Harish Garg *Editor*

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Theory and Applications

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Preface

With the complexity of the socio-economic environment, today's decision-making is one of the most notable ventures, where the mission is to decide the best alternative under the numerous known or unknown criteria. In cognition of things, people may not possess a precise or sufficient level of knowledge of the problem domain and hence they usually face uncertainties in their preferences over the objects. To address it, a theory of fuzzy set, introduced by Lotfi A. Zadeh in 1965, had enclosed a lot of ground with excellent achievements in almost all branches of science. Since its appearance, many application and extensions of it have been developed which found in both theoretical and practical studies from engineering area to arts and humanities, and from life sciences to physical sciences.

In this book, a new extension of the fuzzy sets, entitled as Pythagorean fuzzy sets, is introduced by eminent researchers with several applications. In this set, the performance of the cognitive in terms of fuzzy environment is considered with the help of degrees of membership and non-membership.

This book consists of three parts. The first part involves five chapters presenting contribution on the information measures of Pythagorean fuzzy sets, such as correlation coefficients, divergence measure, similarity measures and isomorphism ranking methods between different sets. The second part contains seven chapters. These chapters include Pythagorean fuzzy decision-making methods and different applications to the real-life problems. Finally, the last part which contains four chapters on the theory of the extension of Pythagorean fuzzy sets and their applications to the decision-making process.

Chapter "A Survey on Recent Applications of Pythagorean Fuzzy Sets: A State-of-the-Art Between 2013 and 2020" in the first part of the book is to conduct a deep survey on the recent applications of the Pythagorean fuzzy sets. This chapter presents a comprehensive literature review to classify, analyse and interprets the existing research to identify the research trends for the applications of the PFSs is presented. Also, the insights regarding the future research directions, challenges and limitations are given. This literature review also analyzes the chronological development of the extensions of the fuzzy set. Chapter "Some New Weighted Correlation Coefficients Between Pythagorean Fuzzy Sets and Their Applications" defines the correlation and weighted correlation coefficients between the pairs of the Pythagorean fuzzy sets. By utilizing these correlation coefficients, it presents an approach to solve the medical diagnosis and pattern recognition problems. Chapter "Parametric Directed Divergence Measure for Pythagorean Fuzzy Set and Their Applications to Multi-criteria Decision-Making" proposes novel parametric directed divergence measures of order α and degree β to solve the decision making problems. A problem related to investment plan is taken to demonstrate it. Chapter "Some Trigonometric Similarity Measures Based on the Choquet Integral for Pythagorean Fuzzy Sets and Applications to Pattern Recognition" introduces several trigonometric similarity measures based on the Choquet integral for Pythagorean fuzzy sets by using the trigonometric functions cosine and cotangent. An application related to pattern recognition and medical diagnoses are discussed with the proposed similarity measures. Chapter "Isomorphic Operators and Ranking Methods for Pythagorean and Intuitionistic Fuzzy Sets" describes the isomorphism between three pairs of fuzzy sets, namely intuitionistic fuzzy sets and Pythagorean fuzzy sets, interval-valued intuitionistic fuzzy sets and interval-valued Pythagorean fuzzy sets, dual hesitant fuzzy sets and dual hesitant Pythagorean fuzzy sets from three aspects: operational laws, aggregation operators and ranking methods.

In the second part of the book, Chapter "A Risk Prioritization Method Based on Interval-Valued Pythagorean Fuzzy TOPSIS and Its Application for Prioritization of the Risks Emerged at Hospitals During the Covid-19 Pandemic" presents a risk prioritization approach by using extended techniques for order preferences by similarity to ideal solution (TOPSIS) under interval-valued Pythagorean fuzzy environment. The approach is applied for the case of prioritizing the risks that emerged at hospitals during the Covid-19 pandemic. Chapter "Assessment of Agriculture Crop Selection Using Pythagorean Fuzzy CRITIC–VIKOR Decision-Making Framework" presents a new hybrid Pythagorean fuzzy model with CRITIC and VIKOR methods named as PF-CRITIC-VIKOR and employs to solve the Kharif crop selection problem. In this model, the criteria weights are computed by the CRITIC approach and the preference order of Kharif crops is evaluated by VIKOR model, which provides easy mathematical steps with accurate and consistent results for assessing the crops. In addition, entropy measures are utilized to assess to compute the experts' importance degrees. Chapter "Choquet Integral Under Pythagorean Fuzzy Environment and Their Application in Decision Making" introduces Pythagorean fuzzy Choquet integral operators, which not only consider the importance of elements or their ordered positions but also consider the interaction among the criteria or ordered positions in criteria of decision making process. A case of sustainable solid waste management problem of between the major cities in Malaysia is presented to illustrate the application of the proposed aggregation operators. Chapter "On Developing Pythagorean Fuzzy Dombi Geometric Bonferroni Mean Operators with Their Application to Multicriteria Decision Making" introduces Pythagorean fuzzy Dombi geometric Bonferroni mean and Pythagorean fuzzy weighted Dombi geometric Bonferroni mean operators. Based on these aggregation operators, it presents an approach for multi-criteria decision-making problems under the Pythagorean fuzzy environment. Chapter "Schweizer–Sklar Muirhead Mean Aggregation Operators Based on Pythagorean Fuzzy Sets and Their Application in Multi-criteria Decision-Making" is on the exploration of the Schweizer–Sklar (SS) operations based on

Pythagorean fuzzy set and studied their score function, accuracy function. Based on these SS operations, Muirhead mean (MM) operators namely, Pythagorean fuzzy Muirhead mean (PFMM) and Pythagorean fuzzy weighted Muirhead mean (PFWMM) are defined for the Pythagorean fuzzy numbers to aggregate the opinions of different decision makers. Later on, based on this PFWMM operator, a decision-making algorithm is introduced to solve the multi-attribute decision-making algorithms. In the Chapter "Pythagorean Fuzzy MCDM Method Based on CODAS", COmbinative Distance-based ASsessment (CODAS) method is extended to its Pythagorean CODAS version for handling the impreciseness and vagueness in decision making process. Chapter "A Novel Pythagorean Fuzzy MULTIMOORA Applied to the Evaluation of Energy Storage Technologies" is on to the evaluation of energy storage technologies. As a result of the industrialization and the growing population, energy demand has been increasing in the world. In this chapter, a conventional multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) method extend into its Pythagorean fuzzy version. The proposed method adopts the aggregation approach in which distances are utilized on a fuzzy basis. A practical example that considers the evaluation of energy storage technologies is provided to illustrate the technique.

The third and last part of the book deals on the theory of the extension of Pythagorean fuzzy sets such as Hesitant fuzzy set, Linguistic fuzzy set, soft set and their applications to the solve the decision-making problems. In this part, Chapter "Application of Linear Programming in Diet Problem Under Pythagorean Fuzzy Environment" deals with Pythagorean fuzzy linear programming (PFLP) in which the associated cost and variables are treated as Pythagorean fuzzy numbers. With the aid of the score functions of the Pythagorean fuzzy numbers, a PFLP model is converted into its proportional crisp linear programming. The utility of the method is tested by solving some linear programming problems related to diet problems. Chapter "Maclaurin Symmetric Mean-Based Archimedean Aggregation Operators for Aggregating Hesitant Pythagorean Fuzzy Elements and Their Applications to Multicriteria Decision Making" in this part deals with hesitant Pythagorean fuzzy information, an extension of the Pythagorean fuzzy set, to solve the decision-making problems. In this chapter, weighted Maclaurin symmetric mean (MSM) with Archimedean *t*-conorms and *t*-norms (At-CNs & *t*-CNs) aggregation operators are defined to aggregate the hesitant Pythagorean fuzzy information. Based on the proposed operators, it presents an approach for multi-criteria decision-making problems under the hesitant Pythagorean fuzzy environment. Chapter "Extensions of Linguistic Pythagorean Fuzzy Sets and Their Applications in Multi-attribute Group Decision-Making" extends the linguistic Pythagorean fuzzy sets to dual hesitant linguistic Pythagorean fuzzy sets (DHLPFSs) and probabilistic DHLPFSs (PDHLPFSs) in which each element is represented with a linguistic term. The basic operational laws, ranking method and aggregation operators of DHLPFSs and PDHLPFSs are stated. Based on these, multi-attribute group decision making algorithms are established. The last chapter (Chapter "Pythagorean Fuzzy Soft Sets-Based MADM") describes Pythagorean fuzzy soft sets (PFSSs) with their properties. In this chapter, some notions related to PFSS along with their algebraic properties are defined. The four algorithms, that is, choice value method, PFS-TOPSIS, VIKOR and method of similarity measures, for modeling uncertainties in MADM problems based upon PFSSs are established together with several numerical example.

