 \end{cases} \tag{2.14}$$

Equation (2.14) is substituted into Eq. (2.13), then the 2nd moment of the non-Gaussian random process is expanded as:

$$m_2^{(\mathrm{NG})} = \alpha\left(\mu_1^2 + \sigma_1^2\right) + (1-\alpha)\left(\mu_2^2 + \sigma_2^2\right) \tag{2.15}$$

Similarly, the 3rd moment of the non-Gaussian process is:

$$\begin{aligned} m_3^{(\mathrm{NG})} &= \int_{-\infty}^{\infty} x^3 f_{\mathrm{NG}}(x)\mathrm{d}x = \alpha \int_{-\infty}^{\infty} x^3 f_1(x)\mathrm{d}x + (1-\alpha)\int_{-\infty}^{\infty} x^3 f_2(x)\mathrm{d}x \\ &= \alpha\Psi_3^{(1)}(x) + (1-\alpha)\Psi_3^{(2)}(x) \end{aligned} \tag{2.16}$$

Of which, $\Psi_3^{(1)}$ and $\Psi_3^{(2)}$ are the 3rd-moment origin moment of the Gaussian component 1 and 2, respectively:

$$\begin{cases} \Psi_3^{(1)}(X) = 3\mu_1\Psi_2^{(1)}(X) - 2\mu_1^3 = \mu_1^3 + 3\mu_1\sigma_1^2 \\ \Psi_3^{(2)}(X) = 3\mu_2\Psi_2^{(2)}(X) - 2\mu_2^3 = \mu_2^3 + 3\mu_2\sigma_2^2 \end{cases} \tag{2.17}$$

Equation (2.17) is substituted into Eq. (2.16), then the 3rd moment of the non-Gaussian random process is expanded as:

$$m_3^{(\mathrm{NG})} = \alpha\left(\mu_1^3 + 3\mu_1\sigma_1^2\right) + (1-\alpha)\left(\mu_2^3 + 3\mu_2\sigma_2^2\right) \tag{2.18}$$

Similarly, the 4th and 5th moment of the non-Gaussian random process are shown in Eqs. (2.19) and (2.20):

$$m_4^{(NG)} = \alpha\Psi_4^{(1)} + (1-\alpha)\Psi_4^{(2)} \tag{2.19}$$

$$m_5^{(NG)} = \alpha\Psi_5^{(1)} + (1-\alpha)\Psi_5^{(2)} \tag{2.20}$$

For Eq. (2.19):

$$\begin{cases} \Psi_4^{(1)}(X) = 4\mu_1\Psi_3^{(1)}(X) - 6\mu_1^2\Psi_2^{(1)}(X) + 3\mu_1^4 + 3\sigma_1^4 = \mu_1^4 + 6\mu_1^2\sigma_1^2 + 3\sigma_1^4 \\ \Psi_4^{(2)}(X) = 4\mu_2\Psi_3^{(2)}(X) - 6\mu_2^2\Psi_2^{(2)}(X) + 3\mu_2^4 + 3\sigma_2^4 = \mu_2^4 + 6\mu_2^2\sigma_2^2 + 3\sigma_2^4 \end{cases} \tag{2.21}$$

Equation (2.21) is substituted into Eq. (2.19), then the 4th moment of the non-Gaussian random process is:

$$m_4^{(NG)} = \alpha\left(\mu_1^4 + 6\mu_1^2\sigma_1^2 + 3\sigma_1^4\right) + (1-\alpha)\left(\mu_2^4 + 6\mu_2^2\sigma_2^2 + 3\sigma_2^4\right) \tag{2.22}$$

For Eq. (2.20) :

$$\begin{cases} \Psi_5^{(1)} = 5\mu_1\Psi_4^{(1)} - 10\mu_1^2\Psi_3^{(1)} + 10\mu_1^3\Psi_2^{(1)} - 4\mu_1^5 = \mu_1^5 + 10\mu_1^3\sigma_1^2 + 15\mu_1\sigma_1^4 \\ \Psi_5^{(2)} = 5\mu_2\Psi_4^{(2)} - 10\mu_2^2\Psi_3^{(2)} + 10\mu_2^3\Psi_2^{(2)} - 4\mu_2^5 = \mu_2^5 + 10\mu_2^3\sigma_2^2 + 15\mu_2\sigma_2^4 \end{cases} \tag{2.23}$$

