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Teaching for Numeracy Across the Age Range An Introduction



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Teaching for Numeracy Across the Age Range

An Introduction

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Preface

Some years ago, Steen (2007) claimed that ‘Being numerate is one of the few essential skills that students absolutely must master, both for their own good and for the benefit of the nation’s democracy and economic well-being’ (p.16). That statement is as true today as it was in 2007. The digital age and easy access to handheld devices for calculation have not made it any less important for the general population to be competent in understanding and working with numbers.

It is widely accepted now that developing the ability of every individual to understand, utilize and create numerical information in different contexts has to be a priority during the school years and beyond. Poor numeracy skills are known to affect a range of everyday competencies such as the ability to understand daily news bulletins, workshop manuals, medical reports, invoices and household expenditure accounts (Thomson et al., 2020a; Tout, 2021). When students leave school with a poor standard of numeracy this can have devastating social and economic consequences, especially for those from disadvantaged backgrounds (Learning Sciences Institute, 2016). Clear evidence from research suggests that there is a large positive effect on students’ achievement in the curriculum and in their quality of life when competence with numbers is well developed (Cason et al., 2019).

Today, possession of functional number skills is viewed as absolutely essential for all members of the workforce in every industry (National Numeracy Organization, 2017). Numeracy is regarded as a vital competency in many types of employment, and represents a skill set required not only to gain employment but also to progress further within a given field. Increasingly, digital technology in schools and at home is providing a motivating and engaging medium for working with and mastering essential number skills, from early childhood onwards (Mowafi & Abumuhfouz, 2021; NCTM, 2017a; Serhan & Almeqdadi, 2020).

The ability to work confidently, accurately and swiftly with numbers has also been identified as one of the enabling competencies absolutely necessary for success in the STEM subjects—science, technology, engineering and mathematics (Schulz, 2018). In the UK, a policy paper titled *Building our industrial strategy* has acknowledged the link between numeracy, the STEM subjects and productivity in the workplace (HM Government, 2017).

In the current economic, technological and social environment, the need to strengthen numeracy standards in schools and in the general population is more acute than ever before. In the UK, a vision for the immediate future is to produce ‘... a generation of citizens, consumers, students and workers who are as comfortable with numbers as they are with words’ (British Academy, 2015, p.2).

Similarly, in Australia, the Education Council (2015) has stressed a need to establish yardsticks indicating the standard of numeracy that all students should attain before leaving school. Although Australia has a national curriculum, the separate states have also produced their own guidelines and policies on ensuring high numeracy standards. A good example is the Department of Education and Training in the state of Victoria, which has produced an online document titled *Numeracy for all learners*, providing advice on how best to foster students’ conceptual understanding, procedural fluency, reasoning and problem-solving abilities (Victoria State Government, 2020). In the United States, the *Common Core State Standards* for mathematics express the same intention, with the relevant guidelines stressing that from the start of schooling all students should acquire number concepts and skills through a well-sequenced and well-taught curriculum (CCSSI, 2017a).

Chapters in this book explore the nature of numeracy, its component areas of knowledge and skills, and how these can best be taught, applied and maintained in children and adults. The issues covered here range from encouraging basic numeracy development in the preschool years, extending numeracy skills in primary and secondary schools, and adult numeracy. Attention is also given to the learning difficulties that some students experience at any age.

The need for higher standards of numeracy is recognized worldwide, and the author of this book has drawn on relevant research and literature from several countries to provide a comprehensive overview. The extensive reference list will help educators wishing to study certain sub-topics in more detail. At the end of each chapter, many links to other sources of online and print information have been provided.

Taipa, Macao

Peter Stuart Westwood

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Chapter 1

Numeracy: Defined, Described and in Context



The term numeracy encompasses a set of concepts, understandings and skills that are vitally important for many aspects of daily life—at school, in employment, within the family, and for functioning effectively as a contributor, participant and consumer in the community (NALA, 2016). In addition to the role that numeracy plays in everyday functioning, possession of good numeracy skills has been identified as one of the main factors enabling students to follow a particular career path (Holmes et al., 2018; Nicholes, 2019). For example, Jain and Rogers (2019, p. 23) have written that ‘The development of numeracy skills is a core aspect of university preparation, with many university courses requiring a certain level of mathematical literacy.’

Defining Numeracy

The term numeracy appears to have been coined many years ago in the *Crowther Report* in the UK (Central Advisory Council for Education, 1959). At that time, there was no precise definition of numeracy other than it was the ‘mirror image of literacy’, but it was widely understood to mean proficiency in dealing with numbers and situations requiring measurement, estimation and calculation. Some authorities have described this ability as ‘quantitative reasoning’, and that term is certainly useful because it encapsulates the essence of what functional numeracy really involves (Ramful & Ho, 2015).

