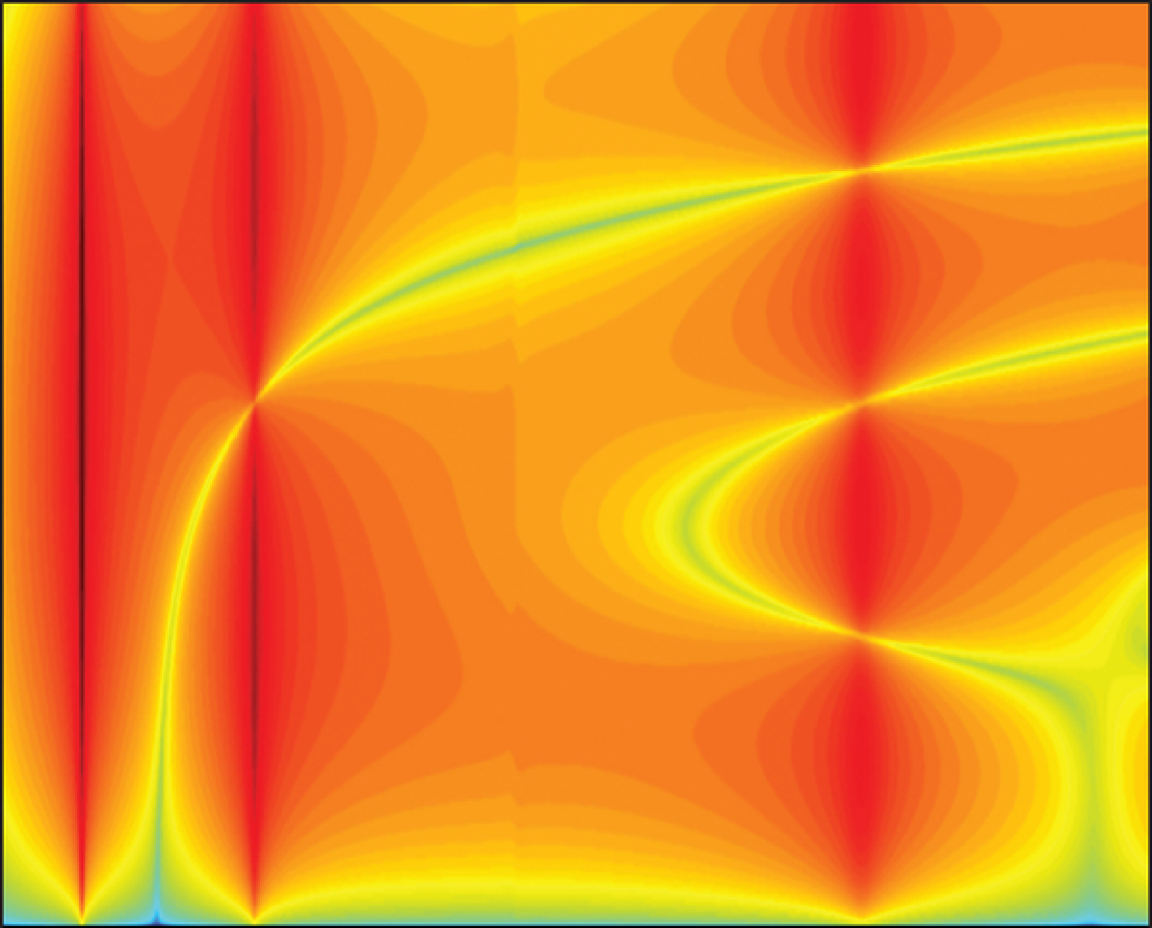

Vibrations of Linear Piezostructures

Andrew J. Kurdila and Pablo A. Tarazaga

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Foreword

The rise of piezoelectric materials as sensors and actuators in engineering systems got started around 1980 and began to make an impact in the world of vibrations about five years after that. Subsequently, it started to explode into the 90s with topics such shunt damping, active control, structural health monitoring and energy harvesting. As a result, the need to document the fundamentals and intricacies of modeling piezoelectric materials in the context of vibrations in book form will well serve a variation of communities. The presentation here puts the topic on a firm mathematical footing.

