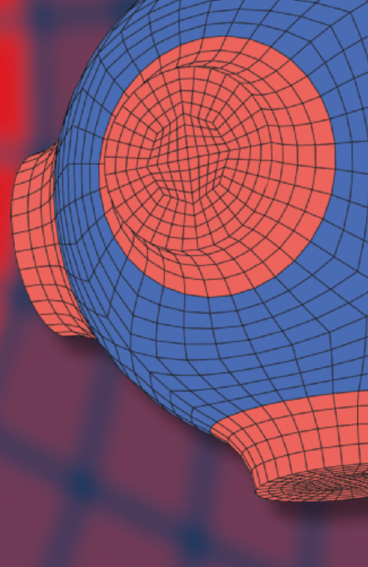
Advanced Structured Materials



Francesco dell'Isola
Simon R. Eugster
Mario Spagnuolo
Emilio Barchiesi *Editors*

Evaluation of Scientific Sources in Mechanics

Heiberg's Prolegomena to the Works of Archimedes and Hellinger's Encyclopedia Article on Continuum Mechanics

Springer

Advanced Structured Materials

Volume 152

Common engineering materials reach in many applications their limits and new developments are required to fulfil increasing demands on engineering materials. The performance of materials can be increased by combining different materials to achieve better properties than a single constituent or by shaping the material or constituents in a specific structure. The interaction between material and structure may arise on different length scales, such as micro-, meso- or macroscale, and offers possible applications in quite diverse fields.

This book series addresses the fundamental relationship between materials and their structure on the overall properties (e.g. mechanical, thermal, chemical or magnetic etc.) and applications.

The topics of *Advanced Structured Materials* include but are not limited to

- classical fibre-reinforced composites (e.g. glass, carbon or Aramid reinforced plastics)
- metal matrix composites (MMCs)
- micro porous composites
- micro channel materials
- multilayered materials
- cellular materials (e.g., metallic or polymer foams, sponges, hollow sphere structures)
- porous materials
- truss structures
- nanocomposite materials
- biomaterials
- nanoporous metals
- concrete
- coated materials
- smart materials

Advanced Structured Materials is indexed in Google Scholar and Scopus.

More information about this series at http://www.springer.com/series/8611

Francesco dell'Isola · Simon R. Eugster ·
Mario Spagnuolo · Emilio Barchiesi
Editors

Evaluation of Scientific Sources in Mechanics

Heiberg's Prolegomena to the Works of Archimedes and Hellinger's Encyclopedia Article on Continuum Mechanics

Springer

Editors
Francesco dell'Isola
DICEAA and International Research Center on Mathematics and Mechanics of Complex Systems
University of L'Aquila
L'Aquila, Italy

Mario Spagnuolo
DICAAR
University of Cagliari
Cagliari, Italy

Simon R. Eugster
Institute for Nonlinear Mechanics
University of Stuttgart
Stuttgart, Germany

Emilio Barchiesi
International Research Center on Mathematics and Mechanics of Complex Systems
University of L'Aquila
L'Aquila, Italy

ISSN 1869-8433 ISSN 1869-8441 (electronic)
Advanced Structured Materials
ISBN 978-3-030-80549-4 ISBN 978-3-030-80550-0 (eBook)
https://doi.org/10.1007/978-3-030-80550-0

This Springer imprint is published by the registered company Springer Nature Switzerland AG
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Preface

This volume is the result of a collaboration among scholars having different competences, albeit the majority of them can be classified to be applied mathematicians or mechanicians.

It is really pitiful how knowledge is being lost so quickly and so systematically in the span of few scholars' generations. A text written in Latin, or even in a modern language different from the modern Lingua Franca *English*, cannot be read by the greatest majority of scholars that are active in present times. This means that a wide part of human knowledge risks to be forgotten and lost, in the worse case, or rediscovered several times, in different places, times and languages, in the best case. In our opinion, this circumstance risks to be very detrimental for the advancement of human knowledge in general, and could cause some regressions in both the human technological capacities and quality of life.

The eldest editor did experience the very sad sensation of realizing how big are the risks of regression in science when he heard an otherwise very clever scholar arguing that «something not written in English is virtually non-existing». Being Italian, he is accustomed to see his mother language treated as a kind of lost dialect whose knowledge is useful only for understanding some ancient songs. For this reason one can understand why the revolutionary works by Gabrio Piola are nearly completely ignored in the modern literature of mechanical sciences. Piola wrote his works in Italian: his ideology considered the concept of Nation so important that he was ready to sacrifice the diffusion of his ideas for it. He could have written them in French, so that the audience of his works would have been somehow larger. However, a reader may comment that French is not English and, therefore, the previously mentioned scholar, whose mother language is American English, would have similarly argued that also if Piola had written in French, unavoidably, his work would have been, very similarly, ignored. And this reader could be considered to be right, because to the same editor it happened to see how a historical paper about mechanics was rejected by a journal with the following argument: «there are too many French excerpts inside it». Needless to say: the editor tried to explain to the Editor-in-Chief that the French sentences were already duly translated into English in the submitted manuscript to prevent any problem that could arise with readers who cannot read French. The answer was: «it is not interesting to discuss about what Lagrange wanted to say, arriving to examine his own words, by checking if his thought was faithfully translated into English». The problem, most likely, was that Lagrange wrote in French and because of that his ideas cannot be so interesting. Even though such a statement was not explicitly expressed, it was implicitly assumed. We believe, instead, that every interesting contribution must be studied, independent of the language it has been written in. This work wants to send the following message: *In Mechanics and also in other sciences, there are very interesting ideas written in languages different from English. These ideas deserve to be translated into English and should not be forgotten.*

There are also two more interesting points that have attracted our attention. The first point concerns the study of the origins of scientific theories as a tool for understanding how novel scientific theories must be formulated. Since we cannot teach to younger generations an infallible method for formulating well-posed and efficient theories capable to predict observed and not yet observed phenomena, we must behave as the ancient Renaissance Maestri teaching an art to their pupil. We must show them how available theories were invented, hoping that this lesson will guide them.

The second point concerns the role of scholars that, while not fully understanding a specific theory, still actively participate to the process of transmitting it. The role of Tartaglia in the transmission of Archimedes' works to posterity is examined as a prototype of many similar behaviors, as observed in many scholars in every age, époque, place and generation. We could list many modern epigones of Tartaglia; but this will be considered to be gossip, or an act of academic political battles. We will refrain from this kind of disputes, as we want to describe the following phenomenon. A scholar, aiming for the sinecure represented by an academic position, strives to prove the world that his intellectual work deserves to be paid by a public institution. Therefore, he tries to make the other scholars believe to have done a great job with his contributions. If he is not as clever as he believes, then he needs to *reformulate*, *translate* or *make precise* what had been written by his predecessors. Tartaglia himself declared, in the title of one of his works, that «here I make clear what was not possible to understand in the original Greek works». Now, Heiberg, in his monumental work gathering all the available opus of Archimedes, proved that Tartaglia was not able to write in correct Latin. Therefore, we believe that it is almost impossible that Tartaglia could have translated from the Doric Greek of Archimedes to Latin a text that, in addition, is very difficult, as it contains complex mathematical concepts. Unfortunately, this argument was buried in the Prolegomena of Heiberg's Archimedes Edition.

