



# INTERMITTENT DEMAND FORECASTING

CONTEXT, METHODS AND APPLICATIONS

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WILEY



## Intermittent Demand Forecasting



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Context, Methods and Applications

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*For Jan and Rachel*





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## Preface

The images on the front of this book highlight a crucial tension for all advanced economies. There is a desire to travel more and consume more, but also a growing awareness of the detrimental effects that this is having on the environment. There is a belated realisation that those of us living in countries with developed economies need to consume less and waste less.

Waste can occur at all stages of the supply chain. Consumers may buy food they never eat or clothes they never wear. Retailers and wholesalers may order goods from manufacturers that never sell. These wastages can be significantly reduced by better demand forecasting and inventory management. Some items conform to regular demand patterns and are relatively easy to forecast. Other items, with irregular and intermittent demand patterns, are much harder.

Wastage can be addressed by changes in production, moving away from built-in obsolescence and towards products that can be maintained and repaired economically. For this to be an attractive proposition, spare parts need to be readily available. Unfortunately, these items are often the most difficult to forecast because many of them are subject to the sporadic nature of intermittent demand. Although there have been significant advances in intermittent demand forecasting over recent decades, these are not all available in commercial software. In the final chapter of this book, we highlight the progress that has been made, including methods that are freely available in open source software.

The reasons for the slow adoption of new forecasting methods and approaches in commercial software are varied. We believe that one of the reasons is a lack of appreciation of the benefits that may accrue. Because intermittent demand items are so difficult to forecast, it may be thought that *highly accurate* forecasting methods can never be found. This may be true. However, it is possible to find *more accurate* methods, which can contribute towards significant improvements in inventory management.

There is also a need for greater awareness of the methods that have been developed in recent years. Information on them is scattered amongst a variety of academic journals, and some of the articles are highly technical. Therefore, we have set ourselves the challenge of synthesizing this body of knowledge. We have endeavoured to bring together the main strands of research into a coherent whole, and assuming no prior knowledge of the subject.

There are various perspectives from which demand forecasting can be addressed. One option would be to take an operations management view, with a focus on forecasting and planning processes. Another would be to take a more statistical perspective, starting with

mathematical models and working through their properties. While some of our material has been influenced by these orientations, the dominant perspective of this book is that of operational research (OR). The start point of OR should always be the real-life situation that is encountered. This means that it is essential to gain an in-depth understanding of inventory systems and how forecasts inform these decisions. Such an appreciation enables a sharper focus on forecasting requirements and the appropriate criteria for a ‘good forecast’.

In this book, the first three chapters focus on the inventory management context in which forecasting occurs, including the inventory policies and the service level measures that are appropriate for intermittent demand. Recognising the interconnection between inventory policies, demand distributions, and forecasting methods, the next two chapters focus on demand distributions, including evidence from studies of real-world data. The following two chapters concentrate on forecasting methods, with discussion of practical issues that must be addressed in their implementation. We then turn to the linkage between forecasts and inventory availability, and review how forecast accuracy should be measured and how its implications for inventories should be assessed. We also look at how stock keeping units should be classified for forecasting purposes, and examine methods designed specifically to address maintenance and obsolescence. The next two chapters deal with methods that can tackle more challenging demand patterns. We conclude with a review of forecasting software requirements and our views on the way forward.

We are grateful to those pioneers who inspired us to study this subject, and who have given us valuable advice over the years, especially John Croston, Roy Johnston, and Tom Willemain. We would like to express our thanks to those who commented on draft chapters of this book: Zied Babai, Stephen Disney, Robert Fildes, Thanos Goltsos, Matteo Kalchschmidt, Stephan Kolassa, Nikos Kourentzes, Mona Mohammadipour, Erica Pastore, Fotios Petropoulos, Dennis Prak, Anna-Lena Sachs, and Ivan Svetunkov; and to Nicole Ayiomamitou and Antonis Siakallis who helped with the figures.