The authors are uniquely qualified to provide a sophisticated analytical framework with an eye for applications. Professor Kurdila has nearly four decades of experience in modeling of multi-physics systems. He authored two other books, one on structural dynamics, and several research monographs. Professor Tarazaga is an experienced creator of piezoelectric solutions to vibration and control problems. Both are well published in their respective research areas of research. Their combined expertise in researching vibratory systems integrated with piezoelectric materials enables this complete and detailed book on the topic. This allows for a formal theoretical background which will enable future research.

Daniel J. Inman
Ann Arbor, Michigan

Preface

The goal of this book is to provide a self-contained, comprehensive, and introductory account of the modern theory of vibrations of linearly piezoelectric structural systems. While the piezoelectric effect was first investigated by the Curies in the 1880*s*, and systematically investigated in the field of acoustics and the development of sonar during the First World War, it is only much more recently that we have seen the widespread interest in mechatronic systems that feature piezoelectric sensors and actuators. Many of the early, now classical, texts present piezoelectricity from the viewpoint of a material scientist such as in [22] or [53]. Others are difficult, if not impossible, to obtain since they are out of print. Older editions of the excellent text [20] are currently selling for prices in excess of $600 on sites such as Amazon.com. Moreover, it is also quite difficult to find treatments of piezoelectricity that systematically cover all the relevant background material from first principles in continuum mechanics, continuum electrodynamics, or variational calculus that are necessary for a comprehensive introduction to vibrations of piezoelectric structures. The authors know of no text that assimilates all this requisite supporting material into one source. One text may give an excellent overview of piezoelectric constitutive laws, but neglect to discuss variational methods. Another may cover variational methods for piezoelectric systems, but fail to review the first principles of electrodynamics, and so forth. A large, substantive literature on various aspects of piezoelectricity has evolved over the past few years in archival journal articles, but much of this material has never been systematically represented in a single text.

This book has evolved from the course notes that the authors have generated while offering courses in active materials, smart systems, and piezoelectric materials over the past decade at various research universities. Most recently, the authors have taught active materials and smart structures courses that feature piezoelectricity at Virginia Tech to a diverse collection of first year graduate students. So much time was dedicated to the particular systems that include piezoelectric components that this textbook emerged. The backgrounds of the students in our classes have varied dramatically. Many students have not had a graduate class in vibrations, continuum mechanics, advanced strength of materials, nor electrodynamics. For this reason, the notes that evolved into this book make every effort to be self-contained. Admittedly, this text covers in one chapter what other courses may cover over one or two semesters of dedicated study. As an example, Chapter 3 reviews the fundamentals of continuum mechanics for this text, a topic that is covered in other graduate classes at an introductory level during a full semester. So, while the presentation attempts to be comprehensive, the pace is sometimes brisk.

While preparing this text, we have tried to structure the material so that it is presented at the senior undergraduate or first year graduate student level. It is intended that this text provide the student with a good introduction to the topic, one that will serve them well when they seek to pursue more advanced topics in other texts or in their research. For example, this text can serve as a introduction to the fundamentals of modeling piezoelectric systems, and it can prepare the student specializing in energy harvesting when they consult a more advanced text such as [21].

This text begins in Chapter 2 with a review of the essential mathematical tools that are used frequently throughout the book. Topics covered include frames, coordinate systems, bases, vectors, tensors, introductory crystallography, and symmetry. Chapter 3 then gives a fundamental summary of topics from continuum mechanics. The stress vector and tensor is defined, Cauchy's Principle and the equilibrium equations are derived. The strain tensor is defined, and an introduction to constitutive laws for linearly elastic materials is also covered in this chapter. Chapter 4 provides the student the required introduction to continuum electrodynamics that is essential in building the theory of linear piezoelectricity in subsequent chapters. The definitions of charge, current, electric field, electric displacement, and magnetic field are introduced, and then Maxwell's equations of electromagnetism are studied.

