

INTERMITTENT DEMAND FORECASTING

CONTEXT, METHODS AND APPLICATIONS

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Table of Contents

<u>Cover</u>

<u>Title Page</u>

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Dedication

<u>Preface</u>

<u>Glossary</u>

About the Companion Website

<u>1 Economic and Environmental Context</u>

1.1 Introduction

1.2 Economic and Environmental Benefits

<u>1.3 Intermittent Demand Forecasting Software</u>

1.4 About this Book

<u>1.5 Chapter Summary</u>

Technical Note

2 Inventory Management and Forecasting

2.1 Introduction

2.2 Scheduling and Forecasting

2.3 Should an Item Be Stocked at All?

2.4 Inventory Control Requirements

2.5 Overview of Stock Rules

2.6 Chapter Summary

Technical Notes

<u>3 Service Level Measures</u>

3.1 Introduction

3.2 Judgemental Ordering

<u>3.3 Aggregate Financial and Service Targets</u>

<u>3.4 Service Measures at SKU Level</u>

3.5 Calculating Cycle Service Levels

<u>3.6 Calculating Fill Rates</u>

3.7 Setting Service Level Targets

3.8 Chapter Summary

Technical Note

<u>4 Demand Distributions</u>

4.1 Introduction

4.2 Estimation of Demand Distributions

4.3 Criteria for Demand Distributions

4.4 Poisson Distribution

4.5 Poisson Demand Distribution

4.6 Incidence and Occurrence

4.7 Poisson Demand Incidence Distribution

4.8 Bernoulli Demand Occurrence Distribution

4.9 Chapter Summary

Technical Notes

5 Compound Demand Distributions

5.1 Introduction

5.2 Compound Poisson Distributions

5.3 Stuttering Poisson Distribution

5.4 Negative Binomial Distribution

5.5 Compound Bernoulli Distributions

5.6 Compound Erlang Distributions

5.7 Differing Time Units

5.8 Chapter Summary

Technical Notes

<u>6 Forecasting Mean Demand</u>

6.1 Introduction

6.2 Demand Assumptions

6.3 Single Exponential Smoothing (SES)

6.4 Croston's Critique of SES

6.5 Croston's Method

6.6 Critique of Croston's Method

6.7 Syntetos-Boylan Approximation

6.8 Aggregation for Intermittent Demand

6.9 Empirical Studies

6.10 Chapter Summary

Technical Notes

7 Forecasting the Variance of Demand and Forecast Error

7.1 Introduction

7.2 Mean Known, Variance Unknown

7.3 Mean Unknown, Variance Unknown

7.4 Lead Time Variability

7.5 Chapter Summary

Technical Notes

<u>8 Inventory Settings</u>

8.1 Introduction

8.2 Normal Demand

8.3 Poisson Demand

8.4 Compound Poisson Demand

8.5 Variable Lead Times

<u>8.6 Chapter Summary</u>

Technical Notes

9 Accuracy and Its Implications

9.1 Introduction

9.2 Forecast Evaluation

9.3 Error Measures in Common Usage

9.4 Criteria for Error Measures

<u>9.5 Mean Absolute Percentage Error and its</u> <u>Variants</u>

9.6 Measures Based on the Mean Absolute Error

9.7 Measures Based on the Mean Error

9.8 Measures Based on the Mean Square Error

9.9 Accuracy of Predictive Distributions

9.10 Accuracy Implication Measures

9.11 Chapter Summary

Technical Notes

10 Judgement, Bias, and Mean Square Error

10.1 Introduction

10.2 Judgemental Forecasting

10.3 Forecast Bias

10.4 The Components of Mean Square Error

10.5 Chapter Summary

Technical Notes

11 Classification Methods

11.1 Introduction

11.2 Classification Schemes

11.3 ABC Classification

11.4 Extensions to the ABC Classification

11.5 Conceptual Clarifications

11.6 Classification Based on Demand Sources

11.7 Forecasting-based Classifications

<u>11.8 Chapter Summary</u>

Technical Notes

12 Maintenance and Obsolescence

12.1 Introduction

12.2 Maintenance Contexts

12.3 Causal Forecasting

12.4 Time Series Methods

12.5 Forecasting in Context

<u>12.6 Chapter Summary</u>

Technical Notes

13 Non-parametric Methods

13.1 Introduction

13.2 Empirical Distribution Functions

13.3 Non-overlapping and Overlapping Blocks

13.4 Comparison of Approaches