In this volume, we present an annotated translation of Heiberg's Prolegomena together with a description of the sociological phenomena involved in the transmission of knowledge. The importance of these phenomena is enormous if we want to understand how novel theories were formulated at first, in order to train the younger generations of scientists. The phenomenon of science transmission has many interesting aspects and we cannot hope to deal with all of them. A particularly important one concerns the role of encyclopedias. They allow for a synthetic account for large bodies of knowledge and are very precious for younger generations of scholars, when a global understanding of the state of the art in a scientific field is required. Hellinger's encyclopedia article describing the state of the art of continuum mechanics in 1913 is astonishing. It proves that continuum mechanics had been blocked in its development by the fact that the article was written in German and that no translation into English was available. In fact, the lucid analysis by Hellinger had been ignored by too many scholars in Mechanics simply because it was written in German. Probably the fact that the author was Jewish increased the speed of erasure of his contribution from the consciousness of scholars in Mechanics. The great scholarly work by Hellinger could have given a stronger momentum to continuum mechanics if it had been properly evaluated by the scientific community. Unfortunately, it had been ignored and mentioned only in a critical way by the few authors who believed that this reference was necessary. The motivations of this sociological phenomenon deserves to be understood, if one wants to organize Academia in a more efficient way.

Rome, Stuttgart
May 2021

Francesco dell'Isola
Simon R. Eugster
Mario Spagnuolo
Emilio Barchiesi

Contents

1 The study of the genesis of novel mathematical and mechanical theories provides an inspiration for future original research 1
Mario Spagnuolo, Francesco dell'Isola and Antonio Cazzani
1.1 Introduction 1
1.2 The process of knowledge transmission: a sociological problem that needs to be studied by using the scientific method 2
1.3 An epistemological intermezzo: inductivism versus falsificationism 13
1.4 From the world reality to its mathematical model and from the model to the replacement of the world reality 22
1.5 Reconstruction, partly conjectural, of the birth and decline of the mathematical models for planetary motion 24
1.6 The postulations of Mechanics, forces and their materializations . 40
1.7 Conclusive remarks 60
References 66

2 Translation of Heiberg's Prolegomena 75
Mario Spagnuolo, Francesco dell'Isola, Beatrice Gerber and Antonio M. Cazzani
2.1 Translators' preface 75
2.2 Prolegomena. On the Archimedean codices 76

3 Hellinger's 1913 encyclopedia article on the fundamentals of the mechanics of continua 99
Simon R. Eugster
3.1 Translator's preface 99
3.2 Die allgemeinen Ansätze der Mechanik der Kontinua 102
3.3 Translator's commentaries 290
References 305

4 The loss and recovery of the works by Piola and the Italian tradition of Mechanics 315
Mario Spagnuolo, Alessandro Ciallella, and Daria Scerrato

4.1 Orthodoxy in Continuum Mechanics: social phenomenology and its naive explanations 315
4.2 In other scientific groups Variational Principles remained the mainly used conceptual tool 318
4.3 How Piola's works were transmitted and how they were – locally in space and time – lost in the Mechanics literature 320
4.4 Mathematical difficulties inherent to Variational Formulation 327
4.5 Conclusions 332
References 333

5 A partial report on the controversies about the Principle of Virtual Work: from Archytas of Tarentum to Lagrange, Piola, Mindlin and Toupin. 341
Emilio Barchiesi, Alessandro Ciallella, and Daria Scerrato
5.1 Some «forgotten» – but not «lost» – sources in mechanical sciences. 342
5.2 An Italian secondary source: Vailati. While underestimating Hellenistic mechanics, he recognizes in it two different ways for studying Statics problems. 345
5.3 The Principle of Virtual Work as formulated by Archytas of Tarentum in the Mechanical Problems. 349
5.4 D'Alembert: the rediscovery of the Principle of Virtual Velocities (or Virtual Work) 352
5.5 The formulation of Continuum Mechanics by Lagrange 355
5.6 The controversy between Poisson and Piola about the deduction of the equation of the equilibrium of fluids: Piola's contact interactions in continua 356
5.7 Navier, Cauchy, Poisson, and Saint-Venant *versus* Lagrange, Piola, and George Green *or* postulations based on Balance Laws *versus* postulations based on the Principle of Virtual Velocity. 360
5.8 Nationalistic Science *or* How Piola's legacy has been blurred because of writing in Italian and counter-posing Italian science to French science. 362
5.9 The formulation of N-th Gradient Continuum Mechanics by Piola: an ignored result that is still topical after more than 150 years 364
5.10 Research perspectives as suggested by the lesson given by History of Mechanics 367
References 369

Chapter 1
The study of the genesis of novel mathematical and mechanical theories provides an inspiration for future original research

Mario Spagnuolo, Francesco dell'Isola and Antonio Cazzani

1.1 Introduction

This introductory Chapter is conceived in order to make explicit the motivations that led the authors and the editors to work on this volume. The reader will find additional arguments and considerations on some of the epistemological and methodological questions discussed here in the Chapter referred to in [41]: we will however try to present self-consistent reasonings, so that one is not expected to complement this chapter with other readings, if she/he does not wish. The question we want to face is simply stated as follows: Can we find a meta-theory teaching us to formulate a set of specific theories each of them being suitable to describe a well-precise set of phenomena? Unfortunately, it seems that to this question there are not fully satisfactory answers yet. There is not, in fact, any kind of *algorithm* following which one can construct a reasonably efficient theory: whatever it may be said by the supporters of *Data Science* it is not possible, in this moment of the scientific development, to replace the creativity act of a scholar in formulating a model with any kind of *Big Data* algorithm. We do not want to say that such a possibility is precluded to humankind: after having invented robots that relieved us from the greatest part of manual work, it is possible, if not likely, that in future we will be relieved by Artificial Intelligence from the greatest part, or maybe from all, the intellectual work. What has to be clear is that, notwithstanding the trends and pretensions of many, the ambitious program of replacing human mind in its formulation of mathematical models is by

Mario Spagnuolo, e-mail: mario.spagnuolo@unica.it
DICAAR, Università degli Studi di Cagliari, Cagliari, Italy.
International Research Center on Mathematics and Mechanics of Complex Systems, University of L'Aquila, L'Aquila, Italy.
Francesco dell'Isola, e-mail: francesco.dellisola.aquila@gmail.com
DICEAA and International Research Center on Mathematics and Mechanics of Complex Systems, University of L'Aquila, L'Aquila, Italy.
Antonio M. Cazzani, e-mail: antonio.cazzani@unica.it
DICAAR, Università degli Studi di Cagliari, Cagliari, Italy.

F. dell'Isola et al. (eds.), *Evaluation of Scientific Sources in Mechanics*,
Advanced Structured Materials 152, https://doi.org/10.1007/978-3-030-80550-0_1

far out of the reach of present times science. We will not dare to give some timeline indication concerning the occurrence of such a gigantic innovation, as the second author did remember very well when a famous scientist, who was his professor in electronics, announced that it was not conceivable the construction of a computer which could beat a human champion in a chess game. It was not earlier than 1983, and *Deep Blue* in 1996 did manage in the endeavor. We want to underline that we are, however, confident that such a progress will occur, and, that it will revolutionize our life, possibly our species biology and, surely, it will open a new era in *Natural History*. However, exactly as Eugenics did not represent any true advancement of science (nothing even barely comparable with modern molecular genetics, whose successes seem to be limitless), present time Data Science seems to be a fashion that is simply exploited by some scholars who are trying to get more academic (and maybe economical) power. The situation, as realistically presents itself in the current historical moment, is really clear: one can teach to young generations how to formulate a scientific model in only one way, that is by showing them how successful scientific models were at first formulated. This aim motivates the entire content of this work.

1.2 The process of knowledge transmission: a sociological problem that needs to be studied by using the scientific method

The present work has been produced by the collaboration of scholars whose competences are relatively varied. However, all of them never accepted a deleterious concept that is at the basis of modern organization of scientific research: that which lead to fragmentation of knowledge into hyper-specialized sub disciplines rigidly divided by sharp boundaries.