Linear piezoelectricity is covered in Chapter 5. The discussion begins by introducing a physical example of the piezoelectric effect in one spatial example, and subsequently giving a generalization of the phenomenon in terms of piezoelectric constitutive laws. The initial-boundary value problem of linear piezoelectricity is then derived from the analysis of Maxwell's equations and principles of continuum mechanics. While the equations governing any particular piezoelectric structure can be derived in principle from the initial-boundary value problem of linear piezoelectricity, it is often possible and convenient to derive them directly for a problem at hand. Chapter 6 discusses the application of Newton's equations of motion for several prototypical piezoelectric composite structural systems. Chapter 7 provides a detailed account of how variational techniques can be used, instead of Newton's method, for many linearly piezoelectric structures. In some cases the variational approach can be much more expedient in deriving the governing equations. This chapter starts with a review of variational methods and Hamilton's Principle for linearly elastic structures. The approach is then extended by formulating Hamilton's Principle for Piezoelectric Systems and Hamilton's Principle for Electromechanical Systems. Several examples are considered, including the piezoelectrically actuated rod and Bernoulli–Euler beam, as well as the electromechanical systems that result when these structures are connected to ideal passive electrical networks. The book finishes in Chapter 8 with a discussion of approximation methods. Both modal approximations and finite element methods are discussed. Numerous example simulations are described in the final chapter, both for the actuator equation alone and for systems that couple the actuator and sensor equations.

June, 2017

Andrew J. Kurdila
Pablo A. Tarazaga

Acknowledgments

This book is the culmination of research carried out and courses taught by the authors over the years at a variety of institutions. The authors would like to thank the various research laboratories and sponsors that have supported their efforts over the years in areas related to active materials, smart structures, linearly piezoelectric systems, vibrations, control theory, and structural dynamics. These sponsors most notably include the Army Research Office, Air Force Office of Scientific Research, Office of Naval Research, and the National Science Foundation. We likewise extend our appreciation to the institutes of higher learning that have enabled and supported our efforts in teaching, research, and in disseminating the fruits of teaching and research: this volume would not have been possible without the infrastructure that makes such a sustained effort possible. In particular, we extend our gratitude to the Aerospace Engineering Department at Texas A&M University, the Department of Mechanical and Aerospace Engineering at the University of Florida, and most importantly, the Department of Mechanical Engineering at Virginia Tech. We extend our appreciation to the many colleagues that have worked with us over the years in areas related to active materials and smart structures. In particular, we thank Dr. Dan Inman for his support and for being a source of inspiration.

We also would like to specifically thank Dr. Vijaya V. N. Sriram Malladi and Dr. Sai Tej Paruchuri for their tireless efforts in editing and correcting the draft manuscript. Their meticulous attention to detail, suggestions and tireless effort has made this book a better version from its original draft. Additionally, we would like to thank our students Dr. Sheyda Davaria, Dr. Mohammad Albakri, Manu Krishnan, Mostafa Motaharibidgoli who have worked through the manuscript in order to improve its clarity. We would also like to also thank Sourabh Sangle, Murat Ambarkutuk, Lucas Tarazaga and Vanessa Tarazaga for their help in proofreading the last draft of the document. Finally, we would like to acknowledge anyone else not mentioned that contributed to the manuscript, including the students in our classes who provided valuable input throughout the years.

And, of course, we thank our families for their continued support and encouragement in efforts just like this one over the years.

Andrew J. Kurdila
Pablo A. Tarazaga

Blacksburg, VA
February, 2021

List of Symbols

Symbol	Description
Vectors and Tensors	
δ_{ij}	Kronecker delta function
ϵ_{ijk}	Levi-Civita permutation tensor
$\boldsymbol{g}_i$	generic basis vector
$\boldsymbol{R}, r_{ij}$	rotation matrix and its components
$V \otimes V \otimes \cdots \otimes V$	vector space of n^{th} order tensors
$\boldsymbol{g}_i \otimes \boldsymbol{g}_j$	tensor product of $\boldsymbol{g}_i$ and $\boldsymbol{g}_j$
χ_Ω	characteristic function of Ω
$\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$	lattice parameters
α, β, γ	unit cell or lattice angles
Ω	domain
$\partial\Omega$	boundary of Ω
Electrodynamics	
c	speed of light
ϵ_0	electric permitivity of free space
μ_0	magnetic permeability of free space
i	current
$\boldsymbol{j}, \boldsymbol{j}_f, \boldsymbol{j}_b, \boldsymbol{j}_p$	total, free, bound, and polarization current density
$\rho_e, \rho_{e,f}, \rho_{e,b}$	total, free, and bound charge density
ϕ	electric potential
$\boldsymbol{\Psi}$	magnetic vector potential
$\boldsymbol{E}, E_i$	electric field vector and its components
$\boldsymbol{p}, p_i$	dipole moment and its components
$\boldsymbol{P}, P_i$	polarization and its components