We hope that this book will provide a useful resource of ideas, techniques, and methods for the research on the theory and applications of Pythagorean fuzzy sets. We are grateful to the referees for their valuable and highly appreciated works contributed to select the high quality of chapters published in this book. We would like to also thank the Springer Nature and its team for supporting throughout its publishing.

Patiala, India Harish Garg

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Pythagorean Fuzzy Information Measures

A Survey on Recent Applications of Pythagorean Fuzzy Sets: A State-of-the-Art Between 2013 and 2020

Muhammet Deveci, Levent Eriskin, and Mumtaz Karatas

Acronyms

AHP	Analytic Hierarchy Process
ARAS	Fuzzy Additive Ratio Assessment
CoCoSo	Combined Compromise Solution
CODAS	Combinative Distance-Based Assessment
COPRAS	Complex Proportional Assessment
CRITIC	Criteria Importance Through Inter-criteria Correlation
DEA	Data Envelopment Analysis
DM	Decision-Maker
DNMA	Double Normalization-Based Multiple Aggregation
EDAS	Evaluation Based on Distance from Average Solution
ELECTRE	ELimination Et Choix Traduisant la Realité
FS	Fuzzy Sets
GRA	Gray Relational Analysis
IFS	Intuitionistic Fuzzy Set
MABAC	Multi-attributive Border Approximation Area Comparison
MAIRCA	Multi-Attribute Ideal Real Comparative Analysis
MCDM	Multi-Criteria Decision-Making
MOORA	Multi-Objective Optimization on the basis of Ratio Analysis
PROMETHEE	Preference Ranking Organization METHod for Enrichment of Evaluations

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PFS	Pythagorean Fuzzy Set
PFN	Pythagorean Fuzzy Number
QUALIFLEX	QUALItative FLEXible
TODIM	An acronym in Portuguese of interactive and multi-criteria decision-making
TOPSIS	Technique for Order of Preference by Similarity to Ideal Solution
VIKOR	Višekriterijumsko Kompromisno Rangiranje
WASPAS	Weighted Aggregated Sum Product Assessment
WDBA	Weighted Distance-Based Approximation

1 Introduction

The theory of fuzzy sets (FSs) known as type-1 fuzzy sets, which characterize the uncertainties by membership functions, was introduced by Zadeh [1]. Due to its potential to address uncertainty, it has achieved a great success in various fields [2]. Several extensions of fuzzy sets in the literature have been proposed by various researchers such as type-2 fuzzy sets [3], interval type-2 fuzzy sets [4], intuitionistic fuzzy sets [5], neutrosophic sets [6], hesitant fuzzy sets [7], Pythagorean fuzzy sets [8], picture fuzzy sets [9], q-rung Orthopair fuzzy sets [10], and so on. These sets have been successfully applied in most of the decision-making problems under uncertain environment such as personnel selection [11], supplier selection [12], evaluation of airline service quality [13], health technology assessment [14, 15], factory site selection [16], energy storage method selection [17], and offshore wind farm site selection [18] problems.

Since FSs can only express the vagueness, they do not have the ability to handle hesitation inherent in human thinking [19, 20]. In order to define the hesitations more clearly, intuitionistic fuzzy sets (IFSs) were developed by Atanassov [5], which are important generalization of the fuzzy sets. This approach uses the degree of membership and non-membership to model vagueness and imprecision while the sum of the two membership degrees must be less than or equal to 1. The main contribution of the IFSs is their ability to deal with the hesitancy that may exist due to imprecise information [2]. However, if the sum of *(membership)*+*(non-membership)* is >1, the IFSs fail to overcome this situation. Therefore, Pythagorean fuzzy sets (PFSs) were proposed to address this shortcoming of IFS.

Yager and Abbasov [8] pioneered the PFSs to extend the IFSs which are represented by the degree of membership and non-membership. PFSs are successful extensions of the IFSs and a new tool to cope with uncertainty regarding the degree of memberships. The sum of the two degrees can be less or more than 1, however, the sum of the squares of two degrees is ≤ 1. PFSs are very successful in dealing with vagueness and imprecision involving human thoughts and subjective judgments [21]. When PFSs are compared to IFSs, we observe that they provide more flexibility

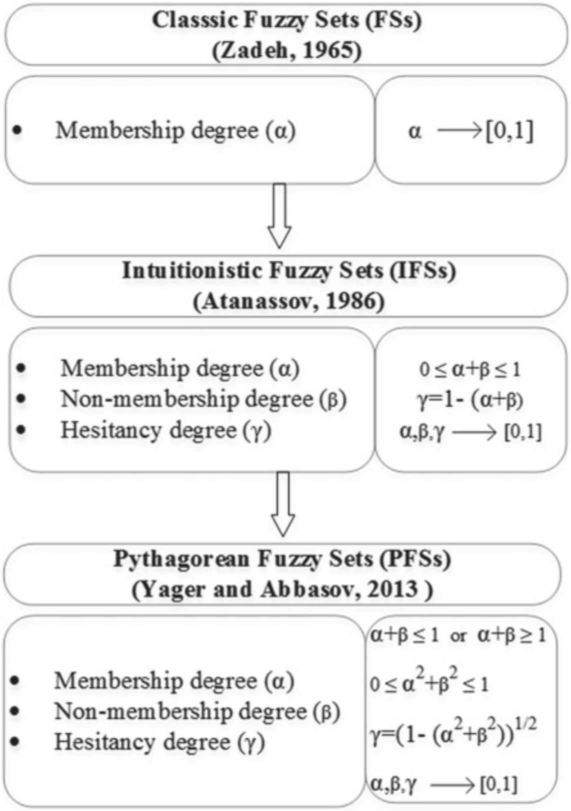


Fig. 1 The development of PFSs and the extensions of classical fuzzy sets

and power to express the uncertainty, since the space of PFSs membership degrees is larger than the space of IFSs (see Fig. 2) [22]. For example, a decision-maker (DM) may provide his/her evaluation for the degree of membership of the element $\hat{x} \in \hat{X}$ with 0.7 and provide his/her evaluation for the degree of non-membership of the element $\hat{x}$ with 0.6. Since the sum of these two values (1.3) is greater than 1, PFSs are preferred for modeling the membership degrees because IFSs cannot meet this condition. Having these properties, PFSs have attracted the attention of many researchers and been applied to many real-life multi-criteria decision-making (MCDM) problems in recent years [23, 24]. The main properties and historical development processes of the FSs, IFSs, and PFSs are illustrated in Fig. 1.

Since PFSs are extensions of the IFSs, they naturally involve the metric space of the IFSs. Moreover, PFSs not only have advantages of the IFSs, but also provide a wider search space to reflect the agreement, disagreement, and hesitancy in decision-making [22].