Later, the Cockcroft Committee in the UK published a report titled *Mathematics counts: A report into the teaching of mathematics in schools* (1982). Therein it was stated that a ‘numerate’ person possesses two attributes: a confidence in dealing with numbers in order to cope with the practical mathematical demands of everyday life; and a capacity to understand quantitative information such as that provided in tables, graphs, diagrams, charts, medical reports and in the media. A recent definition of numeracy is that it represents one’s ability to ‘use numbers in context to assist decision making’ (Diaz et al., 2020).

The original *National Numeracy Strategy* for schools in the UK (DfEE, 1999), now absorbed into the 2003 *Primary National Strategy*, recognized that numeracy involves confidence and competence in working with numbers and measures, and requires an understanding of the number system, a repertoire of computation skills, and an ability to apply numbers to solve problems in a variety of contexts. Later, Foster and Beddie (2005) indicated that numeracy includes not only practical arithmetic skills, but also the ability to communicate quantitative information to others in speech and writing, and to apply common sense estimation and approximation when handling numbers.

The documents referred to above have all provided very clear and acceptable ways of viewing the characteristics of numeracy. It must be noted, however, that there is still no universally accepted definition—a situation that can lead to confusion (Geiger et al., 2015). For example, this lack of definition has resulted in teachers and the community asking the question, ‘Is numeracy simply the same as being good at math?’ The answer to this question should be ‘no’. The terms ‘numeracy’ and ‘mathematical ability’ are not identical in meaning, even though there has been a tendency to use the terms interchangeably in many articles and books (and even more frequently during school staffroom discussions). Mathematical competence comprises much more than simply acquiring and using number skills, because mathematics is a diverse discipline with many branches both applied and theoretical. Many of the concepts within mathematics go well beyond simply understanding and using numbers. However, proficiency in working with numbers represents a key competency that underpins successful performance within a very broad range of topics in mathematics (Tout, 2021).

In this book, the term ‘numeracy’ is used with a deliberately narrow meaning to refer to understanding and applying knowledge and skills involved in measurement, calculation, estimation and quantitative problem-solving. These areas of knowledge and skill are applied not only in mathematics lessons but also across the curriculum and in daily life (Bennison et al., 2020; Forgasz & Hall, 2019). In this respect, the book continues the concept of numeracy that was embodied in the original Cockcroft Report and the *National Numeracy Strategy* in the UK.

Numeracy in Context

Turner (2007, p. 28) suggested that ‘Numeracy has become a personal attribute very much dependent on the context in which the numerate individual is operating [and] numeracy will mean different things to different people according to their interests and lifestyles.’ It is true that numeracy skills are used for different purposes in different settings (Gal et al., 2020), for example, Butcher et al. (2002) referred to ‘numeracy for practical purposes’, ‘numeracy for interpreting society’, ‘numeracy for managing one’s money, budgeting, time and measurement’ and ‘numeracy as an aid to studying other subjects’. The term ‘multiple numeracies’ has therefore emerged in the professional literature (Vacher, 2014).

There is no doubt that numeracy is a prerequisite for entry into many types of employment, and then for promotion and progression through the ranks (Gravemeijer et al., 2017). While different fields of work may require quite specific types of number knowledge (nursing vs. carpentry vs. engineering vs. accountancy), they all depend on a firm foundation of concepts and skills that need to be established in the primary school years. For this reason, schools are expected to develop young children's number concepts and skills as early as possible in order to provide a firm foundation for all later learning.

Numeracy in the Curriculum

The material below provides a brief introduction and overview of numeracy and mathematics curriculum across early childhood and primary school years. Later chapters expand upon this topic and provide more guidance on teaching.

Early Years Curriculum

Beginning in early childhood, math curricula usually focus on three major areas: (i) developing number sense; (ii) establishing counting skills (using the number-word sequence correctly and accurate enumeration with objects); (iii) recognizing number symbols, understanding and using simple addition and subtraction, and written recording of simple operations ($3 + 4 = 7$). Most of the experiences provided to develop these areas involve the children using real objects and visual representations. Later, children begin to learn the meaning of place value in base-ten and develop basic skills in simple mental and written arithmetic (Aunio & Räsänen, 2016; Cheng et al., 2017). At the same time, these skills and understandings are used in solving age-appropriate problems and when carrying out quantitative investigations.

The topics listed above do represent a very reasonable starting point for operationalizing the concept of numeracy, and they highlight areas in which children with learning difficulties may require extra teaching. When children transition to primary school, these skills and concepts will be extended and enriched as new material is taught and as children move from a concrete stage of reasoning to the stage of being able to deal with symbols and abstract ideas. This goal is achieved best when schools use teaching methods, materials and activities that are age-appropriate and that ensure the children are successful in completing the tasks they are required to undertake. It is vital that young learners develop a confident and positive attitude towards working with numbers. It is essential also that the curriculum in the early years has a good sequence and continuity of content (McMahon & Whyte, 2020).

In order to become numerate, a young learner needs to develop a conceptual understanding of number and number relationships, and also become fluent in calculating with numbers. Each aspect of numeracy tends to have its own specific vocabulary