Symbol	Description
$\boldsymbol{D}, D_i$	electric displacement vector and its components
$\boldsymbol{B}, B_i$	magnetic field and its components
$\boldsymbol{m}, m_i$	magnetic dipole moment and its components
$\boldsymbol{M}, M_i$	magnetization or magnetic polarization
$\boldsymbol{H}, H_i$	magnetic field intensity and its components
$\boldsymbol{E}_e, E_{e,j}$	"external" electric field induced by free charge
$\boldsymbol{E}_i, E_{i,j}$	"internal" electric field induced by polarization charge
Elasticity	
ρ_m	mass density
$\boldsymbol{f}_b, f_{bi}$	body force and its components
$\boldsymbol{u}, u_i$	displacement field and its components
$\boldsymbol{T}, T_{ij}$	second order stress tensor and its components
$\boldsymbol{S}, S_{ij}$	second order linear strain tensor and its components
$\boldsymbol{C}, C_{ijkl}$	fourth order material stiffness tensor and its components
$\partial\Omega_u$	boundary of Ω on which u_i is prescribed
$\partial\Omega_T$	boundary of Ω on which T_{ij} is prescribed
$\overline{u}_i$	prescribed displacements on $\partial\Omega_u$
$\overline{\tau}_i$	prescribed stress vector on $\partial\Omega_T$
$\overline{u}_{i,0}$	initial condition on u_i in Ω
$\overline{v}_{i,0}$	initial condition on $\frac{\partial u_i}{\partial t}$ in Ω
$\mathcal{U}_0$	strain energy density
$\mathcal{V}$	strain energy or potential energy
$\mathcal{T}$	Kinetic energy
W	Work
δW	Virtual work
$\mathcal{S}, \mathcal{M}$	Beam shear force and bending moment
$\mathcal{M}_i$	Plate bending moment per unit length
$\mathcal{S}_i$	Plate shear force per unit length
κ	beam area moment
$C_{11}^E I$	Beam bending stiffness
Piezoelectricity	
$\partial\Omega_\phi$	boundary of Ω on which ϕ is prescribed
$\partial\Omega_D$	boundary of Ω on which D_i is prescribed
$\overline{\phi}$	prescribed potential ϕ on $\partial\Omega_\phi$

Symbol	Description
$\overline{\sigma}$	prescribed charge distribution on $\partial\Omega_D$
$\mathcal{Q}$	heat
$\mathcal{U}$	internal energy
$\mathcal{E}$	total electromechanical energy
$\mathcal{H}$	electric enthalpy density
$\mathcal{V}_{\mathcal{H}}$	electric enthalpy
V	voltage
Q	abstract (vector) space of generalized coordinates
$\mathcal{F}$	time varying trajectories that take value in Q
C^E_{ijkl}	4^{th} order tensor, constant $\boldsymbol{E}$, Eq. 5.19
d_{nij}	3^{rd} order piezoelectric tensor in Eq. 5.19
ϵ^S_{ij}	2^{nd} order tensor, constant $\boldsymbol{S}$, in Eq. 5.19
C^D_{ijkl}	4^{th} order tensor, constant $\boldsymbol{D}$, Eq. 5.14
e_{nij}	3^{rd} order piezoelectric tensor in Eq. 5.14
β^S_{ij}	2^{nd} order tensor, constant $\boldsymbol{S}$, in Eq. 5.14
Θ	absolute temperature
s	entropy
$J(q)$	functional $J : \mathcal{F} \to \mathbb{R}$
$\mathcal{A}(q)$	Action integral of Hamilton's Principle
$\mathcal{A}_{\mathcal{H}}(q)$	Action integral, Hamilton's Principle of Piezoelectricity
$DJ(q,p)$	Gateaux derivative at $p \in Q$ in the direction $q \in Q$
$\delta(\cdot)$	(virtual) Variation operator
$\mathcal{V}_{em}$	Electromechanical potential
$\mathcal{V}_e$	Electrical potential of ideal capacitors
$\mathcal{V}_m$	Magnetic potential of ideal inductors
$\mathfrak{L}$	Lagrangian density

1

Introduction

1.1 The Piezoelectric Effect

In the most general terms, a material is piezoelectric if it transforms electrical into mechanical energy, and vice versa, in a reversible or lossless process. This transformation is evident at a macroscopic scale in what are commonly known as the direct and converse piezoelectric effects. The direct piezoelectric effect refers to the ability of a material to transform mechanical deformations into electrical charge. Equivalently, application of mechanical stress to a piezoelectric specimen induces flow of electricity in the direct piezoelectric effect. The converse piezoelectric effect describes the process by which the application of an electrical potential difference across a specimen results in its deformation. The converse effect can also be viewed as how the application of an external electric field induces mechanical stress in the specimen.