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## Glossary

|                  |   |
|------------------|---|
| ADIDA            | aggregate–disaggregate intermittent demand approach         |
| AIC              | Akaike information criterion                                |
| AR               | autoregressive  |
| ARIMA            | autoregressive integrated moving average                    |
| ARMA             | autoregressive moving average                               |
| APE              | absolute percentage error                                   |
| BO               | backorder   |
| BoM              | bill of materials   |
| BS               | Brier score   |
| CDF              | cumulative distribution function                            |
| CFE              | cumulative forecast error                                   |
| CSL              | cycle service level (all replenishment cycles)              |
| CSL <sup>+</sup> | cycle service level (replenishment cycles with some demand) |
| CV               | coefficient of variation                                    |
| EDF              | empirical distribution function                             |
| ERP              | enterprise resource planning                                |
| FMECA            | failure mode, effects, and criticality analysis             |
| FR               | fill rate   |
| FSS              | forecast support system                                     |
| FVA              | forecast value added  |
| HES              | hyperbolic exponential smoothing                            |
| INAR             | integer autoregressive                                      |
| INARMA           | integer autoregressive moving average                       |
| INMA             | integer moving average                                      |
| IP               | inventory position  |
| KS               | Kolmogorov–Smirnov (test)                                   |
| LTD              | lead-time demand  |
| MA               | moving average  |
| MAD              | mean absolute deviation                                     |
| MAE              | mean absolute error   |
| MAPE             | mean absolute percentage error                              |
| MAPEFF           | mean absolute percentage error from forecast                |
| MASE             | mean absolute scaled error                                  |

|       |   |
|-------|---|
| ME    | mean error                                |
| MMSE  | minimum mean square error                 |
| MPE   | mean percentage error                     |
| MPS   | master production schedule                |
| MRO   | maintenance, repair, and operations       |
| MRP   | material requirements planning            |
| MSE   | mean square error                         |
| MSOE  | multiple source of error                  |
| MTO   | make to order                             |
| MTS   | make to stock                             |
| NBD   | negative binomial distribution            |
| NN    | neural network                            |
| NOB   | non-overlapping blocks                    |
| OB    | overlapping blocks                        |
| OUT   | order up to                               |
| PIS   | periods in stock                          |
| PIT   | probability integral transform            |
| RMSE  | root mean square error                    |
| rPIT  | randomised probability integral transform |
| S&OP  | sales and operations planning             |
| SBA   | Syntetos–Boylan Approximation (method)    |
| SBC   | Syntetos–Boylan–Croston (classification)  |
| SCM   | supply chain management                   |
| SES   | single (or simple) exponential smoothing  |
| SKU   | stock keeping unit                        |
| SLA   | service level agreement                   |
| SMA   | simple moving average                     |
| sMAPE | symmetric mean absolute percentage error  |
| sMSE  | scaled mean square error                  |
| SOH   | stock on hand                             |
| SOO   | stock on order                            |
| SSOE  | single source of error                    |
| TSB   | Teunter–Syntetos–Babai (method)           |
| VZ    | Viswanathan–Zhou (method)                 |
| WMH   | Wright Modified Holt (method)             |
| WSS   | Willemain–Smart–Schwarz (method)          |



## About the Companion Website

This book is accompanied by a companion website.

**[www.wiley.com/go/boylansyntetos/intermittentdemandforecasting](http://www.wiley.com/go/boylansyntetos/intermittentdemandforecasting)**



This website includes:

- Datasets (with accompanying information)
- Links to R packages





# 1

## Economic and Environmental Context

### 1.1 Introduction

Demand forecasting is the basis for most planning and control activities in any organisation. Unless a forecast of future demand is available, organisations cannot commit to staffing levels, production schedules, inventory replenishment orders, or transportation arrangements. It is demand forecasting that sets the entire supply chain in motion.

Demand will typically be accumulated in some pre-defined ‘time buckets’ (periods), such as a day, a week, or a month. The determination of the length of the time period that constitutes a time bucket is a very important decision. It is a choice that should relate to the nature of the industry and the volume of the demand itself but it may also be dictated by the IT infrastructure or software solutions in place. Regardless of the length of the time buckets, demand records eventually form a time series, which is a sequence of successive demand observations over time periods of equal length.

On many occasions, demand may be observed in every time period, resulting in what is sometimes referred to as ‘non-intermittent demand’. Alternatively, demand may appear sporadically, with no demand at all in some periods, leading to an intermittent appearance of demand occurrences. Should that be the case, contribution to revenues is naturally lower than that of faster-moving demand items. Intermittent demand items do not attract much marketing attention, as they will rarely be the focus of a promotion, for example. However, they have significant cost implications for a simple reason: there are often many of them!