Having all these properties, PFSs have enormous potential for modeling uncertainty inherent in most of the real-life MCDM problems. To the best knowledge of the authors, there exists no comprehensive review regarding the theory and application areas, the PFSs that will help researchers extract quick and meaningful information. To that end, this study attempts to present a comprehensive survey of application areas and methods that are based on PFSs. In particular, upon collecting a number of quantitative data, such as the year, number of citations, origin country of the articles included in our work, we focus on the areas to which they are applied as well as the MCDM methods and tools they are implemented to. With this multi-dimensional survey approach, we seek to provide a deeper understanding and awareness of how previous research has incorporated the PFSs in different problem domains. This also enables us to examine the most common advantages and challenges observed in PFSs-based applications.

The rest of this paper is structured as follows. Section 2 introduces the basic concepts, aggregation operators, and distance measures of the PFSs. The results of the comprehensive survey are presented in Sect. 3. Finally, Sect. 4 concludes and provides insights regarding advantages, challenges, limitations, and future research directions of the PFSs.

2 Basic Concepts and Operators of Pythagorean Fuzzy Sets

In this section, some basic concepts, operators, and distance measures about PFSs are reviewed.

2.1 Basic Concept of the Pythagorean Fuzzy Sets

First introduced by [5], IFSs are one of the extensions of the classical fuzzy sets to address uncertainty.

Definition 1 Let a set $\hat{F}$ in $\hat{X} = \{\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n\}$ be a finite universe of discourse. IFSs F can be defined as follows:

$$\hat{F} = \left\{ \langle \hat{x} : \alpha_{\hat{F}}(\hat{x}), \beta_{\hat{F}}(\hat{x}) \rangle | \hat{x} \in \hat{X} \right\}, \tag{1}$$

where $\alpha_{\hat{F}}, \beta_{\hat{F}} : \hat{X} \to [0, 1]$ denote, respectively, the degree of membership and the degree of non-membership of the element $\hat{x} \in \hat{X}$, and $0 \leq \alpha_{\hat{F}}(\hat{x}) + \beta_{\hat{F}}(\hat{x}) \leq 1$. The pair $(\alpha_{\hat{F}}(\hat{x}), \beta_{\hat{F}}(\hat{x}))$ can be called as intuitionistic fuzzy number (IFN) and each IFN can be simply expressed as $\hat{\theta} = \left(\alpha_{\hat{\theta}}, \beta_{\hat{\theta}}\right)$, where $\alpha_{\hat{\theta}}, \beta_{\hat{\theta}} : \hat{X} \to [0, 1]$ and $\alpha_{\hat{\theta}} + \beta_{\hat{\theta}} \leq 1$.

The degree of hesitation $\gamma_{\hat{F}}(\hat{x})$ of the element $\hat{x}$ to $\hat{F}$ can be defined as follows: $\gamma_{\hat{F}}(\hat{x}) = 1 - (\alpha_{\hat{F}}(\hat{x}) + \beta_{\hat{F}}(\hat{x}))$. $\gamma_{\hat{F}} : \hat{X} \to [0, 1]$ and if $\gamma_{\hat{F}}(\hat{x}) = 0$, the IFS A is close to a fuzzy set.

IFSs consist of three membership degrees that include membership, non-membership, and hesitancy degrees. However, in some instances, when the sum of $\alpha_{\hat{F}}(\hat{x}) + \beta_{\hat{F}}(\hat{x})$ is greater than >1, the requirement of IFSs is not met. Obviously, a new extension of IFSs is needed because it cannot address this situation. Hence, the PFSs have been proposed by [8] as an extention to the IFSs. The PFSs are a new tool to handle vagueness regarding the degree of membership [25].

Definition 2 Let $\hat{X}$ be a nonempty set. A PFS $\hat{P}$ can be defined by [26] as follows:

$$\hat{P} = \left\{ \langle \hat{x} : \alpha_{\hat{P}}(\hat{x}), \beta_{\hat{P}}(\hat{x}) \rangle | \hat{x} \in \hat{X} \right\}, \tag{2}$$

where $\alpha_{\hat{P}}, \beta_{\hat{P}} : \hat{X} \to [0, 1]$ denote the degree of membership and the degree of non-membership of the element $\hat{x} \in \hat{X}$ to $\hat{P}$, respectively. The following condition must be satisfied for every $\hat{x} \in \hat{X}$:

$$0 \leq \alpha_{\hat{P}}(\hat{x})^2 + \beta_{\hat{P}}(\hat{x})^2 \leq 1. \tag{3}$$

The degree of hesitation $\gamma_{\hat{P}}(\hat{x}) : \hat{X} \to [0, 1]$ of $\hat{x}$ to $\hat{P}$ can be defined as follows:

$$\gamma_{\hat{P}}(\hat{x}) = \sqrt{1 - (\alpha_{\hat{P}}(\hat{x})^2 + \beta_{\hat{P}}(\hat{x})^2)}. \tag{4}$$

If the value of $\gamma_{\hat{P}}(\hat{x})$ is small, then the information about $\hat{P}$ is more precise [27].

Pythagorean fuzzy number (PFN) can be also expressed as $\hat{P} = \left(\alpha_{\hat{P}}, \beta_{\hat{P}}\right)$ and each PFN can be simply expressed as $\hat{\delta} = \left(\alpha_{\hat{\delta}}, \beta_{\hat{\delta}}\right)$ by [28].

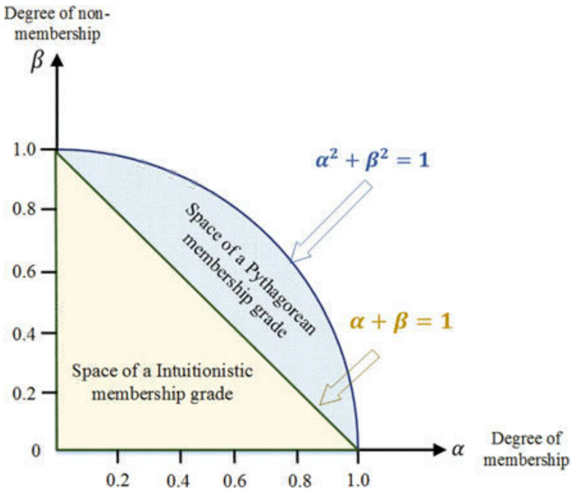
$$\gamma_{\hat{\delta}}(\hat{x}) = \sqrt{1 - (\alpha_{\hat{\delta}}(\hat{x})^2 + \beta_{\hat{\delta}}(\hat{x})^2)} \quad \text{and} \quad 0 \leq \alpha_{\hat{\delta}}(\hat{x})^2 + \beta_{\hat{\delta}}(\hat{x})^2 \leq 1, \tag{5}$$

where $\alpha_{\hat{\delta}}, \beta_{\hat{\delta}} : \hat{X} \to [0, 1]$.

The geometric interpretations of the space of a Pythagorean and intuitionistic membership grades are depicted in Fig. 2 (adopted by [8, 28]). The main difference between PFN and IFN is that they have different constraints, and if an element $\hat{x}$ in $\hat{P}$ is IFN, then it must also be a PFN. However, not all PFNs are IFNs.

In many real-world situations, DMs prefer to use PFNs instead of PFSs to state their evaluation values for alternatives in terms of evaluation criteria [29]. For instance, the evaluation value of Alternative A_i $\hat{\delta}_{ij} = \hat{P}(0.9, 0.3)$ expressed by the DM demonstrates the membership degree as PFN. It should be noted that the alternative A_i is a great alternative in regards to Criterion C_j as 0.9, and meanwhile alternative A_i is not great alternative as 0.4, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ state the alternatives and criteria, respectively.