13.5 Resampling Methods

13.6 Limitations of Simple Bootstrapping

13.7 Extensions to Simple Bootstrapping

13.8 Chapter Summary

Technical Notes

14 Model-based Methods

14.1 Introduction

14.2 Models and Methods

<u>14.3 Integer Autoregressive Moving Average</u> (INARMA) Models

14.4 INARMA Parameter Estimation

14.5 Identification of INARMA Models

14.6 Forecasting Using INARMA Models

14.7 Predicting the Whole Demand Distribution

14.8 State Space Models for Intermittence

14.9 Chapter Summary

Technical Notes

15 Software for Intermittent Demand

15.1 Introduction

15.2 Taxonomy of Software

15.3 Framework for Software Evaluation

15.4 Software Features and Their Availability

15.5 Training

15.6 Forecast Support Systems

15.7 Alternative Perspectives

15.8 Way Forward

15.9 Chapter Summary

Technical Note

<u>References</u>References

<u>Author Index</u>

<u>Subject Index</u>

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List of Tables

Chapter 3

Table 3.1 Order comprising five order lines.Table 3.2 Distribution of demand over one week.Table 3.3 Probability distribution of total demandover two weeks.

Table 3.4 Cumulative distribution of total demand over two weeks.

Table 3.5 Distribution of total demand over two weeks conditional on non-zero...

Table 3.6 Fill rates per time period.

Table 3.7 Distribution of lumpy demand over one week.

Table 3.8 Traditional fill rate calculation (S = 7 and S = 8; R + L = 3).

Table 3.9 Sobel's fill rate calculation (S = 7, L = 2, R = 1).

Chapter 4

Table 4.1 Triangular distribution example.

<u>Table 4.2 Poisson probabilities ($\lambda = 2$).</u>

Table 4.3 Calculation of chi-square goodness of fit statistic.

Table 4.4 Example of demand incidences.

Table 4.5 Weekly demand data.

Table 4.6 Sequence of demand occurrences (1) and non-occurrences (0).

Table 4.7 Observed and estimated order incidences over four weeks.

Table 4.8 Critical values of the chi-square distribution for degrees of freed...

Chapter 5

Table 5.1 Poisson ($\lambda = 2$) and stuttering Poisson ($\lambda = 1$ and $\rho = 0.5$) probabilities (prob) an...

Table 5.2 Calculation of negative binomial probabilities ($\mu = 4$ and $\sigma^2 = 10$).

Table 5.3 Percentages of SKUs with strong fit (demand per period).

Table 5.4 Percentages of SKUs with strong fit (lead time demand).

Table 5.5 Variables to be forecasted for four demand distributions.

Table 5.6 'Stars and bars' diagrams.

Chapter 6

Table 6.1 SES bias (issue points only, R = 1) as a percentage of average demand.

Table 6.2 Intermittent demand series (first eight periods).

Table 6.3 Series of demand sizes and demand intervals.

Table 6.4 Intermittent demand series (first 10 periods).

<u>Table 6.5 Intermittent series (after demand</u> <u>occurrence in period zero).</u>

Table 6.6 Croston's bias as a percentage of average demand.

Table 6.7 Bias correction factors.

Table 6.8 Bias of SES ($\alpha = 0.2$) as a percentage of average demand conditional on de...

Chapter 7

Table 7.1 Updating of mean and variance using <u>SES.</u>

Table 7.2 Updating of variance over protection interval: scaled and direct.

Table 7.3 Distributions of demand over gamma distributed lead times.

Chapter 8

Table 8.1 Safety factors for CSL targets, normal demand.

<u>Table 8.2 Safety factors for fill rate (FR) targets,</u> <u>normal demand.</u>

Table 8.3 Asymmetric effect of under- and overforecasting.

Table 8.4 Adjusted safety factors for cycle service levels.

Table 8.5 Cycle service level for Poisson demand $((R+L)\lambda = 3\lambda = 1.2)$.

Table 8.6 Fill rate for Poisson distributed demand.

Table 8.7 Cycle service levels for stuttering Poisson distributed demand.

Table 8.8 Weighted cumulative probabilities.