Fragmentation of culture brings to inability to deal with complexity

Albeit nearly all authors and editors can be defined to be (Applied) Mathematicians, Physicists (or Mechanicians) or Engineering Scientists, all of them were exposed to humanistic culture and greatly value multidisciplinar studies. They all agreed that it is really dangerous, and surely pitiful, that, in the modern more fashionable attitudes of academic milieux, the fundamental unity of human knowledge is being unrecognized as a founding and strong feature of scholarly activity. As a consequence, quickly and systematically, Western Culture has experienced a decay of the capacity of addressing complex problems with a unitary vision of all their various facets.

In the short span of few scholars' generations there was a dramatic change in the perception of the role of a scholar: in present times the fields of expertise are being more and more carefully delimited by boundaries that are more and more difficult to trespass. Therefore one may wonder if, in present days, a scholar like Johan Ludvig Heiberg (1854-1928) would have been produced by the current academic system. Heiberg not only mastered fully the Ancient Greek language, in its main versions, including Doric Greek, together with Latin. What makes his personality really unique is that he has systematically shown to master some not easy parts of mathematics, as he could perfectly understand very deep texts by Archimedes, those contained in the famous Palimpsest. Postponing to the subsequent chapters of this work some relevant and more detailed discussion about this text, we start recalling here that it is a parchment codex palimpsest, containing, after having scratched the first written text, some prayers. How it could be possible that a scholar did manage to abrade a text by the greatest recognized scientist who ever wrote in Greek language has to be studied carefully, and is one of the problems that we signal for future scientific investigations. In fact, the original text is a Byzantine Greek copy of a compilation of works by many authors among whom Archimedes seems to be the most prominent. This original text contained two previously unknown very important texts by Archimedes i.e. the *Stomachion* and the *Method of Mechanical Theorems* (which is shortly referred to as *The Method*). Moreover, it contains also the only surviving original Greek manuscript of the celebrated Archimedean work on *On Floating Bodies*. Heiberg, following the tradition of Western Culture, did translate the Greek Archimedean text into Latin, whose role of *lingua franca* had been recognized nearly universally. Unfortunately, after the 1906 Heiberg's discovery and the subsequent publication of his *Archimedis Opera Omnia* (i.e. The complete works of Archimedes), and after a short period in which French seemed to have replaced Latin, English has established itself as the modern, possibly even more universal, *lingua franca*.

As a consequence of this sudden change, a text written in Latin (but unfortunately also if it was written in any living language different from English) became not readable by nearly every modern scholar (sometimes even by some professor of Latin Language!). As it always happened when a change of the used language in science occurred, there is a very likely phenomenon that systematically occurs: a large part of the knowledge accumulated in the old language is forgotten and lost. Another part of this knowledge resurfaces in the new dominant language (the part of Archimedean results that resurfaced thanks to Tartaglia gives an example of such a phenomenon, see the following chapters) and is *rediscovered* several times. These rediscoveries occur in several different space locations, in different times (many anachronisms may be explained in this way) and also in different languages. Useless to say, this process of systematic rediscovery slows down a lot the advancement of science and is really detrimental, as it systematically causes regressions in technology.

An example of the rediscovery of a body of knowledge lost because of linguistic barriers that we will examine concerns the works by Gabrio Piola (see [46, 47, 40, 42, 43]). Piola's work were nearly completely ignored for a long period and had been recovered because of a series of fortuitous events. Piola's works were written in Italian and because of the wrong choice of the used language their diffusion was strongly limited. In this work, we will prove that there are, also in mechanical science, very interesting ideas that were originally written in different languages than English.

The aim of the present work is to prove that, differently from what has been too often conjectured, scientific knowledge transmission is not a simple process: the vision of science as a continuous and endless progress from less advanced to more advanced stages has been falsified even too many times in the history of science. Surely there are the problems related to linguistic barriers, when the *lingua franca* changes because of one of the many possible social reasons. Many scientific ideas were lost in translation! However, there are also some psychological and barely *survival* mechanisms that causes erasures, loss and deformation of the scientific knowledge in its transmission process. These mechanisms play a crucial role in the advancement of science, whatever may be believed by some *right-thinking* scholars. These scholars believe that one has to avoid the consideration, when studying knowledge transmission processes, of the social phenomena related to jealousy, revenge, inflated self-esteem, bare ignorance, arrogance, need of earning from academic positions, every form of nepotism and use of scientific knowledge for getting any form of power. Many are embarrassed when the existence of these social and psychological mechanism are evoked and when one expresses the opinion that they may play a crucial role in the rise of any form of Dark Ages. In fact, their consideration is considered not politically correct and trying to take into account their influence in history of science a form of mental disorder of the kind of paranoia. Instead, exactly as Alfred Kinsey has scientifically shown how important is sexuality in human life and in the psychopathology of humankind, we believe that the social forces that are shaping human psychology are of great relevance in the mechanisms that produce scientific research. Such an obvious statement, as a similarly obvious consequence, implies that it is possible that a deep scientific theory, a useful body of knowledge or an effective mathematical model may be erased, lost, or, in the best case, forgotten for a while in a scientific group, simply because of a series of socio-psychological reasons which are completely unrelated to their absolute scientific merit. Aforementioned *right-thinking* scholars will claim that science is *objective* and that even considering the possibility of any influence of the dark side of human mind on its development is harmful for humankind. This reasoning may be considered equivalent to believing that one can defeat an epidemics simply ignoring its existence: an action whose consequences are well-known. The story of the struggles of Tartaglia (1499-1557) to persuade all his contemporaries that he could understand and translate the Archimedean works, as reconstructed objectively by Heiberg, gives us a paradigmatic and incontrovertible evidence that our thesis is very well-grounded.

Why to try to establish how and when a scientific theory was first formulated? Difficulties in this endeavour

In the discussion that we will develop in our work, we will focus on at least two important aspects of the considered question. The first aspect concerns the importance of study of the true origins of the scientific theories. One may argue that the value of a scientific theory resides in its predictive capacity, and that it is enough to supply a whatsoever rigorous and precise formulation for it. If one accepts this point of view then, when a theory is formulated in a way or in another equivalent way then she/he can choose the preferred way based on any reasonable and useful criterion. Our point of view is, instead, that if one wants to learn how to formulate a completely new theory, a theory that was never formulated before, she/he has to learn, in absence of the meta-theory invoked and dreamed before, how the successful theories have been formulated first, and how they were subsequently developed. To see how old and established theories were born may be of use in the process of inventing a completely new one. Indeed, we do not have, presently, any way to supply to younger generations any other well-working method for teaching them how to build theories that are efficiently capable to give the correct predictions for both observed and not-yet-observed phenomena.

As a consequence we are obliged to follow the educational methods of those ancient *Renaissance Maestri*, who trained their pupils to sculpture or painting simply by showing them as the *Maestro* was painting or sculpting. Unfortunately, there are very few great *Maestri* alive in a certain historical moments and, moreover, their workshops are already full of pupils. Therefore one has to show to those young scholars, who aspire to invent something original, how the available theories, relevant in the chosen disciplines, were first conceived and developed: in this way we hope that the lesson given by great scholars example will guide new generations. For this reason a presentation of available theories must follow, as carefully as possible, the original invention process that led their inventors to get them[1].

The second aspect, on which our analysis will particularly focus, concerns the process of transmission of knowledge from competent scholars to competent scholars via intermediate scholars who are not so competent. Albeit the transmission of science is based on written texts, the role of the scholars participating to the editing of the texts and using them as textbooks for their young pupils cannot be neglected. When the books were handwritten, their relatively enormous economical value introduced a further selection filter in knowledge transmission: the economical costs imposed a selection of what could be copied and what deserved oblivion. In this choice the Archimedean Palimpsest was sacrificed for a book of prayers against diseases, a subject that seemed more “practical” than abstract mathematics. A scholar choosing what kind of textbook deserves to be transmitted plays a relevant role also in the era of printed books: many books are not reprinted and remain in fewer and fewer exemplars in the storages of libraries, virtually disappearing from the attention of younger generations. In our (unfortunate) époque of citations metrics another method has been conceived to condemn to oblivion certain textbooks, authors or theories: it is enough to forget to cite them, and soon nobody will find these works in the *mare magnum* of modern literature, which is literally overflown with too many repetitive and not original papers and textbooks.