Fig. 2 Comparison of spaces of Pythagorean and Intuitionistic membership grades



2.2 *Principal Operations*

Let $\hat{\delta}_1 = \left(\alpha_{\hat{\delta}_1}, \beta_{\hat{\delta}_1}\right)$ and $\hat{\delta}_2 = \left(\alpha_{\hat{\delta}_2}, \beta_{\hat{\delta}_2}\right)$ be two PFNs in the set. Their basic operations for PFNs can be expressed as follows [8, 26, 28, 30, 31]:

Definition 3 Three basic operations are initially defined by [26] as follows:

1. $\hat{\delta}_1 \cap \hat{\delta}_2 = P(max\{\alpha_{\hat{\delta}_1}, \alpha_{\hat{\delta}_2}\}, min\{\beta_{\hat{\delta}_1}, \beta_{\hat{\delta}_2}\})$.
2. $\hat{\delta}_1 \cup \hat{\delta}_2 = P(min\{\alpha_{\hat{\delta}_1}, \alpha_{\hat{\delta}_2}\}, max\{\beta_{\hat{\delta}_1}, \beta_{\hat{\delta}_2}\})$.
3. $\hat{\delta}^{\eta} = P(\alpha_{\hat{\delta}}, \beta_{\hat{\delta}})$.

Definition 4 Four operations for PFNs are recreated by [28] as follows:

1. $\hat{\delta}_1 \oplus \hat{\delta}_2 = \left(\sqrt{\alpha^2_{\hat{\delta}_1} + \alpha^2_{[1.]\hat{\delta}_2} - \alpha^2_{\hat{\delta}_1} \cdot \alpha^2_{\hat{\delta}_2}}, \beta_{\hat{\delta}_1} \cdot \beta_{\hat{\delta}_2}\right)$.
2. $\hat{\delta}_1 \otimes \hat{\delta}_2 = \left(\alpha_{\hat{\delta}_1} \cdot \alpha_{\hat{\delta}_2}, \sqrt{\beta^2_{\hat{\delta}_1} + \beta^2_{\hat{\delta}_2} - \beta^2_{\hat{\delta}_1} \cdot \beta^2_{\hat{\delta}_2}}\right)$.
3. $\rho\hat{\delta} = \left(\sqrt{1 - (1 - \alpha^2_{\hat{\delta}})^{\rho}}, \beta^{\rho}_{\hat{\delta}}\right), \rho > 0$.
4. $\hat{\delta}^{\rho} = \left(\alpha^{\rho}_{\hat{\delta}}, \sqrt{1 - (1 - \beta^2_{\hat{\delta}})^{\rho}}\right), \rho > 0$.

Definition 5 The corresponding operations are also described by [25] as follows:

1. $\hat{\delta}_1 \ominus \hat{\delta}_2 = \left(\sqrt{\frac{\alpha^2_{\hat{\delta}_1} - \alpha^2_{\hat{\delta}_2}}{1 - \alpha^2_{\hat{\delta}_2}}}, \frac{\beta_{\hat{\delta}_1}}{\beta_{\hat{\delta}_2}}\right)$, if $\alpha_{\hat{\delta}_2} \leq \alpha_{\hat{\delta}_1}$, $\beta_{\hat{\delta}_1} \leq min\left\{\beta_{\hat{\delta}_2}, \frac{\beta_{\hat{\delta}_2}\gamma_{\hat{\delta}_1}}{\gamma_{\hat{\delta}_2}}\right\}$.
2. $\hat{\delta}_1 \oslash \hat{\delta}_2 = \left(\frac{\alpha_{\hat{\delta}_1}}{\alpha_{\hat{\delta}_2}}, \sqrt{\frac{\beta^2_{\hat{\delta}_1} - \beta^2_{\hat{\delta}_2}}{1 - \beta^2_{\hat{\delta}_2}}}\right)$, if $\beta_{\hat{\delta}_2} \leq \beta_{\hat{\delta}_1}$, $\alpha_{\hat{\delta}_1} \leq min\left\{\alpha_{\hat{\delta}_2}, \frac{\alpha_{\hat{\delta}_2}\gamma_{\hat{\delta}_1}}{\gamma_{\hat{\delta}_2}}\right\}$.

2.3 Score and Accuracy Functions

In this section, a variety of score functions are presented from the literature to emphasize the significance of PFNs.

Definition 6 [28] Let $\hat{\delta}$ be a PFN. The score function of $\hat{\delta}$ is described as follows:

$$Score(\hat{\delta}) = (\alpha_{\hat{\delta}})^2 - (\beta_{\hat{\delta}})^2, \tag{6}$$

where $-1 \leq Score(\hat{\delta}) \leq 1$.

Based on the defined score function of PFNs, the comparison laws of the PFNs are expressed as follows:

Definition 7 [28] Let $\hat{\delta}_1 = \left(\alpha_{\hat{\delta}_1}, \beta_{\hat{\delta}_1}\right)$ and $\hat{\delta}_2 = \left(\alpha_{\hat{\delta}_2}, \beta_{\hat{\delta}_2}\right)$ be two PFNs. $Score(\hat{\delta}_1)$ and $Score(\hat{\delta}_2)$ are expressed as follows:

1. If $Score(\hat{\delta}_1) > Score(\hat{\delta}_2)$, then $\hat{\delta}_1 > \hat{\delta}_2$.
2. If $Score(\hat{\delta}_1) < Score(\hat{\delta}_2)$, then $\hat{\delta}_1 < \hat{\delta}_2$.
3. If $Score(\hat{\delta}_1) = Score(\hat{\delta}_2)$, then $\hat{\delta}_1 \sim \hat{\delta}_2$.

Example 1 Let $\hat{\delta}_1 = \left(\sqrt{7}/5, 3/8\right)$ and $\hat{\delta}_2 = \left(\sqrt{2}/3, 2/7\right)$ be two PFNs. We can calculate the score values of $Score(\hat{\delta}_1)$ and $Score(\hat{\delta}_2)$ according to Definition 7.

$Score(\hat{\delta}_1) = (\sqrt{7}/5)^2 - (3/8)^2 = 1/7$, and $Score(\hat{\delta}_2) = (\sqrt{2}/3)^2 - (2/7)^2 = 1/7$. The results show that $Score(\hat{\delta}_1) = Score(\hat{\delta}_2)$, then $\hat{\delta}_1 \sim \hat{\delta}_2$.

Definition 8 To solve this problem for the equality case, the accuracy function has been proposed by [25]. The accuracy function is expressed as follow:

$$Accuracy(\hat{\delta}) = (\alpha_{\hat{\delta}})^2 + (\beta_{\hat{\delta}})^2, \tag{7}$$

where $Accuracy(\hat{\delta}) \in [0, 1]$.

1. If $Score(\hat{\delta}_1) > Score(\hat{\delta}_2)$, then $\hat{\delta}_1 > \hat{\delta}_2$.
2. If $Score(\hat{\delta}_1) = Score(\hat{\delta}_2)$, then
   (a) If $Accuracy(\hat{\delta}_1) > Accuracy(\hat{\delta}_2)$, then $\hat{\delta}_1 > \hat{\delta}_2$.
   (b) If $Accuracy(\hat{\delta}_1) = Accuracy(\hat{\delta}_2)$, then $\hat{\delta}_1 \sim \hat{\delta}_2$.