Service or spare parts are very frequently characterised by intermittent demand patterns. These items are essentially components or (sub-) assemblies contributing to the build-up of a final product. However, they face ‘independent demand’, which is demand generated directly from customers, rather than production requirements for a particular number of units of the final product. In the after-sales environment (or ‘aftermarket’), we deal exclusively with ‘independent demand’ items. Service parts facing intermittent demand may represent a large proportion of an organisation’s inventory investment. In some industries, this proportion may be as high as 60% or 70% (Syntetos 2011). The management of these items is a very important task which, when supported by intelligent inventory control mechanisms, may yield dramatic cost reductions.

Industries that rely heavily on after-sales support, including the automotive, IT, and electronics sectors, are dominated by intermittent demand items. The contributions of the after-sales services to the total revenues of organisations in these industries have been reported to be as high as 60% (Johnston et al. 2003). This signifies an opportunity not only to reduce costs but also to increase revenues through a careful balancing of keeping enough in stock to satisfy customers but not so much as to unnecessarily increase inventory investments. There are tremendous economic benefits that may be realised through the reappraisal of managing intermittent demand items.

There are also significant environmental benefits to be realised by such a reappraisal. Because of their inherent slow movement, intermittent demand items are at the greatest risk of obsolescence. The problem is exacerbated by the greatly reduced product life cycles in modern industry. This affects the planning process for all intermittent demand items (both final products and spare parts used to sustain the operation of final products). Better forecasting and inventory decisions may reduce overall scrap and waste. Furthermore, the sustained provision of spare parts may also reduce premature replacement of the original equipment.

The area of intermittent demand forecasting has been neglected by researchers and practitioners for too long. From a business perspective, this may be explained in terms of the lack of focus on intermittent demand items by the marketing function of organisations. However, the tough economic conditions experienced from around 2010 onwards have resulted in a switch of emphasis from revenue maximisation to cost minimisation. This switch repositions intermittent demand items as the focus of attention in many companies, as part of the drive to dramatically cut down costs and remain competitive. In addition, the more recent emergence of the after-sales business as a major determinant of companies' success has also led to the recognition of intermittent demand forecasting as an area of exceptional importance.

Following a seminal contribution in this area by John Croston in 1972, intermittent demand forecasting received very little attention by researchers over the next 20 years. This was in contrast to the extensive research conducted on forecasting faster-moving demand items. Research activity grew rapidly from the mid-1990s onwards, and we have now reached a stage where a comprehensive body of knowledge, both theoretical and empirical, has been developed in this area. This book aims to provide practitioners, students, and academic researchers with a single point of reference on intermittent demand forecasting. Although there are considerable openings for further advancements, the current state of knowledge offers organisations significant opportunities to improve their intermittent demand forecasting. Numerous reports, to be discussed in more detail later in this chapter, indicate that intermittent demand forecasting is one of the major problems facing modern organisations. Specialised software packages offer some forecasting support to companies but they often lag behind new developments. There are great benefits that have not yet been achieved in this area, and we hope that this book will make a contribution towards their realisation.

There are three main audiences for this book:

1. Supply chain management (SCM) practitioners, broadly defined, who wish to realise the full benefits of managing intermittent demand items.

2. Software designers wanting to incorporate new developments in forecasting into their software.
3. Students and academics wishing to learn and incorporate into their curricula, respectively, the state of the art in intermittent demand forecasting.

In summary, business pressures to reduce costs and environmental pressures to reduce scrap (often introduced in the form of prescribed policies imposed by national governments or the EU for example) render intermittent demand items, and forecasting their requirements, one of the most important areas in modern organisations.

There are great benefits associated with forecasting intermittent demand more accurately, and those benefits are far from being realised. This may be explained by the well reported innovation–adoption gap, which arises from the divergence between innovations and real-world practices. Organisational practices typically lag behind software developments, and software developments typically lag behind the state of the art in the academic literature. It is the aim of this book to bridge these gaps and show how intelligent intermittent demand forecasting may result in significant economic and environmental benefits.