Finally, an influential compiler of a textbook, having many students may influence many of them with his biased choices. In the milieux of mechanical sciences there are many textbooks that were very successful in transmitting the correct ideas to

[1] The second author is greatly indebted to Prof. Roberto Stroffolini (Università di Napoli Federico II) for having shown him how such a teaching method has to be pursued.

clever students, albeit it is clear that their compilers did not understand very much the scientific results that they had carefully copied from reliable sources. There are, also, examples of textbooks that deformed the true intent of their sources, imposing to too many younger scholars wrong points of view or making for them every original research extremely difficult. We will fully describe, under the guidance of the authoritative Heiberg's analysis, how Tartaglia did manage to have a relevant role in the *translation* in the *language* used by Western Science of some of the most relevant works by Archimedes. Albeit this may seem rather simple (and most likely also very useful), we will not try to establish any relationship between the publishing (and survival) strategy chosen by Tartaglia and that chosen by (too) many more modern scientists. The need of getting a salary seems to allow for any kind of deplorable choice, while Tartaglia features a "representative" scholar, belonging to a specific kind. This kind of scholar is observed nearly ubiquitously in history of science: one can find examples of it in any social group, language, scientific discipline, historical period, geographical location and economical and political organization.

Instead of looking for specific examples of such kind of scholar, we will try to phenomenologically describe their behavior, the effects of their existence on science transmission and on its accumulation and loss. We will try to apply the scientific method in our phenomenological description and in our first efforts of looking for a model of it. The phenomenology can be shortly resumed as follows: in the competition that they need to accept in order to have recognized their own scientific capacities, many scholars systematically want to ignore any signal indicating that they are not original enough to deserve an academic position. They badly need the sinecure that they believe to be associated to it, and therefore try to prove, in any possible way, that they do deserve highly ranked positions. If they meet somebody indicating how weak their scientific skills are, then they may react in two different ways: i) they start believing that there is a conspiracy against them or ii) albeit they may understand that the criticism against them is well-founded, they manage to persuade themselves that since there are so many incompetent scholars, then their own exclusion from academia is not moral. These scholars, either if they are conscious of their weaknesses or if they sincerely believe to be clever enough for their ambitions, try to make their best to persuade all other scholars that they can be considered original thinkers. Sometimes, exactly as it was done by Tartaglia, these self-proclaimed scientists *reformulate*, *make rigorous*, *translate*, *clarify* or *make more precise* works that they have found in the literature. Exactly as Tartaglia included in the title of one of his presumed translations the following statement: "here I make clear what was not possible to understand in Archimedes works", his epigones manage to declare that they "clarified" the previously "obscure" theories, while in fact they are completely misunderstanding them.

To unveil the real contribution of Tartaglia (and his encyclopaedic or polymath epigones) to science is not easy

The capacity of some scholars in avoiding any discussion about the merit of their scientific contributions is legendary. They manage to bend even mathematical argument to their aims, making any discussion about what they claim to have discovered completely useless. One has to avoid any effort in trying to prove that a single specific scholar is not producing any original contribution or any original view in presenting already known results. Instead it is very useful to describe from a general point of view the kind of effect that the existence of the aforementioned type of scholars has on science transmission and development. If this phenomenology is understood then, most likely, some countermeasures can be acted to limit the unavoidable impact of such scholars on the destinies of science.

Albeit this information seems to have been somehow forgotten, Heiberg happened to discover, while reordering and preparing for his edition the whole available texts by Archimedes, that, in reality, the only merit one can attribute to Tartaglia, for what concerns the appreciation of Archimedes work, is purely propagandistic. Tartaglia contributed to revive the interest in Archimedes. Heiberg, while prefacing his Complete Works by Archimedes, gathered all necessary evidence to prove that Tartaglia's capacity in writing in a correct Latin was rather scarce. One can deduce therefore that he could never have the possibility to translate, from the Doric Greek used by Archimedes into Latin, a complex text of advanced mathematics.

Heiberg's argument seems to us very detailed, serious and careful: unfortunately, this argument was buried in the Prolegomena of the famous Archimedes Edition. We could say it was buried since this Prolegomena (as well as the whole translation of Greek text) was written in Latin. While there are many valuable translations into English of Heiberg's Latin text, the Prolegomena, to our knowledge, were never translated into any modern language. Therefore, we were motivated to translate in this work aforementioned Prolegomena and to add our own comments to it, in order to highlight those aspects of the phenomenology of knowledge to which we are particularly interested. The sociological and cultural phenomena that are surfacing from this reading deserve, in our opinion, a great attention.

Their importance cannot, indeed, be underestimated: if one wants to describe carefully the process of birth of a novel theory she/he must establish exactly when, how and in which formulation, it was first conceived. This description is essential for pedagogical aims: younger generations of scientists must learn how to formulate novel theories by looking at the invention process of the most successful ones. The phenomenon of science transmission is rather complex and manifold: one can find many of its aspects that are of great relevance. One that plays an important role concerns the systematizing and paradigmatic role of Encyclopaedias and encyclopaedic compilations. Because of their true nature, they gather many important aspects of knowledge into a well-organized and unitary way, by using a common formalism and vision. Moreover, they give a synthetic account of all human knowledge, in the most ambitious projects, or for a specific group of disciplines, in other cases. Encyclopaedias supply a precious support for subsequent generations of scholars, as they supply a global understanding of the state of the art, in a given group of scholars, place and époque. By sacrificing some technical details, they resume large bodies of knowledge in an agile presentation and indicate where the interested scholar can find the details that she/he may need. However, the existence of encyclopaedic summaries makes more difficult to understand if a certain scholar did really master her/his discipline, or if she/he did simply adsorb superficially one of the available Compendia.

Our attention has been attracted, in this context, by the 1913 Hellinger's Entry of the German Encyclopaedia of Mathematics whose aim was to give an overview of the then current state of the art in Continuum Mechanics and list some research perspectives that seemed promising to the author. This text has not been translated into English until recently (see [69, 70, 71]) and proves that, in fact, Continuum Mechanics has been "frozen" because of the establishment of English as the novel *lingua franca*, and by the incapacity of the community of experts in Mechanics to read French, Italian or German.

The summary and the analysis presented by Hellinger is really clear and far reaching. The research perspectives, read by somebody in 2021, seem to be even visionary: only recently some of them are being developed. It is remarkable that Hellinger could forecast the main directions of future development of Continuum Mechanics with such a large anticipation. The question therefore is: why Hellinger's work has been removed by the list of the most used sources of the 20th century by the great majority of scholars in Mechanics? A partial answer is that it was written in German. Moreover, the author was Jewish and, unfortunately, this did not help the diffusion of his work in the milieu of German speaking Mechanicians, at least until the end of Second World War.