2.4 Distance Measures

This subsection examines some widely used distance measures such as Hamming distance, Euclidean distance, and Taxicab distance. The distance between PFNs ($\hat{\delta}_1$ and $\hat{\delta}_2$) can be calculated with different measures as follows [21]:

Definition 9 [28] The Hamming distance measure is described as follow:

$$\phi^{ZX}(\hat{\delta}_1, \hat{\delta}_2) = \frac{1}{2}\Big(\Big|(\alpha_{\hat{\delta}_1})^2 - (\alpha_{\hat{\delta}_2})^2\Big| + \Big|(\beta_{\hat{\delta}_1})^2 - (\beta_{\hat{\delta}_2})^2\Big| + \Big|(\gamma_{\hat{\delta}_1})^2 - (\gamma_{\hat{\delta}_2})^2\Big|\Big). \quad (8)$$

Definition 10 [32] The Euclidean distance measure is described as follow:

$$\phi^{RXG}(\hat{\delta}_1, \hat{\delta}_2) = \left\{\frac{1}{2}\Big[\Big((\alpha_{\hat{\delta}_1})^2 - (\alpha_{\hat{\delta}_2})^2\Big)^2 + \Big((\beta_{\hat{\delta}_1})^2 - (\beta_{\hat{\delta}_2})^2\Big)^2 + \Big((\gamma_{\hat{\delta}_1})^2 - (\gamma_{\hat{\delta}_2})^2\Big)^2\Big]\right\}^{\frac{1}{2}}. \quad (9)$$

Definition 11 [33] The Taxicab distance measure is described as follow:

$$\phi^{B}(\hat{\delta}_1, \hat{\delta}_2) = \frac{1}{2}\Big(\big|\alpha_{\hat{\delta}_1} - \alpha_{\hat{\delta}_2}\big| + \big|\beta_{\hat{\delta}_1} - \beta_{\hat{\delta}_2}\big| + \big|\gamma_{\hat{\delta}_1} - \gamma_{\hat{\delta}_2}\big|\Big). \quad (10)$$

Definition 12 [34] The Generalized distance measure is described as follow:

$$\phi^{C}_{\tau}(\hat{\delta}_1, \hat{\delta}_2) = \left[\frac{1}{2}\Big(\Big|(\alpha_{\hat{\delta}_1})^2 - (\alpha_{\hat{\delta}_2})^2\Big|^{\tau} + \Big|(\beta_{\hat{\delta}_1})^2 - (\beta_{\hat{\delta}_2})^2\Big|^{\tau} + \Big|(\gamma_{\hat{\delta}_1})^2 - (\gamma_{\hat{\delta}_2})^2\Big|^{\tau}\Big)\right]^{\frac{1}{\tau}}, \quad (11)$$

where τ is a distance parameter that satisfies $\tau \geq 1$. It degenerates to Hamming distance as Eq. (8) and Euclidean distance as Eq. (9) when $\tau = 1$ and $\tau = 2$, respectively.

Definition 13 [35] Another generalized distance measure is described as follows:

$$\phi^{LZ}_{\tau}(\hat{\delta}_1, \hat{\delta}_2) = \left[\frac{1}{4}\Big(\big|\alpha_{\hat{\delta}_1} - \alpha_{\hat{\delta}_2}\big|^{\tau} + \big|\beta_{\hat{\delta}_1} - \beta_{\hat{\delta}_2}\big|^{\tau} + \big|\gamma_{\hat{\delta}_1} - \gamma_{\hat{\delta}_2}\big|^{\tau} + \big|\psi_{\hat{\delta}_1} - \psi_{\hat{\delta}_2}\big|^{\tau}\Big)\right]^{\frac{1}{\tau}}, \quad (12)$$

where $\psi_{\hat{\delta}_1}$ and $\psi_{\hat{\delta}_2}$ denote on a range of to 1 how fully the strengths $\gamma_{\hat{\delta}_1}$ and $\gamma_{\hat{\delta}_2}$, respectively.

2.5 *Pythagorean Fuzzy Aggregation Operators*

Some main aggregation operators are presented in this section. Before moving further for a discussion of Pythagorean fuzzy aggregation operators, we would like to emphasize the importance of the aggregation concept in the decision-making process. In this regard, Fig. 3 shows a typical MCDM process for a ranking problematic. For a ranking problematic involving multiple criteria to consider, the first step is to determine the evaluation criteria. Then individual field experts evaluate these criteria based on their own expertise. Essentially, all these evaluations contribute to the overall information content of the problem. One significant task at this stage is to aggregate all these information content provided by the individual experts. The way we aggregate the information content may have dramatic impact on the results of the decision, hence, choosing the proper aggregation operator that hinders information loss is of utmost importance. Finally, the aggregated information is mapped into

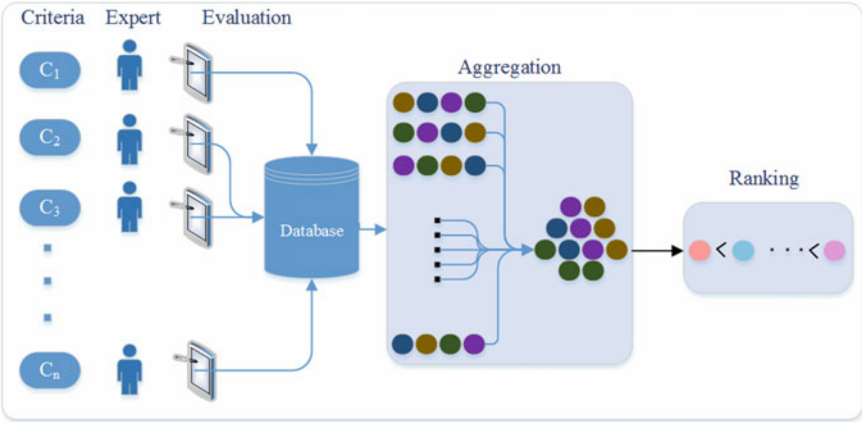


Fig. 3 Multi-criteria decision-making process for a ranking problematic

a score for each alternative which is used for ranking. Aforementioned decision-making process reveals that aggregation operators are important components of the MCDM methodologies which deserve further attention.

Let $\hat{P}_i = (\hat{\alpha}_i, \hat{\beta}_i)(i = 1, 2, \ldots, m)$ be PFNs and $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ be the weight vector of $\hat{P}_i$, and $\sum_i^m \omega_i = 1$.

Definition 14 [26] An $O - PFWA$ (original Pythagorean fuzzy weighted averaging) is defined by

$$O - PFWA(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\sum_{i=1}^{m} \omega_i \hat{\alpha}_i, \sum_{i=1}^{m} \omega_i \hat{\beta}_i \right). \tag{13}$$

If ω=$(\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m})$, the $O - PFWA$ operator degenerates into the O-Pythagorean fuzzy average (O-PFA) operator:

$$O - PFWA(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\frac{\sum_{i=1}^{m} \hat{\alpha}_i}{m}, \frac{\sum_{i=1}^{m} \hat{\beta}_i}{m} \right). \tag{14}$$

Definition 15 [26] An $O - PFWG$ (orginal Pythagorean fuzzy weighted geometric) is defined by

$$O - PFWG(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\prod_{i=1}^{m} \hat{\alpha}_i^{\omega_i}, \prod_{i=1}^{m} \hat{\beta}_i^{\omega_i} \right). \tag{15}$$

If $\omega = (\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m})$, the $O - PFWG$ operator degenerates into the O-Pythagorean fuzzy geometric (O-PFA) operator:

$$O - PFWG(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\prod_{i=1}^{m} \hat{\alpha}_i^{\frac{1}{m}}, \prod_{i=1}^{m} \hat{\beta}_i^{\frac{1}{m}} \right). \tag{16}$$

Definition 16 [36] An $PFWA$ (Pythagorean fuzzy weighted averaging) operator is presented by

$$PFWA(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\sqrt{1 - \prod_{i=1}^{m} (1 - (\hat{\alpha}_i)^2)^{\omega_i}}, \prod_{i=1}^{m} (\hat{\beta}_i)^{\omega_i} \right). \tag{17}$$

If $\omega = (\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m})$, the PFWA operator degenerates into the PFA operator:

$$PFA(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\sqrt{1 - \prod_{i=1}^{m} (1 - (\hat{\alpha}_i)^2)^{\frac{1}{m}}}, \prod_{i=1}^{m} (\hat{\beta}_i)^{\frac{1}{m}} \right). \tag{18}$$

Definition 17 [36] An $PFWG$ (Pythagorean fuzzy weighted geometric) operator is defined by

$$PFWA(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\prod_{i=1}^{m} (\hat{\alpha}_i)^{\omega_i}, \sqrt{1 - \prod_{i=1}^{m} (1 - (\hat{\beta}_i)^2)^{\omega_i}}. \right) \tag{19}$$

If $\omega = (\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m})$, the PFWA operator degenerates into the PFA operator:

$$PFWA(\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_m) = \left(\prod_{i=1}^{m} (\hat{\alpha}_i)^{\frac{1}{m}}, \sqrt{1 - \prod_{i=1}^{m} (1 - (\hat{\beta}_i)^2)^{\frac{1}{m}}} \right). \tag{20}$$

Apart from these aforementioned operators, there are other operators such as novel neutrality operations-based Pythagorean fuzzy geometric aggregation [37] and new logarithmic operational laws and their aggregation operators [38] in the literature.