Table 8.9 Adjusted safety factors for fill rates.

<u>Table 8.10</u>**CSL**⁺ <u>component calculations for Poisson</u> <u>distributed demand.</u>

Table 8.11 CSL⁺ calculations for Poisson distributed demand.

Table 8.12 Fill rate calculations for Poisson demand.

Table 8.13 **CSL**⁺ component calculations for stuttering Poisson demand.

Chapter 9

Table 9.1 Mean error, mean square error, mean absolute error, and mean absolu...

Table 9.2 Forecast value added (FVA) example.

Table 9.3 MAPEFF and sMAPE for intermittent demand.

Table 9.4 MAE : Mean ratios for multiple series.

Table 9.5 Mean absolute error for zero forecasts.

Table 9.6 Mean error (ME) and mean absolute error (MAE).

Table 9.7 Scaled mean error for multiple series.

Chapter 10

Table 10.1 Reported usage of forecast methods in practice.

Table 10.2 Judgemental adjustments: effect on cycle service levels.

Table 10.3 Cumulative forecast error (CFE).

Table 10.4 Mean square error (frequent zeroes).

Chapter 13

Table 13.1 Cumulative frequency percentages.

Table 13.2 Three-month overlapping blocks (OB) and non-overlapping blocks (NO...

Table 13.3 Resampling from previous observations.

<u>Table 13.4 VZ resampling method (R + L = 3).</u>

Table 13.5 Most recent 10 observations from Table 13.2.

Table 13.6 Conditional probabilities of demand occurrence.

Table 13.7 Simple bootstrapping with Markov chain extension..

<u>Table 13.8 Theta function calculation (m = 2, y = 2), overlap of one period.</u>

Chapter 14

Table 14.1 INAR(1) process example.

Table 14.2 INMA(1) process example.

Table 14.3 Four simplest INARMA models.

Table 14.4 Empirical evidence on model identification.

Table 14.5 Conditional probabilities of demand at time $t + 1_{(d_{t+1})}$ given demand at t...

Table 14.6 Cumulative conditional probabilities attime $t + 1_{(d_{t+1})}$ given demand at ...

Table 14.7 Cumulative probabilities of demand over two periods $(d_{t+1} + d_{t+2})$, given dem...

Chapter 15

Table 15.1 Software implementation.

List of Illustrations

Chapter 1

Figure 1.1 Intermittent and lumpy demand.

Chapter 2

Figure 2.1 Bill of materials (BoM) example.

<u>Figure 2.2 Periodic review and continuous review</u> <u>systems.</u> Figure 2.3 Continuous review (s, Q) and (s, S) policies for unit sized transactions.

<u>Figure 2.4 Periodic review (R, S) policy.</u>

Chapter 3

Figure 3.1 Comparison of CSL and CSL⁺.

Figure 3.2 Exchange curve.

Figure 3.3 RightStock Inventory Strategist.

Chapter 4

<u>Figure 4.1 Monthly demand time series for an automotive SKU.</u>

Figure 4.2 Demand frequencies for an automotive <u>SKU.</u>

<u>Figure 4.3 Demand relative frequencies with</u> <u>triangle superimposed.</u>

<u>Figure 4.4 Actual relative frequencies and</u> <u>triangular probabilities.</u>

Figure 4.5 Poisson distribution for varying mean () values.

<u>Figure 4.6 Poisson probabilities and actual relative</u> <u>frequencies.</u>

<u>Figure 4.7 Variance and mean of weekly order</u> <u>frequencies.</u>

Chapter 5

<u>Figure 5.1 Geometric distribution ($\rho = 0.2, 0.5, 0.8$).</u>

<u>Figure 5.2 Standard deviation and mean of demand</u> <u>sizes. Source: Johnston et ...</u>

<u>Figure 5.3 Frequency distribution of order sizes.</u> <u>Source: Johnston et al. 20...</u> Figure 5.4 Logarithmic distribution (p = 0.33, 0.66, 0.99).

<u>Figure 5.5 Exponential distributions. (a) Probability</u> <u>density; (b) Cumulativ...</u>

Figure 5.6 Erlang distributions.

<u>Figure 5.7 Normal distribution (poor approximation).</u>

<u>Figure 5.8 Normal distribution (better approximation).</u>

Chapter 6

Figure 6.1 Weights of previous observations. (a) $\alpha = 0.3$. (b) $\alpha = 0.1$.