Such an erasure resulted in a great damage to the advancement of Mechanical Sciences. The loss of the consciousness of Hellinger's analysis in the German speaking Mechanics community had rather singular effects. Indeed, while many authors showed to be aware of the results presented in his work, the information about the fundamental fact that these results were, for the first time, obtained by using variational principles was lost. Therefore, exactly as it was done by Tartaglia, the secondary sources from Hellinger (some of them emigrated in the USA, together with their authors) presented some reworks of Hellinger's Compendium in such a way that it was impossible to get from them any hint about the heuristic method used for finding the presented results. These Compendia were presented as if they were a completely original contribution of their authors, who seemed to have had an *out of the blue* inspiration. This feeling impresses on the readers the false belief that science is an *epic* endeavor where few, particularly gifted scientists, *wake up one morning* and without any apparent cause, simply because they are geniuses, manage to invent a novel theory. In fact, any theory is the result of a choral work of generations of scholars: what is found in some modern textbooks in Continuum Mechanics is the elaboration, hiding the variational procedure first used for finding them, of the contribution to the discipline given by many scholars, starting from Lagrange [108, 107, 20], Piola [46, 47, 40, 128, 151], the Cosserat brothers [38, 4, 8, 58, 120], and continuing with Sedov [137, 135, 136], Toupin [144] and Mindlin [116], among many others [68, 76, 88, 87, 105, 72].

The Entry by Hellinger represents a deep scientific contribution to Mechanics, as it originally reorganizes, with the rigor of a gifted mathematician, all results available up to 1913. It could have given an impressive impulse to the development of 20th century Continuum Mechanics if only it had been understood by the scientific community.

It has to be said that there is a possible misuse of the Encyclopaedic Entries, and this misuse concerned also that by Hellinger: indeed, the results presented in this kind of Compendia may be *adsorbed* and *reworked* by Tartaglia's epigones, who will present them from different, and sometimes twisted, points of view. The existence of Encyclopaedic Entry make possible the existence of so-called polymath scholars: these scholars, who probably have the access to Encyclopaedia Entry, are claimed to have a universal knowledge. Instead, most probably, they simply had access to a, very often lost, Encyclopaedic Entry. In particular Hellinger's work is surely the starting point of the reworking of Continuum Mechanics as presented by those scholars who do refuse Variational Postulation. Knowing in advance the correct results it is easy to deduce them by a series of ad hoc postulates, claiming that they are *induced* by experimental evidence. We will more diffusely present this point in the following sections of this Chapter. Of course this misuse was not intended by Helinger when he conceived his Entry. Unfortunately, until very recently, as a direct source this Entry was completely ignored. We could find a few fugitive mentions of it, where it has been rather harshly criticized. What we have just described is another of the sociological phenomena that must be studied and understood. Understanding it will have an important consequence: thanks to the obtained insight one can find operative methods for organizing the recruitment of academic bodies in a more efficient way.

Archimedes: "The Method of Mechanical Theorems" is an authoritative source confirming our thesis

To our knowledge, Archimedes is the first known scientist who described explicitly a heuristic way for finding novel theories, theorems and mathematical models. Archimedes' mastering of the concept of "model" of physical reality by using mathematical deductive theories has pushed us to conjecture that his epistemological vision may be considered, in essence, to be that of a falsificationist.

This statement may need further deepening: here we consider sufficient to quote, once more, and to give some further few comments, what Archimedes wrote at the beginning of his "The Method of Mechanical Theorems, for Eratosthenes". The Archimedean text was written in Doric Greek, and it is a difficult issue to decide which English translation transmits more faithfully the original ideas and spirits. It seems that the scholarly work of those who are capable to understand mathematics, physics, model theory and Doric Greek is very useful also nowadays. The English text which we are going to reproduce here is the final result of many transformations: the Greek text found by Heiberg was translated by Heiberg himself into (Modern) Latin (in his celebrated Edition of Archimedes' Works). Heiberg's Latin text was then translated into Dutch by E. J. Dijksterhuis in 1938 and then into English by C. Dikshoorn in 1956 (see [56]). Notwithstanding this subsequent translations, we manage to see clearly the original ideas of Archimedes. Instead, Hellinger's ideas [69, 70, 71] have been reformulated into English [146] without citing them directly, in a way that blurs the original spirit. There are also some hints about the way in which Hellenistic culture organized science in this text. Archimedes starts his "cover letter" by recognizing to Eratosthenes a scholarly preeminence but only as a "manager of scientific research" and as "editor-in-chief" of the publications and manuscripts produced by the library of Alexandria: "Since, as I said, I know that you are diligent, an excellent teacher of philosophy, and greatly interested in any mathematical investigations that may come your way, I thought it might be appropriate to write down and set forth for you in this same book a certain special method, by means of which you will be enabled to recognize certain mathematical questions with the aid of mechanics."

This preamble may be interpreted as a kind of *captatio benevolentiae*. Now, from all sources we know how great was the fame that Archimedes enjoyed also during his life. Why did he need to be so careful in sending his paper to Eratosthenes? One can conjecture that also in Hellenistic scientific milieux it was possible to observe a phenomenon that to a much larger extent has been developed later: the diffusion of culture happens to be controlled by few powerful scholars, whose decision can greatly influence the destiny of any scientific work, including those written by outstanding persons, as Archimedes was already recognized to be. The existence of "well-established" scientific personalities who had the power to control what can be published or what must be bound to oblivion seems to be therefore attested already at the époque of the library of Alexandria, and seems to be an unavoidable side effect of any form of organization of Big Science.

Eratosthenes of Cyrene (c. 276 – c. 195/194 BC) was probably one of the most influential personality of Hellensitic science. Obviously, having the control and full access to the biggest source of scientific knowledge of antiquity, he is often described as a polymath. It is interesting to remark here that the etymology of the word "polymath" goes back to ancient Greek. The Greek word πολυμαθής can be translated as follows: "[somebody] having learned much". The translation that has been more often used in Latin is: *homo universalis*. i.e., "universal man". We believe that too often polymaths are simply scholars who managed to better reorganize the results found by other, more original, scientists. Very often the compiler of Compendia or Encyclopaedia Entries are this kind of erudite polymath. The most skilled among polymaths, however, are very precious: they allow for the diffusion of specialist theories among a wider set of scholars: we believe that Hellinger, being an original mathematician himself, when accepting to write an Entry about Continuum Mechanics did cleverly master the subject and then could give the best indications about its future paths of development.

Eratosthenes' interests apparently spanned mathematics, poetry, geography, astronomy and music theory. In fact, most likely he was an erudite who managed to persuade the Pharaoh Ptolemy III Euergetes to nominate him as a "chief-librarian" at the Library of Alexandria in the year 245 BC. One has to consider that the choice was really appropriate: as head of such an institution one needs indeed a true and gifted polymath. He was the leader of the group of scientists and technicians that founded scientific geography and he is best known for having directed the group of scholars that obtained a careful calculation of the circumference of the Earth and the tilt of the Earth's axis. He introduced the first global planar projection of the world, by using parallels and meridians. Most likely he has also calculated the distance from the Earth to the Sun and understood the need of the leap day for a precise Calendar. In number theory, the sieve of Eratosthenes, an efficient algorithm for calculating prime numbers is attributed to him. In the entry of the Suda[2] concerning Eratosthenes it is reported that his critics called him Beta (that is: the "second", as beta is the second letter of the Greek alphabet). This scornful attribute had been chosen to underline that he was the second biggest expert in all his domains of competence. On the other hand, without denying this circumstance and even confirming it, his supporters called him *Pentathlos* after the Olympian Athletes competing in the pentathlon, i.e. athletes being "well-rounded" in five different sports. Eratosthenes' approach to science can be positively interpreted by stating that he tried to dominate the complexities of reality (in fact his appointment at the Library required this kind of skills!) and, for this reason, he had to prove to have talents in a large variety of disciplines. He was capable to understand many things and wanted to use every kind of information which he could achieve. As a consequence he could not be the best expert in anything, but he could play a role in transmitting knowledge from one discipline to another. In fact, as reported by Strabo: Eratosthenes was regarded to be a mathematician among geographers and a geographer among mathematicians[3].