While the brothers Pierre and Jacques Curie discovered piezoelectricity in 1880, much the early impetus motivating its study can be attributed to the demands for submarine countermeasures that evolved during World War I. An excellent and concise history, before, during, and after World War I, can be found in [43]. With the increasing military interest in detecting submarines by their acoustic signatures during World War I, early research often studied naval applications, and specifically sonar. Paul Langevin and Walter Cady had pivotal roles during these early years. Langevin constructed ultrasonic transducers with quartz and steel composites. Shortly thereafter, the use of piezoelectric quartz oscillators became prevalent in ultrasound applications and broadcasting. The research by W.G. Cady was crucial in determining how to employ quartz resonators to stabilize high frequency electrical circuits.

Vibrations of Linear Piezostructures, First Edition. Andrew J. Kurdila and Pablo A. Tarazaga.

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A number of naturally occurring crystalline materials including Rochelle salt, quartz, topaz, tourmaline, and cane sugar exhibit piezoelectric effects. These materials were studied methodically in the early investigations of piezoelectricity. Following World War II, with its high demand for quartz plates, research and development of techniques to synthesize piezoelectric crystalline materials flourished. These efforts have resulted in a wide variety of synthetic piezoelectrics, and materials science research into specialized piezoelectrics continues to this day.

1.1.1 Ferroelectric Piezoelectrics

Perhaps one of the most important classes of piezoelectric materials that have become popular over the past few decades are the ferroelectric dielectrics. A ferroelectric can have coupling between the mechanical and electrical response that is several times a large as that in natural piezoelectrics. Ferroelectrics include materials such as barium titanate and lead zirconate titanate, and their unit cells are depicted in Figure 1.1. When the centers of positive and negative charge in a unit cell of a crystalline material do not coincide, the material is said to be polar or dielectric. An electric dipole moment $\boldsymbol{p}$ is a vector that points from the center of negative charge to the center of positive charge, and its magnitude is equal to $|\boldsymbol{p}| = q \cdot \delta$ where q is the magnitude of the charge at the centers and δ is the separation between the centers. The limiting volumetric density of dipole moments is the polarization vector $\boldsymbol{P}$. Intuitively we think of the polarization vector $\boldsymbol{P}$ as measuring the asymmetry of the internal electric field of the piezoelectric crystal lattice. Ferroelectrics exhibit spontaneous electric polarization that can be reversed by the

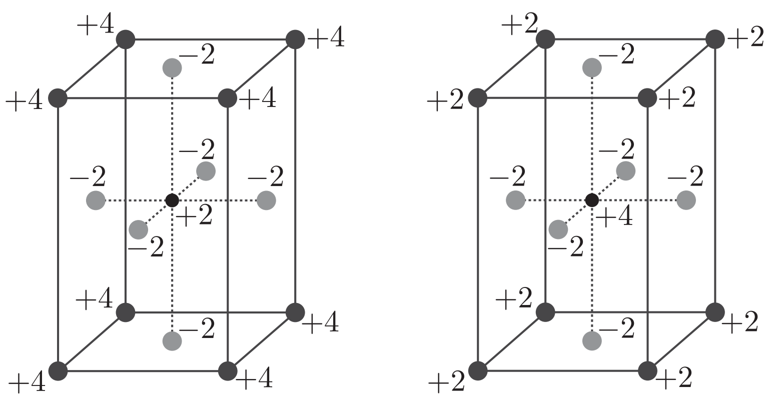

Figure 1.1 Barium titanate and lead zirconate titanate. (Left) Barium titanate $BaTiO_3$ with Ba^{+2} cation at the center, O^{-2} anions on the faces, and Ti^{+4} cations at the corners of the unit cell. (Right) Lead zircanate titanate $PbZr_{\alpha}Ti_{(1-\alpha)}O_3$ with Ti^{+4} or Zr^{+4} cation at the center, $O^{-}2$ anions on the faces, and Pb^{+2} cations at the corners of the unit cell.