3 Literature Review

3.1 Survey Methodology

Our survey mainly focused on journal papers, conference papers, and book chapters addressing PFSs. The search was conducted on two prominent resource libraries for scientific literature which cover most of the PFS applications, namely, ScienceDirect and Scopus. Since it was the work of Yager [26] that paved the way for the

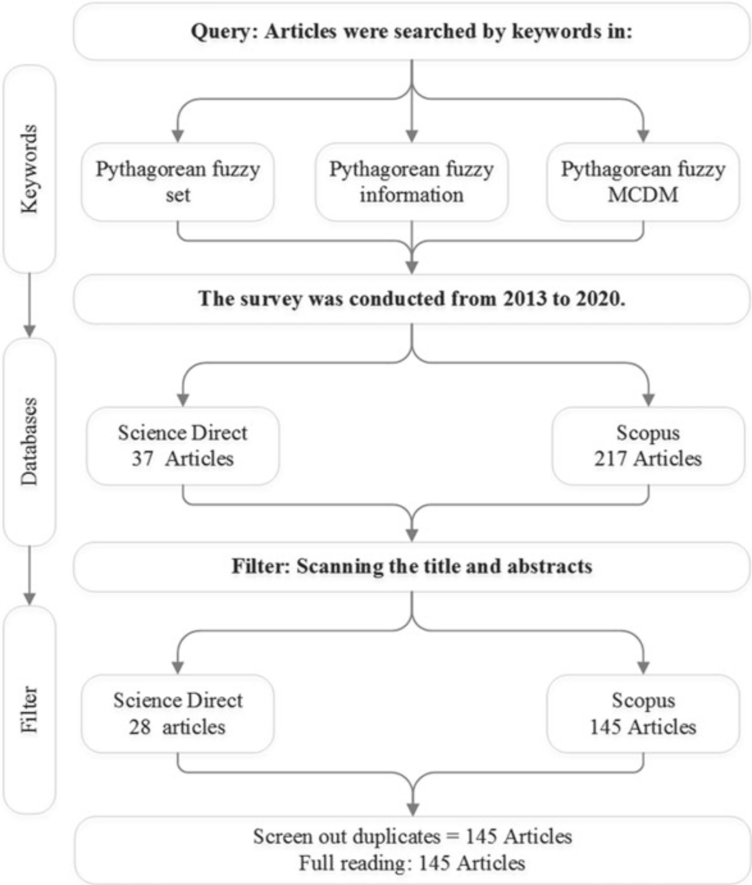


Fig. 4 Flowchart of the literature review on PFSs

PFSs, the search was performed starting from year 2013 until 2020. The keywords "*pythagorean fuzzy information*," "*pythagorean fuzzy MCDM*," and "*pythagorean fuzzy set*" steered the search by examining the titles, abstracts, and keywords of the papers. Among total of 145 papers identified, 136 of the papers are detected with the keyword "*pythagorean fuzzy set*", 46 papers are with the keyword "*pythagorean fuzzy information*", and 1 paper with the keyword "*pythagorean fuzzy MCDM*". Flowchart of the survey methodology is given in Fig. 4.

Figure 5 displays the ratio of studies reached with these keywords. All of the detected papers are found in Scopus database while ScienceDirect database yielded only 28 of the papers.

In an effort to provide as much information as possible from the existing research, we examined the identified papers in various dimensions. These dimensions that are believed to elicit relevant information are given as follows:

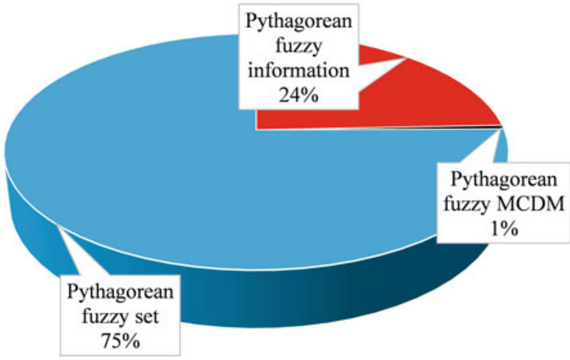


Fig. 5 Percentage of studies with respect to the three keywords used in this study

- **Year**: Year of the publication.
- **Country**: The country where the study was made. If not reported, country of the first author is considered.
- **Citations**: The number of times the publication has been cited.
- **Journal**: The name of the journal where the paper was published.
- **Application area**: The area to which the proposed approach was applied.
- **Methods and tools used**: Other MCDM tools that were used in combination with PFSs.

One of the aims of this survey is to reveal application areas of PFSs and future research directions, hence, we generally focused on and discussed application-oriented papers rather than the theoretical ones.

3.2 Survey Results

In this section, we give quantitative information regarding reviewed papers according to identified review dimensions, firstly. Then, we discuss the literature based on the application area and methods and tools dimensions.

Figure 6 shows the share of each country in the literature of PFSs. The figure shows the number of studies with respect to the origin of country. The results reveal that, China, the origin country of more than half of the 145 studies included in this study, is leading the studies incorporating the PFS concept. China is followed by the origin countries Turkey, Pakistan, Taiwan, and India with the number of studies varying between 9 and 18 out of the 145 studies. Figure 7, on the other hand, draws the attention to the yearly number of PFS related studies published between 2015 and 2020. The bars and solid line in the figure represent the individual yearly published paper numbers and cumulative number of papers, respectively. According to the figure, following [26]'s study in 2013, there is a growing interest in PFS implementations especially after 2017. In particular, 38 and 63 papers have been published in 2018 and 2019, respectively, and 25 papers appear to be published in the first half of 2020.