Equation (2.23) is substituted into Eq. (2.20), then the 5th moment of the non-Gaussian random process is:

$$m_5^{(NG)} = \alpha\left(\mu_1^5 + 10\mu_1^3\sigma_1^2 + 15\mu_1\sigma_1^4\right) + (1-\alpha)\left(\mu_2^5 + 10\mu_2^3\sigma_2^2 + 15\mu_2\sigma_2^4\right) \tag{2.24}$$

In practice, every moment of the non-Gaussian random load is unknown and usually estimated as per the sample records. Presuming the time series of the sample for the zero-mean non-Gaussian process is $x(t)$, then estimation of the ith moment is:

$$\hat{m}_i^{(NG)} = E\left[x^i(t)\right], \quad i = 1, 2, \ldots \tag{2.25}$$

The estimation is replaced by the theoretical value, then we can obtain a system of quintic Equations for $\alpha,\ \mu_1,\ \mu_2,\ \sigma_1,\ \sigma_2$, based on (2.12), (2.15), (2.18), (2.22), and (2.24):

$$\begin{cases} 0 = \alpha\mu_1 + (1-\alpha)\mu_2 \\ \hat{m}_2^{(\mathrm{NG})} = \alpha\left(\mu_1^2 + \sigma_1^2\right) + (1-\alpha)\left(\mu_2^2 + \sigma_2^2\right) \\ \hat{m}_3^{(\mathrm{NG})} = \alpha\left(\mu_1^3 + 3\mu_1\sigma_1^2\right) + (1-\alpha)\left(\mu_2^3 + 3\mu_2\sigma_2^2\right) \\ \hat{m}_4^{(\mathrm{NG})} = \alpha\left(\mu_1^4 + 6\mu_1^2\sigma_1^2 + 3\sigma_1^4\right) + (1-\alpha)\left(\mu_2^4 + 6\mu_2^2\sigma_2^2 + 3\sigma_2^4\right) \\ \hat{m}_5^{(\mathrm{NG})} = \alpha\left(\mu_1^5 + 10\mu_1^3\sigma_1^2 + 15\mu_1\sigma_1^4\right) + (1-\alpha)\left(\mu_2^5 + 10\mu_2^3\sigma_2^2 + 15\mu_2\sigma_2^4\right) \end{cases} \tag{2.26}$$

Solving the above quintic nonlinear equation s can give estimation of the parameters: $\hat{\mu}_1$, $\hat{\mu}_2$, $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\alpha}$. Substituting the estimation into the Eq. (2.11) can get the expression of the PDF of non-Gaussian random load. The above nonlinear equation s can be solved by means of the numerical method or symbolic operation software. It can be seen that, compared with skewness and kurtosis only, the skewness non-Gaussian PDF considers the 5th moment of the non-Gaussian random process, thus improving the description accuracy for the non-Gaussian statistical property.

2.4 Case Analysis

2.4.1 Case of Symmetrical Non-Gaussian Random Process

Two cases are given in the study to verify the validity of the method proposed:

1. Simulated non-Gaussian random signal with small kurtosis;
2. Measured non-Gaussian random vibration signal of vehicle with big kurtosis;

The PDF of the four non-Gaussian random signals (two are simulation signals, and other other two are measured signals) are estimated by GMM. The method proposed is compared with other methods for a quantitative verification of the validity and accuracy.

2.4.1.1 Simulated Signal

The non-Gaussian random signals with symmetrical distribution are obtained by simulation. Figure 2.1 shows that mean $\mu = 0$, variance $\sigma^2 = 1.1976 \times 10^3$, skewness $\gamma_3 = 0$, kurtosis $\gamma_4 == 8.1394$. Figure 2.1a is the time series of the sample; Fig. 2.1b is PSD, a typical broadband non-Gaussian random process.

The sample series shown in Fig. 2.1a is substituted into (2.5), then we can obtain:

$$\begin{cases} \hat{m}_2 = 1.1976 \times 10^3 \\ \hat{m}_4 = 1.1673 \times 10^7 \\ \hat{m}_6 = 2.5042 \times 10^{11} \end{cases} \tag{2.27}$$