His skills placed Eratosthenes in a very privileged position: he could decide what had to be published becoming a book stored in the library of Alexandria, and therefore, considering the importance of this library, which book could be transmitted to future generations. Without any doubt, Eratosthenes belonged to the *timocratic scientific élite*, i.e. the dominant group of intellectuals of his epoch. Archimedes, who usually could not hide his great self-esteem (see [56, 97]), was obliged to treat with great reverence such an important person. He, therefore, called him a "diligent", "an excellent teacher of philosophy", and "greatly interested in any mathematical investigations that may come your way". Archimedes, as modern scholars are often doing when submitting a paper, writes clearly to the editor-in-chief about its motivations:

> *I am convinced that this* [heuristic method] *is no less useful for finding the proofs of these same theorems. For some things, which first became clear to me by the mechanical method,*

---

[2] The Suda is a Byzantine encyclopedia, written during the 10th-century after Christ. It is a Greek lexicon, having 30,000 entries and including many drawings copied from ancient sources, sources which have been, unfortunately, subsequently lost. The name derives probably from the Byzantine Greek word "souda", which means "stronghold [of knowledge]". Eustathius, misunderstanding the etymology of the title, declared that Suda was a deformation of the name Suidas, that was his author's name.

[3] This destiny is bounded to modern mathematical physicists: they are neither mathematicians nor physicists. However they can be useful in allowing for the communication among the two groups.

were afterwards proved geometrically, because their investigation by the said method does not furnish an actual demonstration.

The reader must remember here that the expression "proved geometrically" is a precise calque of the Greek original expression. It has to be understood, in modern language, as follows: "proved with mathematical rigor". Archimedes has a great standard of mathematical rigor. He states that something is "proven" only when he finds a logically precise sequence of statements which can be deduced, one after the other, from his axioms. A heuristic reasoning is NOT a theorem, for every mathematician since the Greek invention of rigorous mathematics. The use of the word "geometry" in Archimedes' text is simply related to the fact that, in Hellenistic science, the theory of real numbers was formulated in terms of geometrical entities like segments, areas and volumes (see e.g. [37]). The argument of Archimedes continues as follows:

For it is easier to supply the proof when we have previously acquired, by the method, some knowledge of the questions than it is to find it without any previous knowledge. That is the reason why, in the case of the theorems, the proofs of which Eudoxus was the first to discover, viz. on the cone and the pyramid, that the cone is one-third [of the volume] *of the cylinder and the pyramid one-third of the prism having the same base and equal height, no small share of the credit should be given to Democritus, who was the first to state the fact about the said figure, though without proof.*

Archimedes is aware of the importance of both the heuristic, creative invention act which leads to the conjecture of a mathematical result and the technical rigorous demonstration which is needed to state that such a theorem is true. He distinguishes between the inventor of a mathematical proof and the discoverer, who is aware of a well-conceived conjecture, whose result is left to be proven. Then, he discusses the specific heuristic procedure, based on his understanding of a problem of mechanics, which led him to calculate the area of a parabolic section:

My own experience is also that I discovered the theorem now published, in the same way as the earlier ones [the theorems conjectured by Democritus and proven by Eudoxus]. *I now wish to describe the method in writing, partly, because I have already spoken about it before, that I may not impress some people as having uttered idle talk*

Archimedes wants to underline that his creative work has to be split into two parts: i) the conjecture of the statement of the theorem, based on a heuristic argument, and ii) the rigorous proof of the theorem, based on a logical procedure, starting from the axioms he has accepted. It has to be remarked here explicitly that Archimedes calculates the aerea of a parabolic section by what will be called later an integration method. For doing so, he needs the rigorous definition of the set of real numbers, which Archimedes attributes to Eudoxus of Cnidus. On the other hand he conjectures that the area of the parabolic section has a certain value by means of an experimental measure. Archimedes, following a habit that is unfortunately too often spread among pure mathematicians, communicated his rigorous proof without any reference to his heuristic mental process. However, he had spoken about it while discussing with his colleagues: he feels the need to describe it in a written form. He is doing this in order to keep his reputation of serious scientist, who is not talking in vain. To keep his own high reputation is not the only reason for which he discloses his way of reasoning:

> *partly because I am convinced that it will prove very useful for mathematics; in fact, I presume there will be some among the present as well as future generations who by means of the method here explained will be enabled to find other theorems which have not yet fallen to our share.*

Archimedes wants to show to future generations how a theorem is conjectured: he is not happy to give the rigorous proof of it, only. As he has not a technique of discovery which can be formally presented to the reader, he explains his own mental process, based on a clear understanding of mechanical phenomena. Finally, he gives us the specific technical details concerning his theorem

> *We will now first write down what first became clear to us by the mechanical method, viz. that any segment of an orthotome*[4] *is larger by one-third than the triangle which has the same base and equal height, and thereafter all the things that have become clear in this way. At the end of the book we will give the geometrical proofs of the theorems whose propositions we sent you on an earlier occasion.*

The few sentences cited above were considered by Heiberg, their modern discoverer, as possibly the most important ones uttered by Archimedes. Archimedes transmits to us the mental process which occurred in his mind during his mathematical creation. Rather seldom such a clear perspective is given in a mathematical text. Hellenistic Mathematics, and also all subsequent mathematical tradition, is characterized and founded on the logical rigor of the presentation. The economy of thought and its precise formulation are considered the prevalent criterion when presenting mathematical results. A mathematical text, since Hellenistic mathematicians, is a sequence of logical conclusions, obtained with correct deduction rules, starting from the accepted hypotheses, conceived in such a way that the theses are related to the hypotheses by a irrefutable reasoning. While this demand of rigor is essential for the development of hard sciences, it is also undoubtedly true that this style of presentation, giving the synthetic final result of the process of demonstration, is ignoring the equally important demand of understanding the reasons which led the mathematician to the presented demonstration and the heuristic method using which this demonstration was found for the first time. Risking to spoil the myth of his own genius, Archimedes reveals spontaneously how himself, before even starting to try to prove his theorems, conjectured their theses and managed to be persuaded that they were true.

1.3 An epistemological intermezzo: inductivism versus falsificationism

Without any hope to succeed in presenting an exhaustive report of the epistemological knowledge that led us to understand how scientific theories are built, for seek of

[4] An old name first used by Menaechmus to designate the particular conic section resulting from cutting a right-angled cone by a plane which is perpendicular to its surface, thus producing a parabola

self-consistency, we sketch here those most fundamental ideas that should guide a mathematical physicist in his scientific practice.

We have been sometimes very surprised in discovering that otherwise very gifted scholars may have a too naive vision about the epistemological concepts which are needed for correctly guiding their scientific research. In general, for what concerns the postulation scheme used in Continuum Mechanics, we have seen too many presentations in which a series of ad hoc postulates are accepted based on *experimental evidence* or even claiming that they are *induced by experience*. These approaches led to an occlusion of Continuum Mechanics in a stage that was already recognized to be too particular in the works of Gabrio Piola [46, 47, 40, 43, 42, 122, 128].

In order to get rid of the limiting scheme of Continuum Mechanics as elaborated by Cauchy and imposed in Engineering Sciences by its undoubted successes in predicting deformative behavior of bodies, it is necessary to resort to a truly falsificationist approach in the comparison of different mathematical models used for *describing* reality.

Relation between Science and Technology: a view back through History of Science

For the kind of analysis we want to conduct, it is of primary importance to ask what is the effective relationship between Science and Technology. Is there a theory that describes the birth, growth and decay of Scientific Theories and Scientific Technology? To get answers in this direction, it is necessary to refer to concepts that are the specific object of History and Philosophy of Science. If thinking about History of Science does not confuse us, because we can easily recognize in it the ordered set of observed facts, discussing Philosophy of Science may induce misunderstandings. We refer to Philosophy of Science as that meta-theory which, by organizing the set of available information about the way in which well-established theories were constructed, tries to supply efficient methodologies apt to formulate new theories. In the perspective of a mathematical physicist, therefore, a Philosophy of Science is indispensable.