Figure 6.2 SES response to a step-change. (a) $\alpha = 0.3$. (b) $\alpha = 0.1$.

Figure 6.3 SES bias for issue points only (R = 1).

Figure 6.4 Forecast initialisation and optimisation.

Figure 6.5 ADIDA forecasting framework.

Figure 6.6 Comparison of model forms.

Chapter 8

Figure 8.1 Standard normal distribution.

Figure 8.2 Normally distributed demand and OUT levels.

Chapter 9

Figure 9.1 Errors and absolute ('Abs') errors.

<u>Figure 9.2 Non-uniform distributions of randomised</u> <u>PITs.</u>

Figure 9.3 Exchange curves.

Chapter 10

Figure 10.1 Cumulative demands and forecasts.

Figure 10.2 Squared error decomposition.

<u>Figure 10.3 (Extended) squared error</u> <u>decomposition.</u>

Chapter 11

Figure 11.1 Customer demand and forecasting.

<u>Figure 11.2 Categorisation of non-normal demand</u> <u>patterns.</u>

Figure 11.3 Categorisation based on sources of demand characteristics.

<u>Figure 11.4 Categorisation by mean square error:</u> <u>SES (issue points, R = 1) vs. C...</u>

<u>Figure 11.5 Categorisation by mean square error:</u> <u>SES vs. SBA.</u>

Chapter 12

<u>Figure 12.1 Maintenance generated demand and</u> <u>forecasting.</u>

Figure 12.2 Life cycle stages.

Figure 12.3 TSB and Croston forecasts.

Figure 12.4 Forecasting in context.

Figure 12.5 Inventory-forecasting interactions.

Chapter 13

Figure 13.1 Intermittent series.

Figure 13.2 Cumulative frequency percentages: three-month overlapping blocks...

<u>Figure 13.3 Proportional reduction in variance of</u> <u>CDF estimates by using OB ...</u> Figure 13.4 Cumulative frequency percentages (OB, NOB, and bootstrap).

Chapter 14

Figure 14.1 Demand transitions from one period to the next.

Intermittent Demand Forecasting

Context, Methods and Applications

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For Jan and Rachel

Preface

The images on the front of this book highlight a crucial tension for all advanced economies. There is a desire to travel more and consume more, but also a growing awareness of the detrimental effects that this is having on the environment. There is a belated realisation that those of us living in countries with developed economies need to consume less and waste less.

Waste can occur at all stages of the supply chain. Consumers may buy food they never eat or clothes they never wear. Retailers and wholesalers may order goods from manufacturers that never sell. These wastages can be significantly reduced by better demand forecasting and inventory management. Some items conform to regular demand patterns and are relatively easy to forecast. Other items, with irregular and intermittent demand patterns, are much harder.

Wastage can be addressed by changes in production, moving away from built-in obsolescence and towards products that can be maintained and repaired economically. For this to be an attractive proposition, spare parts need to be readily available. Unfortunately, these items are often the most difficult to forecast because many of them are subject to the sporadic nature of intermittent demand. Although there have been significant advances in intermittent demand forecasting over recent decades, these are not all available in commercial software. In the final chapter of this book, we highlight the progress that has been made, including methods that are freely available in open source software. The reasons for the slow adoption of new forecasting methods and approaches in commercial software are varied. We believe that one of the reasons is a lack of appreciation of the benefits that may accrue. Because intermittent demand items are so difficult to forecast, it may be thought that *highly accurate* forecasting methods can never be found. This may be true. However, it is possible to find *more accurate* methods, which can contribute towards significant improvements in inventory management.

There is also a need for greater awareness of the methods that have been developed in recent years. Information on them is scattered amongst a variety of academic journals, and some of the articles are highly technical. Therefore, we have set ourselves the challenge of synthesizing this body of knowledge. We have endeavoured to bring together the main strands of research into a coherent whole, and assuming no prior knowledge of the subject.

There are various perspectives from which demand forecasting can be addressed. One option would be to take an operations management view, with a focus on forecasting and planning processes. Another would be to take a more statistical perspective, starting with mathematical models and working through their properties. While some of our material has been influenced by these orientations, the dominant perspective of this book is that of operational research (OR). The start point of OR should always be the real-life situation that is encountered. This means that it is essential to gain an indepth understanding of inventory systems and how forecasts inform these decisions. Such an appreciation enables a sharper focus on forecasting requirements and the appropriate criteria for a 'good forecast'.