In the remainder of this chapter, we first discuss in more detail the potential benefits that may be realised through improved intermittent demand forecasting. We then provide an overview of the current state of supply chain software packages and enterprise resource planning (ERP) solutions with regard to intermittent demand forecasting. This is followed by a section where we elaborate on both the structure of this book and the perspective that we take regarding the material presented here. We close with a summary of the chapter.

## 1.2 Economic and Environmental Benefits

Intermittent demand for products appears sporadically, with some time periods showing no demand at all. Moreover, when demand occurs, the demand size may be constant or variable, perhaps highly so, leading to what is often termed ‘lumpy demand’. Later in this chapter, we discuss why forecasting sporadic and lumpy demand patterns is a very difficult task. Specific characterisations of intermittent demand series are considered in detail in Chapters 4 and 5.

### 1.2.1 After-sales Industry

Intermittent demand items dominate service and repair parts inventories in many industries (Boylan and Syntetos 2010). A survey by Deloitte Research (2006) benchmarked the service businesses of many of the world’s largest manufacturing companies with combined revenues reaching more than \$1.5 trillion; service operations accounted for an average of 25% of revenues. In addition to their contribution to revenues, these items present a distinct opportunity for cost reductions. Maintenance, repair, and operations (MRO) inventories typically account for as much as 40% of the annual procurement budget (Donnelly 2013). Increased revenues and reduced costs naturally lead to increased profits. Many organisations have repeatedly testified to the importance of after-sales services for their businesses

and the profits they generate. Companies such as Beretta, Canon, DAF Trucks, Electrolux, EPTA, GE Oil & Gas, and Lavapiu have reported contributions of the after-sales services to their total profit of up to 50% (Syntetos 2011). Comparable numbers have been reported by Gaiardelli et al. (2007), Kim et al. (2007), and Glueck et al. (2007), while after-sales service has been identified as a key profit lever in the manufacturing sector (Manufacturing Management 2018).

Intermittent demand items are at the greatest risk of obsolescence. Many case studies (e.g. Molenaers et al. 2012) have documented large proportions of ‘dead’ (obsolete) stock in a variety of industries, with serious environmental implications. However, under-stocking situations may be as harmful, given the potentially high criticality of the items involved. In civil aviation, for example, lack of spare parts is one of the major causes of ‘aircraft on ground’ events (problems serious enough to prevent aircraft from flying). Badkook (2016) found that a quarter of the aircraft in an (un-named) airline’s Boeing 777 fleet were affected by such aircraft on ground events over a year.

### 1.2.2 Defence Sector

Defence inventories, which are highly reliant on spare parts, have been repeatedly identified as a high risk area with a direct impact on a nation’s economy. In the United States for example, the Department of Defense (DoD) manages around five million secondary items. These include repairable components, subsystems, assemblies, consumable repair parts, and bulk items. They reported that, as of September 2017, the value of the inventory was \$93 billion (GAO 2019). Although a matter of concern, there had been no substantial reductions in inventory values over the previous decade (being, for example, \$95 billion in 2013 and 2010; GAO 2012, 2015).

A major determinant of the performance of an inventory system is the forecasting method(s) being used to predict demand. Inaccurate forecasts lead to either excess inventory or shortfalls, depending on the direction of the forecast error. Over-forecasting can lead to holding stocks that are simply not needed. According to the US Government Accountability Office (GAO 2011, p. 11), ‘Our recent work identified demand forecasting as the leading reason why the services and DLA [Defense Logistics Agency] accumulate excess inventory’.

Unfortunately, progress in improving forecasting and inventory management has been slow in many industries, with the defence industry being a case in point. The GAO of the United States reported, ‘Since 1990, we have identified DoD [Department of Defense] supply chain management as a high-risk area due in part to ineffective and inefficient inventory management practices and procedures, weaknesses in accurately forecasting the demand for spare parts, and other supply chain challenges. Our work has shown that these factors have contributed to the accumulation of billions of dollars in spare parts that are excess to current needs’ (GAO 2015, p. 2). Progress in inventory management has been made since then, especially with regard to the visibility of physical inventories, receipt processing, and cargo tracking (GAO 2019). These improvements in information systems have led to inventory management being removed from the list of high-risk areas. However, it is notable that no claims have yet been made for corresponding improvements in demand forecasting.