But let's go back to the original question that we believe has a basic importance: what is the relationship between the development of an organized Science and the technological progress of a society? The answer to this question is extremely complex, but we can already get a clear idea by considering on an imaginary time line the focal points of human technological development and then, on the same time line, place the cornerstones of scientific development. What would immediately appear is that for about two million years man has used chops and more or less polished stones for hunting, working skins, cutting wood and other subsistence activities. A few thousand years before Christ, man began to build the first instruments. Gradually technological advances have increased, but there has been an incredible acceleration in correspondence with the birth of Hellenistic Science: the ballista, the Syracusia ship, the astronomical calculator of Antikythera, just to name a few. It has to be remarked that the existence of the disk of Nebra (we will give details about it in the following) seems to indicate that, albeit we do not have any written evidence about it, the great development of the technology related to the Bronze Age may be related to a first elaboration of a proto-Science.

One can follow this imaginary timeline up to the present day by observing how the relationship between scientific development and technological development is inextricably linked. A society that abandons Science, after a suitable time-delay, goes through three successive phases:

i. it no longer produces any kind of new technological development,
ii. it loses the knowledge related to the use of technological tools developed in a previous era of scientific flowering and
iii. transforms (in the best case) such tools into religious objects.

We will see how this decline of Science, and consequently of Technology, is inexorable when certain conditions are created in a given society. One can possibly explain the fall of Western Roman Empire relating it to the loss of awareness about the importance of Hellenistic Science and the related slower, but equally inexorable, loss of technological capacity. We believe that there is an exemplary case that deserves to be shortly discussed here: we mean the use of gravity aqueduct. Hellenistic hydraulics did know a form of the law that has been named after Bernoulli. This theoretical knowledge allows for the conception and construction of the cheaper pressurized aqueduct. In fact, in Pompei we can see a network of pipes distributing the water in the city with a small local pressurized aqueduct. However, building a large aqueduct is not a very frequent need. In the Pergamon Museum in Berlin important parts of a large pressurized aqueduct serving the Pergamon Acropolis are shown. We do not know when the needed theoretical knowledges of hydraulics were lost: for serving Rome, unfortunately, engineers who ignored hydraulics built gravity aqueducts, causing a large economical loss. A sum of such losses most likely made the difference of the destiny between Western and Eastern Roman Empires. One may consider that for some unknown reason the advanced topographic knowledge needed for building a gravity aqueduct were not lost in the passage between Hellenistic and Roman cultures: the reasons for which Romans did manage to preserve a part of Engineering Sciences (Topography) while loosing another part (Hydraulics) may be related to an arbitrary choice of a librarian who could not understand the mathematically difficult arguments in Hydraulics while could catch the simpler reasonings used in Topography, probably because this last can be synthesized using drawings and simple Euclidean Geometry.

An interesting philosophical question that arises spontaneously when we try to organize the phenomenology of scientific progress of human societies is to wonder if the path of human history is a progression of stages that has been repeated many times, independently by different groups, in the same order or if each progress has occurred only once and then it has consequently widespread. This distinction between social determinism and diffusionism finds its basis in the thought of Giambattista Vico, who wrote

Similar ideas that originate from entire peoples unknown to each other must have a common basis of truth.

We tend, differently by what appears in Vico's thought, towards a diffusionist approach. This approach explains better the phenomena related to the scientific flowering which occurred in the Renaissance. Is there really anyone who can believe that the Renaissance evolved in a completely autonomous way? Can anyone really continue to deny the very strong influences that Hellenistic thought had on Renaissance thought? And if there are still few who deny such influences, why, instead, are there still so many who deny the importance of Hellenistic Science and even deny it a classification as a truly "modern" Science?

The library of Cardinal Bessarion is the first fundamental part of Marcian Library in Venice and was constituted mainly by Greek codices. Based on the *transport* of Hellenistic Science via Greek manuscripts arriving in Europe, the main characters of Italian Renaissance started the re-discovery of ancient Science not always recognizing their debt towards their sources.

Approaches to Science: Falsificationism or Inductivism?

In the formulation of a scientific theory at least two alternative approaches can be used. Following the standard nomenclature in the literature, they are called inductivism and falsificationism. We are aware of the fact that more sophisticated conceptual frames have been adopted in Philosophy of Science. However, discussing only these two approaches is enough for our aims. Both of these visions can be traced back throughout the History of Science. As far as we will discuss in the following of this chapter, we are interested in how they were declined in Hellenistic thought, as we will analyze the development and decline of the models introduced for the description of the motion of the planets, and how they were used within the group of scientists who in the 19th century and later developed modern Continuum Mechanics.

As for Hellenistic Science, as we shall see, unfortunately surviving sources are so rare that it is difficult to tell in which form the debate on inductivism and falsificationism took place among Hellenistic scientists. The echoes of this debate, however, are resonating in a significantly later period: Proclus (412-485) discusses the nature of epicycles (we will see below the details of the deferent-epicle model) and asks himself whether they exist or are pure mathematical hypotheses in his treatise Hypotyposis (i.e. Exposition of Astronomical Hypotheses). As we will see, for scientists of the Hellenistic age, as Apollonius of Perga who first introduced planet models using deferents and epicycles, it was obvious that these were simple mathematical objects and that they are not objects in the physical world. They loose every meaning if not contextualized in the model where they were introduced. As Proclus is a post-scientific philosopher, he seems to report about an ancient debate and, being completely unable to fully understand its content, he manages to deny the validity of both positions. However, Proclus claims to be a follower of the philosophical thought of the Platonic school: therefore, he should be able to see a difference between mathematical and physical objects, albeit believing that one can experience, in the world of mathematical ideas, some experiences leading mathematicians to mathematical theorems.

When Platonism is adopted in the development of mathematical thought, then extreme positions are generated. In fact, according to Hardy [96], mathematical platonism is based on the statement

> *Mathematical reality lies outside of us and our function is to discover and observe it and the theorems we prove* [...] *are simply the accounts of our observations.*

According to mathematical Platonism, then, physicists discover physical reality while mathematicians deal with mathematical reality. As we will see with examples taken from both the development of models for the motion of the planets and the development of modern Continuum Mechanics, it is very dangerous to confuse, or even identify, mathematical entities with the physical entities of which they are assumed to be models. Moreover, there are some mathematical entities for which one cannot find any physical correspondence: these mathematical entities are useful only in the logical development of the formulated mathematical model. When one confuses the mathematical model with the physical reality, it may happen that, instead of concluding that the specific model is not suitable to describe physical evidence, one could believe that reality is not self-consistent and may arrive at the conclusion that nature is intrinsically paradoxical. This ontological point of view should be avoided if one wants to have any hope to describe and predict physical phenomenology.

The confusion between models and physical reality is carefully avoided by Platonic mathematicians: therefore, such a philosophical position is not impeaching the needed distinction between mathematical objects and the physical objects they are modeling. Once one has distinguished between mathematical models and real objects, it is easy to confute the so-called inductivist vision of Philosophy of Science.