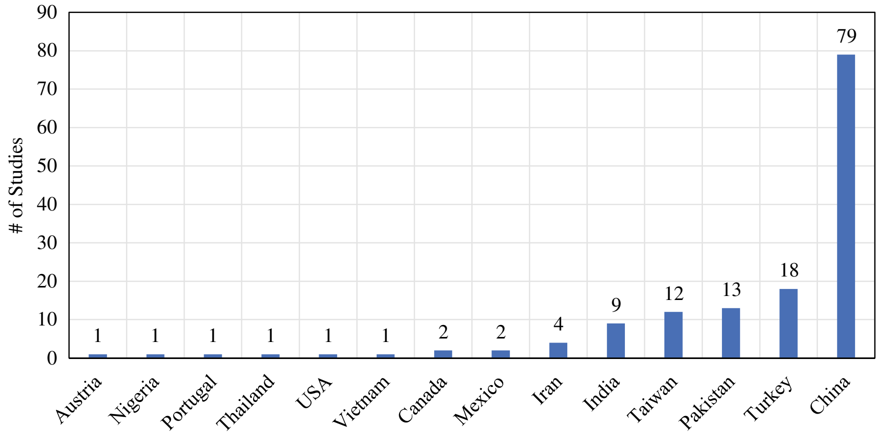


Fig. 6 Number of studies with respect to countries

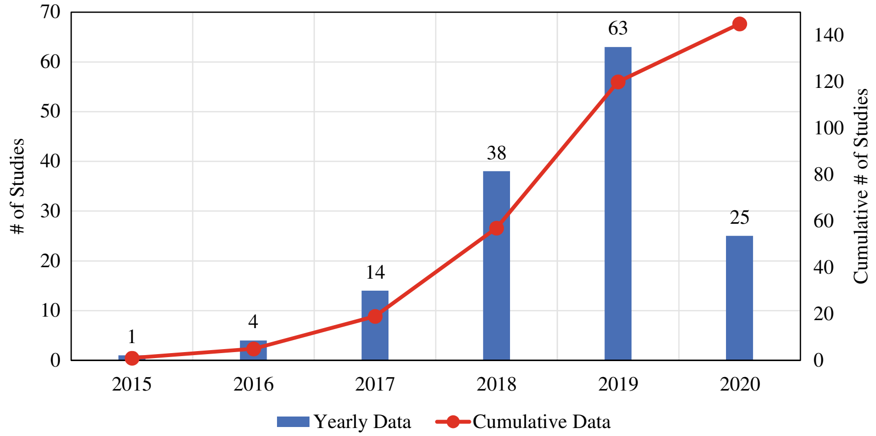


Fig. 7 Yearly and cumulative yearly number of studies.

Table 1 reports the breakdown of yearly number of papers published with respect to the origin country. Once again, the table reveals the growing interest in PFS implementations starting after 2017. It is noteworthy that China is constantly improving its contribution to the body of knowledge related to the PFS implementations.

Figure 8 depicts the frequency of citations for the 145 studies included in this paper. The solid line shows the cumulative percentage values. According to the results, more than half of these studies are cited by less than or equal to 10 papers, and approximately a third of them have been cited for between 11 and 50 times. Although the concept of PFSs is relatively new to the literature, the number of citations collected by studies which incorporate PFSs shows that there is increasing attention and awareness around the world of the capability of PFSs.

Table 1 The number of yearly PFSs-based studies with respect to different countries

Country	2015	2016	2017	2018	2019	2020	Sum
Austria	–	–	–	1	–	–	**1**
Canada	–	1	1	–	–	–	**2**
China	1	3	11	17	35	12	**79**
India	–	–	1	2	4	2	**9**
Iran	–	–	1	–	2	1	**4**
Mexico	–	–	–	1	1	–	**2**
Nigeria	–	–	–	–	1	–	**1**
Pakistan	–	–	–	4	4	5	**13**
Portugal	–	–	–	–	1	–	**1**
Taiwan	–	–	–	7	4	1	**12**
Thailand	–	–	–	–	–	1	**1**
Turkey	–	–	–	6	9	3	**18**
USA	–	–	–	–	1	–	**1**
Vietnam	–	–	–	–	1	–	**1**
Sum	**1**	**4**	**14**	**38**	**63**	**25**	**145**

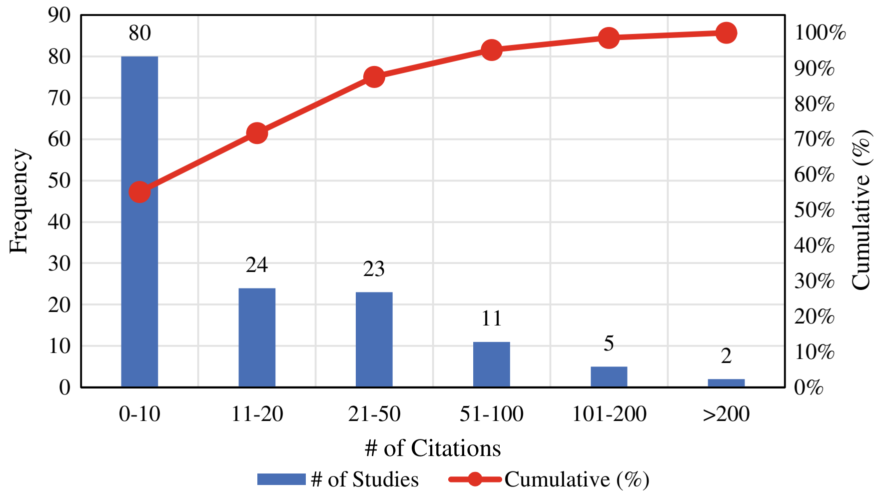


Fig. 8 Histogram for the number of citation

In Table 2 the papers are categorized with respect to the journals they are published in for each year between 2015 and 2020. Note that the journals are listed in decreasing order of the number of articles published (given in the last column). Among the 53 journals listed, IEEE Access, Mathematics, and Soft Computing are the top three journals with 14, 11, and 10 articles published, respectively, in the last three years. Applied Soft Computing, Complexity, and Symmetry are the next three journals with 7 published articles in the PFS domain. We also observe that most of the journals published articles concerning PFSs are peer-reviewed journals in engineering, computational science, information, and mathematics domains. We also note that 4 of the studies out of 145 are conference papers; hence, we do not list them in Table 2.

As emphasized previously, the concept of membership and non-membership degree provides an effective means for modeling vagueness and imprecision as addressed by the IFSs. As a successful extension to the IFSs, the PFSs provide a representation on a larger body of membership and non-standard membership grades which enables DMs to take uncertainty into consideration more flexibly [39]. Being superior to other types of fuzzy extensions, the PFSs are better options for modeling real-life phenomenons. In this regard, many real-life MCDM problems from a variety of fields have been addressed by the PFSs.

Table 3 shows the number of yearly studies published with respect to different application areas. The table reveals that MCDM problems pertaining to supply chain management, investment/capital management, risk management, and project selection areas have been frequently addressed by the PFSs-based MCDM approaches. Additionally, the interest in supply chain management applications is growing recently. When we examine the country origins of these studies with respect to the application areas, we observe that researchers from China mainly focused on supply chain management applications, as seen in Table 4. Turkey, as the second leading country in terms of number of publications, generally publishes on the risk assessment area. Practitioners from other countries, on the other hand, show interest on a variety of application areas without focusing on a particular one.

Classification of papers with respect to application areas and years is presented in Table 5. As remarked, the application area where the PFSs are utilized mostly is the supply chain management. A sustainable supply chain management is important for a company to improve long-term performance while meeting economic, social, and environmental objectives. Selecting the most proper supplier in this regard has become a critical decision problematic for a company since it directly affects organization's success. Having multiple criteria to consider most of which involve uncertain information, the PFSs are utilized for choosing a sustainable supplier [27, 40–44], green supplier [45–48], and partner [49–51] extensively.