In this book, the first three chapters focus on the inventory management context in which forecasting occurs, including the inventory policies and the service level measures that are appropriate for intermittent demand. Recognising the interconnection between inventory policies, demand distributions, and forecasting methods, the next two chapters focus on demand distributions, including evidence from studies of real-world data. The following two chapters concentrate on forecasting methods, with discussion of practical issues that must be addressed in their implementation. We then turn to the linkage between forecasts and inventory availability, and review how forecast accuracy should be measured and how its implications for inventories should be assessed. We also look at how stock keeping units should be classified for forecasting purposes, and examine methods designed specifically to address maintenance and obsolescence. The next two chapters deal with methods that can tackle more challenging demand patterns. We conclude with a review of forecasting software requirements and our views on the way forward.

We are grateful to those pioneers who inspired us to study this subject, and who have given us valuable advice over the years, especially John Croston, Roy Johnston, and Tom Willemain. We would like to express our thanks to those who commented on draft chapters of this book: Zied Babai, Stephen Disney, Robert Fildes, Thanos Goltsos, Matteo Kalchschmidt, Stephan Kolassa, Nikos Kourentzes, Mona Mohammadipour, Erica Pastore, Fotios Petropoulos, Dennis Prak, Anna-Lena Sachs, and Ivan Svetunkov; and to Nicole Ayiomamitou and Antonis Siakallis who helped with the figures.

Lancaster and Cardiff January 2021

John E. Boylan Aris A. Syntetos

Glossary

ADIDA

aggregate-disaggregate intermittent demand approach

AIC

Akaike information criterion

AR

autoregressive

ARIMA

autoregressive integrated moving average

ARMA

autoregressive moving average

APE

absolute percentage error

BO

backorder

BoM

bill of materials

BS

Brier score

CDF

cumulative distribution function

CFE

cumulative forecast error

CSL

cycle service level (all replenishment cycles)

 CSL^+

cycle service level (replenishment cycles with some demand)

CV

coefficient of variation

EDF

empirical distribution function

ERP

enterprise resource planning

FMECA

failure mode, effects, and criticality analysis

FR

fill rate

FSS

 $forecast\ support\ system$

FVA

forecast value added

HES

hyperbolic exponential smoothing

INAR

integer autoregressive

INARMA

integer autoregressive moving average

INMA

integer moving average

IP

inventory position

KS

Kolmogorov-Smirnov (test)

LTD

lead-time demand

MA

moving average

MAD

mean absolute deviation

MAE

mean absolute error

MAPE

mean absolute percentage error

MAPEFF

mean absolute percentage error from forecast

MASE

mean absolute scaled error

ME

mean error

MMSE

minimum mean square error

MPE

mean percentage error

MPS

master production schedule

MRO

maintenance, repair, and operations

MRP

material requirements planning

MSE

mean square error

MSOE

multiple source of error

MTO

make to order

MTS

make to stock

NBD

negative binomial distribution

NN

neural network

NOB

non-overlapping blocks

OB

overlapping blocks

OUT

order up to

PIS

periods in stock

PIT

probability integral transform

RMSE

root mean square error

rPIT

randomised probability integral transform

S&OP

sales and operations planning

SBA

Syntetos-Boylan Approximation (method)

SBC

Syntetos-Boylan-Croston (classification)

SCM

supply chain management

SES

single (or simple) exponential smoothing

SKU

stock keeping unit

SLA

service level agreement

SMA

simple moving average

sMAPE

symmetric mean absolute percentage error

sMSE

scaled mean square error

SOH

stock on hand

SOO

stock on order

SSOE

single source of error

TSB

Teunter-Syntetos-Babai (method)

VZ

Viswanathan-Zhou (method)

WMH

Wright Modified Holt (method)

WSS

Willemain-Smart-Schwarz (method)

About the Companion Website

This book is accompanied by a companion website.

www.wiley.com/go/boylansyntetos/intermittentdemandforec



<u>asting</u>

This website includes:

- Datasets (with accompanying information)
- Links to R packages