Inductivism has been considered for too long time as the true scientific method that has to be practiced by diligent scientists. Unfortunately, it is still a commonplace view in many scientific milieux to believe that one can induce from many observations some physical laws, that belong to physical reality and can be established once forever. Such a vision of the scientific method is not efficient and effective to develop scientific theories, as an efficient process like *induction of a physical law* cannot be established. In fact, inductivism is based on the belief that a systematic research approach exists, that involves an inductive reasoning (whatever it may mean) enabling scientists, when applied with due diligence, to *objectively discover* the unique true theory describing every phenomenon. The prescription of inductivism, when examined attentively, presents a very ambiguous clause: the scientist must apply *due diligence*. Therefore, when an induced physical law reveals some limits, naive inductivists are simply stating that the scientist formulating it was not diligent enough. Such a point of view is not at all scientific: how can a scientist know which is the *due diligence* necessary for being sure that his law is “true”? The position of naive inductivists has been ridiculed by Bertrand Russel with his famous anecdote about the inductivist chicken [131, Ch. 6, p. 47]:

> *Domestic animals expect food when they see the person who usually feeds them. We know that all these rather crude expectations of uniformity are liable to be misleading. The man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken.*

In a more picturesque way, Chalmers in [34] reformulates it as follows:

> [We present] *a gruesome example attributed to Bertrand Russell. It concerns a turkey who noted on his first morning at the turkey farm that he was fed at 9 am. After this experience had been repeated daily for several weeks the turkey felt safe, in drawing the conclusion "I am always fed at 9 am". Alas, this conclusion was shown to be false in no uncertain manner when, on Christmas eve, instead of being fed, the turkey's throat was cut. The turkey's argument led it from a number of true observations to a false conclusion, clearly indicating the invalidity of the argument from a logical point of view.*

More seriously and shortly, but maybe in a more effective way, Einstein also criticizes inductivism:

> *Any amount of experiments may prove that I am right; a single experiment can prove that I am wrong* [A. Einstein, letter to Max Born on the December 4th of 1926].

In conclusion, *the idea that theories can be derived from, or established on the basis of, facts* is a statement with an empty meaning, and we believe that the same use of the world "theory" is not appropriate. In fact, a theory is, etymologically a sequence of statements deduced logically from a conjectured set of postulates. The commonplace statement which we have quoted before should be rephrased by introducing instead the world "physical laws" if one could give a meaning to such an expression.

Inductivism was formulated, in our opinion, while misunderstanding Hellenistic sources that stressed the importance of the experimental verification of formulated mathematical theories. Inductivism was developed during four centuries and Francis Bacon was one of its champions. Western Europe's prevailing epistemological approach, in the époque of Bacon, was the so-called scholasticism. Also scholasticism was based presumably on a misunderstanding of Hellenistic sources: the philosophers of this school believed that, based on preconceived beliefs, one could, without any interrogation of experimental evidence, forecast the behavior of physical phenomena. Clearly, scholasticism was accepting only partially what we presume was the true formulation of ancient falsificationism. The falsificationist approach, which consists in conjecturing a model having the aim of describing a set of observed facts, verifies only a posteriori how much can be predicted on the basis of the assumed conjecture.

In fact, falsificationism bases its analysis of natural phenomena, and the corresponding formulation of theories, on the conjecture of some basic postulates from which the scientist must deduce consequences, to be used, when possible, to predict physical phenomena. Therefore, while the stress of scholasticism was presumably focused on the first part of the process of scientific invention as described by ancient falsificationism and neglected the important required check obtained by experiments, inductivism stressed only on experimental evidence, by loosing the deductive part so highly considered in ancient falsificationism. It is clear that the scholars of Middle Ages, having a partial understanding of their sources, could catch only a part of the original complex epistemological vision. This vision has been completely reconstructed only at the beginning of 20th century, when it was necessary in order to formulate really novel physical theories like Quantum Mechanics or General Relativity.

A falsificationist does not try to induce his postulates, he only checks that all the logical consequences of his postulates, for which this is possible, are verified experimentally. Falsificationism has shown to be extremely advantageous in the advancement of scientific progress, compared to a naive inductivism. We claim that one of the first implicit expositions of falsificationism can be found in Archimedes' *Treatise on Method*, in which the Syracusan scientist provides guidance on how to proceed in conjecturing new theories correctly. If it were not for the fact that modern Science is Archimedes' progeny, we could say that Archimedes has all the characteristics of a modern scientist!

Contrary to what History of Science has shown so far, i.e. that only a scientific knowledge produces advances in Technology (therefore, we claim that the only possible way to produce new technological advances is to develop new theories that allow us to observe phenomena never observed before), unfortunately today scientific progress appears to be stuck in the pointless debate on a *data driven* or *theory driven* Science. This debate represents the modern rephrasing of the debate between inductivism and falsificationism, that seems to have been evoked by Proclus.

Proponents of the *data driven* strategy, strengthened by the fact that today there is a relative overabundance of data available and computing capacity, argue that the description of reality can be simply induced by means of the manipulation of experimentally collected data. We will see, in the following, a fundamental example of how even the modern critical interpretation of Hellenistic Science is sometimes given in a *data driven* key. In fact, while Hipparchus of Nicaea conjectured a priori the precession motion of the rotation axis of the Earth, today's modern inductivists, who are data driven, let us believe that Hipparchus *induced* the precession law from a comparison of the positions of certain stars as measured by him and those reported in a star catalog compiled 150 years before him. We believe, and we will describe extensively the reasoning that leads us to this belief, that, instead, Hipparchus first conjectured Earth's axis precession and only after then, based on his conjecture, explained the discrepancies between the two catalogs. Albeit we do not have the relevant sources available (imagine if we could find Hipparchus counterpart of Archimedes' *On the Method*!), we can suppose that, after having seen the motion of a spinning top (see below for more details), Hipparchus, knowing what he was looking for, checked the star catalog for obtaining a confirmation of his conjecture.

The debate between inductivists and falsificationists is being repeated nowadays, for instance, also in the research field devoted to the invention of new materials with properties which are not observed *spontaneously* (i.e. not too frequently) in nature. In this area, which is also discussed extensively in other chapters of this work, a "data driven" strategy is not only impractical, but also conceptually wrong and economically disadvantageous. Therefore, we claim that an awareness of epistemological basic concepts is needed also in nowadays researchers studying basic problems in Engineering Sciences.

Underdetermination of Scientific Theories: a problem for Inductivism?

In the conceptual framework we have discussed up to now, when formulating a new theory, a fundamental role is played by the basic hypotheses, or physical postulates. In the falsificationist approach, starting from the basic hypotheses (postulates), and using rigorous logical procedures, one can deduce consequences that can be confronted with experimental data. It has no sense wondering a priori whether hypotheses are true or false: hypotheses can be only judged on the basis of the comparison between the whole set of their consequences and available experience. Moreover, hypotheses have to be contextualized in the model for which they are formulated. It is a very common misunderstanding the confusion between the hypotheses of a specific model and the hypotheses of another model treating a different aspect of the same physical system. Also if two models are describing the same physical entity, this does not imply that one has to assume the same hypotheses in both of them, if the phenomena to be described are sufficiently different. We present here some paradigmatic examples of this underdetermination of scientific theories.

We do not believe into the inductivist approach, because, obviously, a collection of phenomena concerning a physical system does not uniquely determine *the true and only scientific theory* to be used for describing it. In fact, and as we have stressed before, the used hypotheses may change when choosing a model or another model for the same physical object. A very famous example of the underdetermination of scientific theories is given by Archimedean study of the mechanical behavior of Oceans.

Let us start from a strong ontological statement, clearly accepted by Archimedes: oceans exists and are always the same physical object where tides occur and on which vessels float! Now Archimedes knows that the phenomena involving the floating of vessels can be described by the model of planar surface of oceans. Indeed, Archimedes uses the hypothesis that the surface of seawater is a horizontal plane (in the treatise *On floating bodies*) as a basic one when he wants to establish the stability conditions for ships hull in the vertical configuration. Archimedes had to develop his famous buoyancy law to found this specific theory. However, somewhere else (we conjecture this happened when he was preparing the model for describing tides, that we know has been developed by Seleucus) Archimedes also proves, starting from other postulates, that the surface of the Oceans has to be spherical!

He knew how to use different hypotheses, depending on the different type of phenomena he wanted to describe. Can we find a contradiction between the two models for the surface of Oceans? Is Archimedes, as it is claimed by some modernistic historians of Science, a primitive and confused scholar? In fact, the two visions