Risk assessment is another field where PFSs are commonly applied. Risk assessment enables DMs to identify and analyze potential future events that may impact on assets, projects, environments, or individuals. Even though the results of the assessment can be expressed in terms of qualitative or quantitative fashion, particularly quantitative measures help DMs to identify the amount of tolerability of the risks. Inherent uncertainty in the future events make fuzzy approaches useful alter-

Table 2 The number of yearly PFSs-based studies published in different journals

Journal	2015	2016	2017	2018	2019	2020	Sum
IEEE Access	–	–	–	3	9	2	**14**
Mathematics	–	–	–	6	3	2	**11**
Soft Computing	–	–	–	1	4	5	**10**
Applied Soft Computing	–	1	2	1	3	–	**7**
Complexity	–	–	1	4	1	1	**7**
Symmetry	–	–	–	2	3	2	**7**
International Journal of Fuzzy Systems	–	–	1	1	3	1	**6**
Computers and Industrial Engineering	–	–	–	1	3	1	**5**
Expert Systems With Applications	–	–	–	1	1	2	**4**
IEEE Transactions on Fuzzy Systems	–	1	1	1	–	1	**4**
Information	–	–	–	2	1	1	**4**
Neural Computing and Applications	–	–	–	–	4	–	**4**
Economic Research-Ekonomska Istraživanja	–	–	–	–	2	1	**3**
International Journal of Computational Intelligence Systems	–	–	–	2	1	–	**3**
Applied Intelligence	–	–	–	–	2	–	**2**
Archives of Control Sciences	–	–	1	–	1	–	**2**
Artificial Intelligence Review	–	–	1	–	1	–	**2**
Cognitive Computation	–	–	–	–	2	–	**2**
Computational and Applied Mathematics	–	–	–	–	2	–	**2**
Human and Ecological Risk Assessment	–	–	–	1	1	–	**2**
Information Sciences	–	1	–	1	–	–	**2**
Journal of Ambient Intelligence and Humanized Computing	–	–	–	–	1	1	**2**
Journal of Cleaner Production	–	–	–	1	1	–	**2**
Journal of Experimental and Theoretical Artificial Intelligence	–	–	–	1	1	–	**2**
Journal of Intelligent Systems	–	–	–	1	–	1	**2**
Knowledge and Information Systems	–	–	1	–	1	–	**2**
Safety Science	–	–	–	2	–	–	**2**
Mathematical Problems in Engineering	–	–	–	1	–	–	**1**
Bulletin of the Brazilian Mathematical Society	–	–	–	–	1	–	**1**
Computational and Mathematical Organization Theory	–	–	1	–	–	–	**1**
Discrete Dynamics in Nature and Society	1	–	–	–	–	–	**1**
Engineering Applications of Artificial Intelligence	–	–	–	–	1	–	**1**
EURO Journal on Decision Processes	–	–	–	–	–	1	**1**
IEEE Transactions on Engineering Management	–	–	–	–	1	–	**1**
IEEE/CAA Journal of Automatica Sinica	–	–	–	–	1	–	**1**
Information Fusion	–	–	–	1	–	–	**1**
International Journal of Approximate Reasoning	–	–	–	–	–	1	**1**
International Journal of Hydrogen Energy	–	–	–	–	1	–	**1**
International Journal of Information Technology and Decision-Making	–	1	–	–	–	–	**1**
International Journal of Occupational Safety and Ergonomics	–	–	–	1	–	–	**1**
International Journal of Uncertainty	–	–	–	1	–	–	**1**
International Journal of Intelligent Systems	–	–	–	1	–	–	**1**
Inzinerine Ekonomika-Engineering Economics	–	–	–	–	–	1	**1**
Iranian Journal of Fuzzy Systems	–	–	–	–	1	–	**1**
Journal of Applied Mathematics and Computing	–	–	–	–	–	1	**1**
Journal of Failure Analysis and Prevention	–	–	–	–	1	–	**1**
Journal of Mathematics and Computer Science	–	–	–	–	–	1	**1**
Journal of Natural Gas Science and Engineering	–	–	–	–	1	–	**1**
Journal of Safety Research	–	–	–	–	1	–	**1**
Journal of the Operational Research Society	–	–	–	–	1	–	**1**
New Mathematics and Natural Computation	–	–	–	1	–	–	**1**
Scientia Iranica	–	–	–	1	–	–	**1**
Sustainability	–	–	–	–	1	–	**1**
Sum	**1**	**4**	**9**	**39**	**62**	**26**	**141**

Table 3 The number of yearly PFSs-based studies published with respect to different application areas

Application Area	2015	2016	2017	2018	2019	2020	Sum
Supply Chain Management	–	–	4	6	12	2	**24**
Investment/Capital Management	–	–	1	10	4	3	**18**
Risk Assessment	1	1	1	7	7	–	**17**
Project Selection	–	–	1	4	6	1	**12**
System/Alternative Evaluation	–	–	1	2	4	4	**11**
Product Selection	–	–	3	3	1	1	**8**
Healthcare Management	–	–	–	–	5	3	**8**
Location Selection	–	–	–	1	3	3	**7**
Company Selection	–	1	2	1	3	–	**7**
Information System Management	–	–	1	2	3	1	**7**
Construction Management	–	–	–	1	2	1	**4**
Employee Selection		1	1	–	–	2	**4**
Pattern Recognition	–	–	–	–	4	–	**4**
Emergency Management	–	–	–	1	1	1	**3**
Environment Management	–	–	–	1	2	–	**3**
Military Planning	–	–	–	–	2	1	**3**
Technology Management	–	–	–	–	3	–	**3**
Logistics Management	–	–	–	1	–	1	**2**
Sum	**1**	**4**	**14**	**39**	**62**	**25**	**145**

natives for quantifying these risks. For instance, technological innovation projects play a vital role for high-tech firms to obtain competitive advantage against their rival firms. Hence, evaluating risks associated with potential projects is important for a company [52]. Occupational health and safety risks [53–56], safety risks in gas pipeline construction and mining projects [57–59], personal credit default risks [60], assessment of commercial banks' credit risks [61] are some other examples of PFS applications.

Companies quite often face investment decisions for handling the financial and other assets. In accordance with these decisions, short or long-term strategies for acquiring and disposing of portfolio holdings are determined. In some cases, governments or private companies need to make strategic decisions regarding which technology to invest. All these decisions require numerous factors or criteria to consider in the presence of uncertainty of the future. Having the ability to model uncertain environments very successfully, the fuzzy set theory is the prominent option for addressing investment/capital management decisions. As seen from Table 5, PFSs have frequently been applied to investment/capital management problems such as renewable energy investments [62], financing decision on aggressive/conservative policies of working capital management [63, 64], evaluating Internet companies for investment [22], determining multinational company's future investment group strategies [65,

Table 4 The number of yearly PFSs-based studies published with respect to different application areas and countries

Application Area	Austria	Canada	China	India	Iran	Mexico	Nigeria	Pakistan	Portugal	Taiwan	Thailand	Turkey	USA	Vietnam	Sum
Supply Chain Management	–	–	19	1	–	1	–	2	–	–	–	1	–	–	**24**
Investment/Capital Management	–	–	7	4	1	–	–	2	–	3	–	1	–	–	**18**
Risk Assessment	–	–	8	–	–	–	–	–	–	1	–	8	–	–	**17**
Project Selection	–	–	5	1	3	–	–	1	–	2	–	–	–	–	**12**
System/Alternative Evaluation	–	1	3	–	–	–	–	3	1	1	–	2	–	–	**11**
Product Selection	–	–	6	1	–	–	–	–	–	1	–	–	–	–	**8**
Healthcare Management	–	–	5	1	–	–	–	1	–	1	–	–	–	–	**8**
Location Selection	–	–	4	1	–	–	–	–	–	–	–	2	–	–	**7**
Company Selection	–	–	6	–	–	–	–	1	–	–	–	–	–	–	**7**
Information System Management	–	–	4	–	–	–	–	3	–	–	–	–	–	–	**7**
Construction Management	–	–	–	–	–	–	–	–	–	4	–	–	–	–	**4**
Employee Selection	–	–	2	–	–	–	–	1	–	–	–	1	–	–	**4**
Pattern Recognition	–	–	2	–	–	–	1	–	–	–	–	–	–	1	**4**
Emergency Management	–	–	3	–	–	–	–	–	–	–	–	–	–	–	**3**
Environment Management	–	–	2	–	–	–	–	1	–	–	–	–	–	–	**3**
Military Planning	–	–	3	–	–	–	–	–	–	–	–	–	–	–	**3**
Technology Management	–	–	1	1	–	–	–	–	–	–	–	1	–	–	**3**
Logistics Management	–	–	1	–	–	1	–	–	–	–	–	–	–	–	**2**
Sum	**0**	**1**	**81**	**10**	**4**	**2**	**1**	**15**	**1**	**13**	**0**	**16**	**0**	**1**	**145**